Principles of Mathematics 12

April 1999 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		**Sub-Organizers**
1. Problem Solving	A	Problem Set
2. Patterns and Relations	B	Sequences and Series
	C	Polynomials
	D	Logarithms and Exponents
	E	Quadratic Relations
	F	Quadratic Systems
3. Shape and Space	G	Trigonometry
	H	Geometry

Part A: Multiple Choice

Q	**K**	**C**	**CO**	**PLO**	**Q**	**K**	**C**	**CO**	**PLO**
1.	B	K	2	C1	24.	C	U	2	B2
2.	C	U	2	C3	25.	D	U	2	B4
3.	C	U	2	C6	26.	B	U	2	B4
4.	A	H	2	C4	27.	D	H	2	B4
5.	C	H	2	C1	28.	B	H	1	B6, A1
6.	D	K	2	E5	29.	C	U	3	G1
7.	B	U	2	E2	30.	B	K	3	G5
8.	B	U	2	F5	31.	D	U	3	G2
9.	C	U	2	E5	32.	D	U	3	G3
10.	C	U	2	E6	33.	B	U	3	G2
11.	D	U	2	F1	34.	C	U	3	G5
12.	A	U	2	F1	35.	A	U	3	G8
13.	A	H	2	E7	36.	B	U	3	G7
14.	D	U	2	D5	37.	A	U	3	G7
15.	A	U	2	D5	38.	A	H	3, 1	G9, A7
16.	D	U	2	D1	39.	B	U	3	H2
17.	A	K	2	D2	40.	A	U	3	H2
18.	C	U	2	D6	41.	D	U	3	H2
19.	A	U	2	D5	42.	C	H	3	H4
20.	A	H	2	D5	43.	D	H	1	A3
21.	C	K	2	B2	44.	B	U	1	A3
22.	B	U	2	B4	45.	D	U	1	A3
23.	C	U	2	B5					

Multiple Choice = 45 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1.	1	U	3	2	F2
2.	2	U	3	2	C7
3.	3	U	3	2	E4
4.	4	U	3	3	G9, A7
5.	5	U	3	2	D5, A7
6.	6	U	3	3	H4
7.	7	U	3	1	A3
8.	8	U	4	3	H2

Written Response = 25 marks

Multiple Choice = 45 (45 questions)
Written Response = 25 (8 questions)
EXAMINATION TOTAL = 70 marks

LEGEND:
Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
CO = Curriculum Organizer
PLO = Prescribed Learning Outcome

PART B: WRITTEN RESPONSE

Value: 25 marks **Suggested Time: 45 minutes**

INSTRUCTIONS:	Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.
	If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.
	When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.
	Full marks will NOT be given for the final answer only.

1. Solve the following system algebraically. Express all solutions as ordered pairs. **(3 marks)**

$$x^2 + y^2 = 25$$

$$x = y^2 - 5$$

Solution

$$x^2 + y^2 = 25$$

$$x - y^2 = -5$$

$$x^2 + x = 20$$

$$x^2 + x - 20 = 0 \quad \leftarrow \tfrac{1}{2} \text{ mark}$$

$$(x+5)(x-4) = 0 \quad \leftarrow \tfrac{1}{2} \text{ mark}$$

$\frac{1}{2}$ **mark** $\rightarrow$ $x = -5$ or $x = 4$ $\leftarrow \frac{1}{2}$ **mark**

$x^2 + y^2 = 25$	$x^2 + y^2 = 25$
$25 + y^2 = 25$	$16 + y^2 = 25$
$y^2 = 0$	$y^2 = 9$
$y = 0$	$y = \pm 3$
↑ $\frac{1}{2}$ **mark**	↑ $\frac{1}{2}$ **mark**

$(-5,\ 0)$ $(4,\ 3)$ $(4,\ -3)$ ($\frac{1}{2}$ **mark** deduction if answers not listed as ordered pairs)

1. Solve the following system algebraically. Express all solutions as ordered pairs. **(3 marks)**

$$x^2 + y^2 = 25$$
$$x = y^2 - 5$$

Alternate Solution

$$\left.\begin{aligned}(y^2-5)^2 + y^2 &= 25\\ y^4 - 10y^2 + 25 + y^2 &= 25\\ y^4 - 9y^2 &= 0\end{aligned}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$y^2(y^2 - 9) = 0 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$\frac{1}{2}$ **mark** $\rightarrow$ $y = 0$, $x = \pm 3$ $\leftarrow \frac{1}{2}$ **mark**

$x = y^2 - 5$	$x = 9 - 5$
$= 0 - 5$	$= 4$
$= -5$	

↑ $\frac{1}{2}$ **mark** ↑ $\frac{1}{2}$ **mark**

$(-5,\ 0)$ $(4,\ 3)$ $(4,\ -3)$ ($\frac{1}{2}$ **mark** deduction if answers not listed as ordered pairs)

2. A polynomial function of degree 3 has a zero of -1 and a double zero of 4. Determine this function if it passes through the point $(1,\ 10)$. Answer may be left in factored form. **(3 marks)**

Solution

$$y = \underbrace{k}_{\uparrow}\underbrace{(x+1)(x-4)(x-4)}_{\uparrow}$$

$\frac{1}{2}$ **mark** $\frac{1}{2}$ **mark**

$$k(1+1)(1-4)(1-4) = 10$$ ← **1 mark** ($\frac{1}{2}$ **mark** for $x = 1$ and $\frac{1}{2}$ **mark** for $y = 10$)

$$k(2)(-3)(-3) = 10$$

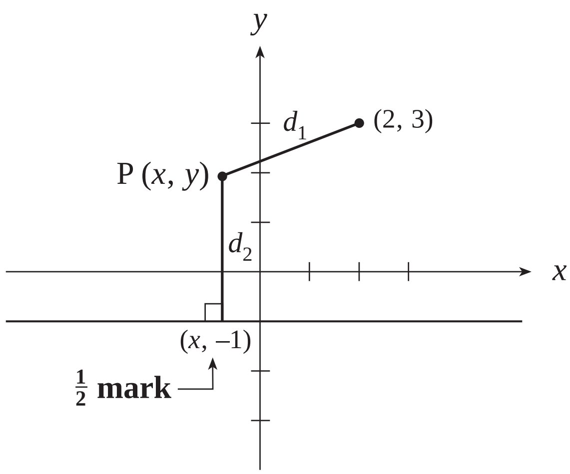
$$18k = 10$$

$$k = \frac{10}{18} = \frac{5}{9}$$ ← $\frac{1}{2}$ **mark**

$$y = \frac{5}{9}(x+1)(x-4)(x-4)$$ ← $\frac{1}{2}$ **mark**

3. A point $P(x,\ y)$ moves such that it is always equidistant from the point $A(2,\ 3)$ and the line $y = -1$. Determine the equation of this locus, in standard form. **(3 marks)**

Solution



$$d_1 = d_2$$

½ mark → $\sqrt{(x-2)^2 + (y-3)^2} = \sqrt{(x-x)^2 + (y+1)^2}$ ← ½ mark

$(x-2)^2 + (y-3)^2 = (y+1)^2$ ← ½ mark

$(x-2)^2 + y^2 - 6y + 9 = y^2 + 2y + 1$ ← ½ mark

$$(x-2)^2 = 8y - 8$$

$y - 1 = \frac{1}{8}(x-2)^2$

or ← ½ mark

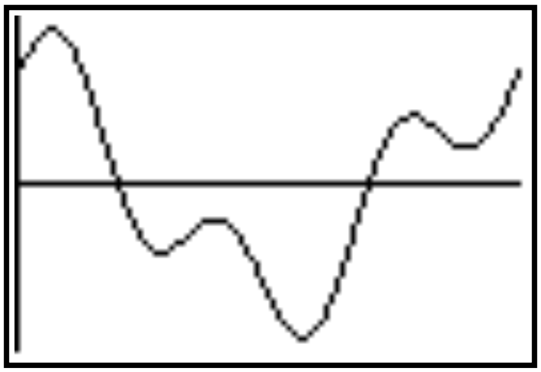
$y = \frac{1}{8}(x-2)^2 + 1$

4. Use a graphing calculator to solve the following equation for x where $0 \le x < 2\pi$. **(3 marks)**

$$2\cos x = -\sin 3x$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window.

Solution



$Y_1 = 2\cos x + \sin 3x$ ← $\frac{1}{2}$ **mark** for equation

← $\frac{1}{2}$ **mark** for graph

$x\ [0,\ 2\pi]$ $y\ [-3,\ 3]$ ← $\frac{1}{2}$ **mark** for window dimensions

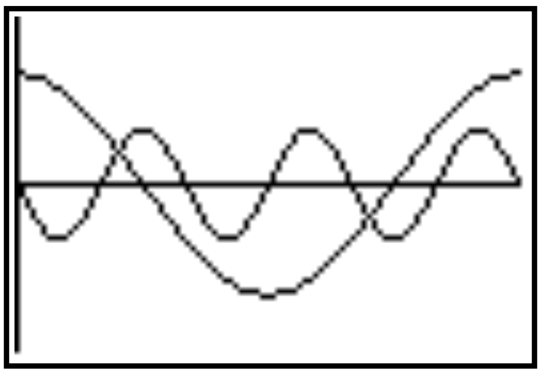
$x = 1.26\ ,\ 4.41$ ← $1\frac{1}{2}$ **marks**

4. Use a graphing calculator to solve the following equation for x where $0 \le x < 2\pi$. **(3 marks)**

$$2\cos x = -\sin 3x$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window.

Alternate Solution



$Y_1 = 2\cos x$

$Y_2 - \sin 3x$

← $\frac{1}{2}$ **mark** for graph of each curve

$x\ [0,\ 2\pi]$ $\quad$ $y\ [-3,\ 3]$ ← $\frac{1}{2}$ **mark** for window dimensions

$x = 1.26\ ,\ 4.41$ ← **$1\frac{1}{2}$ marks**

5. Solve the following system using a graphing calculator. Express all solutions as ordered pairs.

(3 marks)

$$y = 3\log(x+2)+1$$

$$y = 2^{x} - 1$$

Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the function(s) and all intersection points are visible.

Solution

$Y_1 = 3\log(x+2)+1$

$Y_2 = 2^{x} - 1$

$\leftarrow \frac{1}{2}$ **mark** for each graph

$x\ [-5,\ 5]$ $\quad y\ [-3,\ 4]$ $\leftarrow \frac{1}{2}$ **mark** for window dimensions

Therefore the solutions are: $(1.92\ ,\ 2.78)$, $(-1.73,\ -0.70)$ $\leftarrow$ **$1\frac{1}{2}$ marks** for answers

5. Solve the following system using a graphing calculator. Express all solutions as ordered pairs.

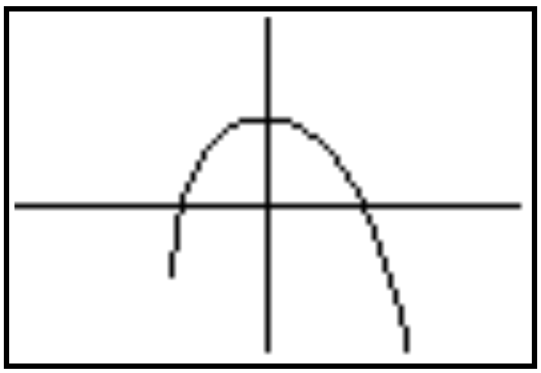
(3 marks)

$$y = 3\log(x+2)+1$$

$$y = 2^x - 1$$

Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the function(s) and all intersection points are visible.

Alternate Solution



$Y_1 = 3\log(x+2)+1-2^x+1$ ← $\frac{1}{2}$ **mark**

← $\frac{1}{2}$ **mark** for graph

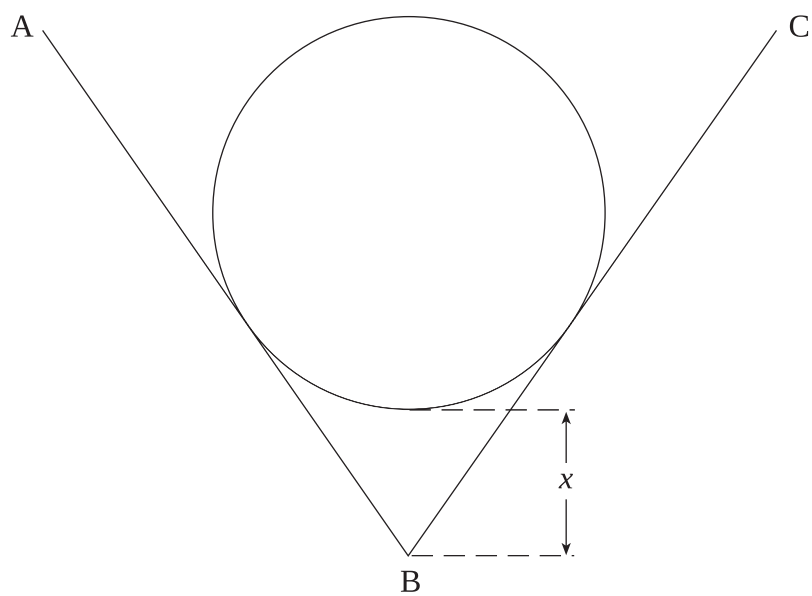
$x\ [-5,\ 5]$ $y\ [-3,\ 4]$ ← $\frac{1}{2}$ **mark** for window dimensions

The zeros of the function are: $x = 1.92$, $x = -1.73$ ← $\frac{1}{2}$ **mark**

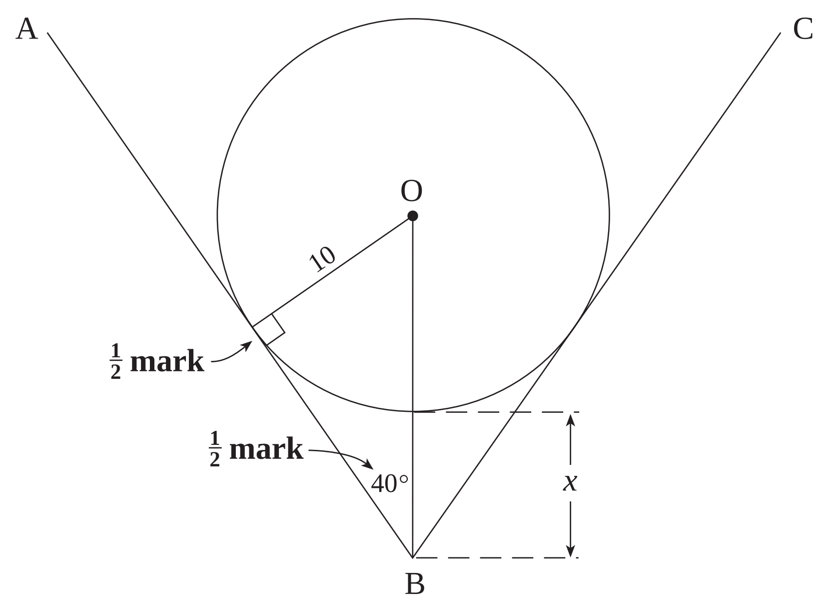
Therefore, by substitution, the solution to the system is:

$(1.92,\ 2.78)$, $(-1.73,\ -0.70)$ ← **1 mark** for obtaining both *y*-values

6. In the diagram below, AB and CB are tangents to a circle with radius 10. If $\angle ABC = 80°$, find x, the shortest distance from B to the circle. **(3 marks)**



Solution



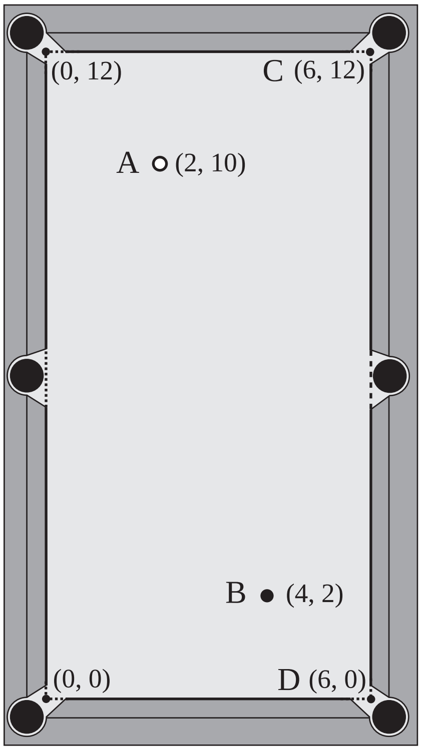
$\sin 40° = \dfrac{10}{OB}$ ← **1 mark**

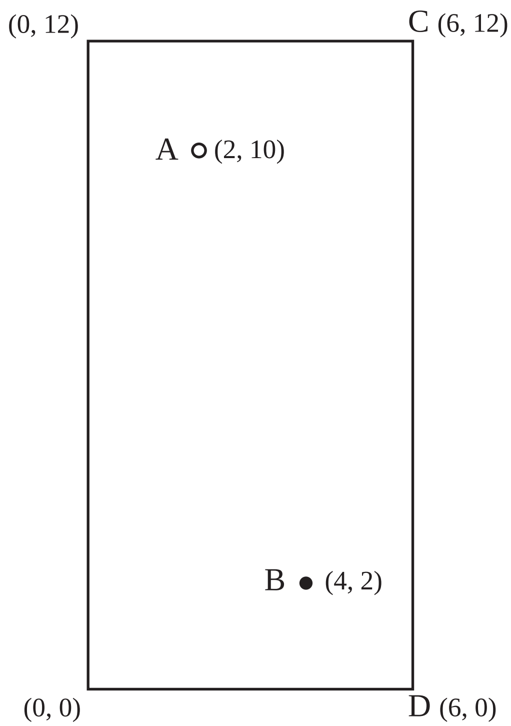
$OB = \dfrac{10}{\sin 40°} = 15.557$ ← **½ mark**

$x = 15.557 - 10$

$= 5.56$ ← **½ mark**

7. Pam and Donna are playing snooker on a 6×12 snooker table. To establish the position of the balls, consider a coordinate system with $(0,\ 0)$ at the bottom left corner and the other corners at $(6,\ 0)$, $(6,\ 12)$ and $(0,\ 12)$. The cue ball is at $\text{A}(2,\ 10)$ and the ball that must be hit is at $\text{B}(4,\ 2)$. If the cue ball at A must bounce off side CD and then hit the ball at B, determine the coordinates of the point on side CD that the cue ball must hit. **(3 marks)**





Solution

$$\frac{4}{8-x} = \frac{2}{x} \qquad \leftarrow \textbf{1 mark}$$

$$4x = 16 - 2x$$

$$6x = 16$$

$$x = \frac{16}{6} = \frac{8}{3} = 2.\dot{6}$$

$$x + 2 = 2.\dot{6} + 2$$

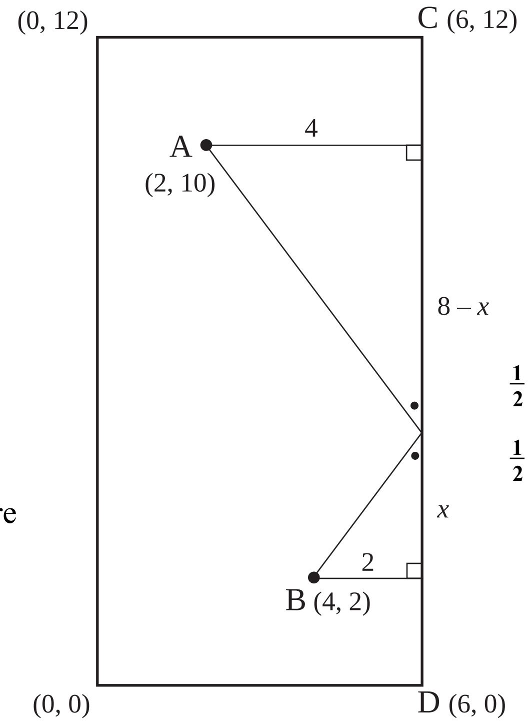
$$= 4.\dot{6}$$

$\therefore$ the coordinates of the point on CD are

$$\left(6,\ 4.\dot{6}\right) \text{ or } \left(6,\ \frac{14}{3}\right)$$

↑ ↑

½ mark each



½ mark for ∠s ≅

½ mark for other numbers with variables

Alternate Solution 1

$$\frac{4}{10-y} = \frac{2}{y-2} \qquad \leftarrow \textbf{1 mark}$$

$$4y = 8 - 20 - 2y$$

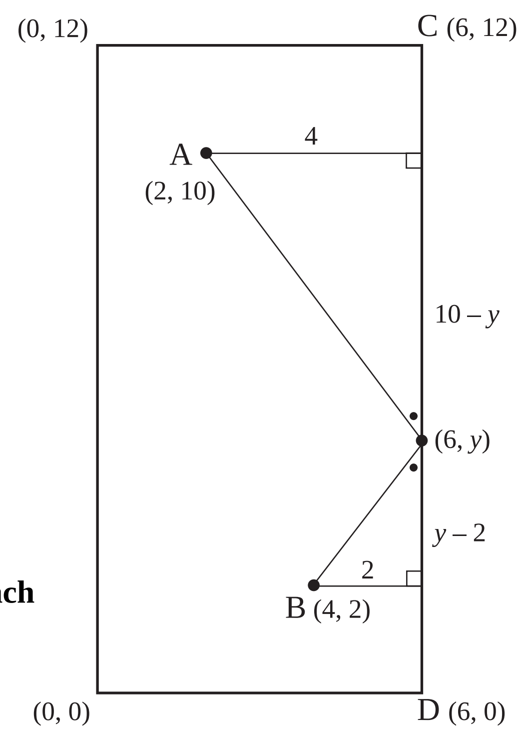
$$6y = 28$$

$$y = \frac{28}{6} = \frac{14}{3}$$

$\therefore$ the point is $\left(6,\ \frac{14}{3}\right)$ or $\left(6,\ 4.\dot{6}\right)$

↑ ↑

½ mark each



½ mark for ∠s ≅

½ mark for other numbers with variables

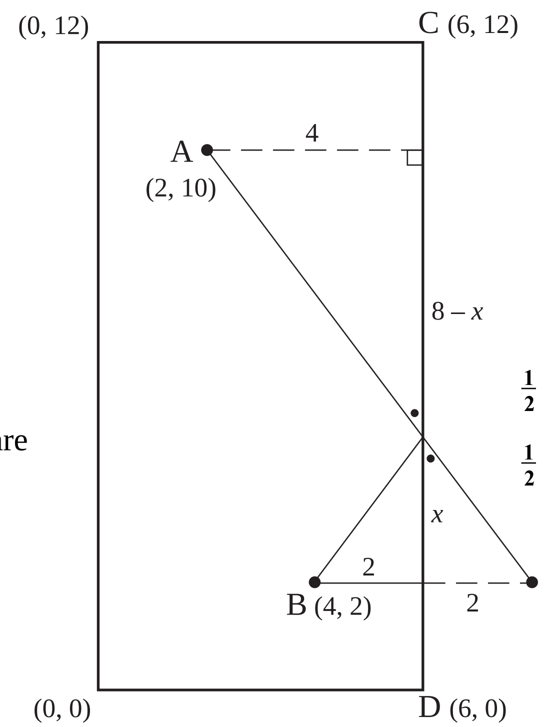
Alternate Solution 2

$$\frac{4}{8-x} = \frac{2}{x} \quad \leftarrow \textbf{1 mark}$$

$$4x = 16 - 2x$$

$$6x = 16$$

$$x = \frac{16}{6} = \frac{8}{3} = 2.\dot{6}$$



$\frac{1}{2}$ **mark** for ∠s ≅

$\frac{1}{2}$ **mark** for other numbers with variables

∴ the coordinates of the point on CD are

$$\left(6,\ 4.\dot{6}\right) \text{ or } \left(6,\ \frac{14}{3}\right)$$

↑ ↑

$\frac{1}{2}$ **mark each**

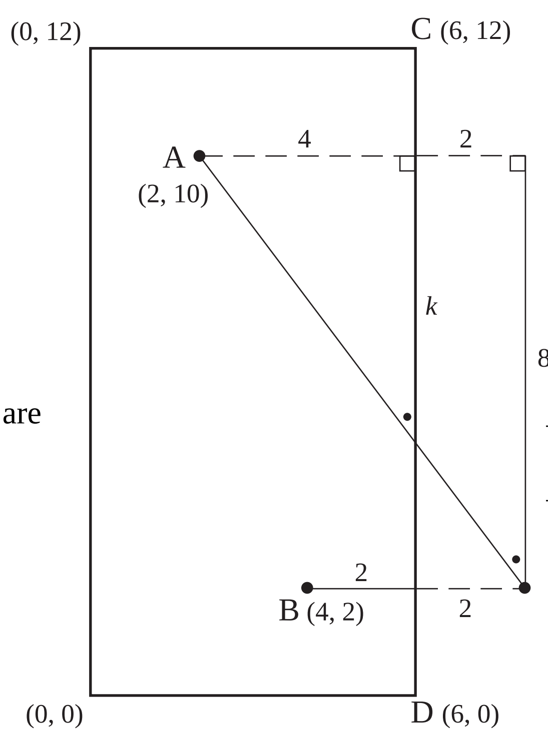
Alternate Solution 3

$$\frac{8}{6} = \frac{k}{4} \quad \leftarrow \textbf{1 mark}$$

$$32 = 6k$$

$$k = \frac{16}{3}$$

$$10 - \frac{16}{3} = \frac{14}{3}$$



$\frac{1}{2}$ **mark** for ∠s ≅

$\frac{1}{2}$ **mark** for other numbers with variables

∴ the coordinates of the point on CD are

$$\left(6,\ \frac{14}{3}\right)$$

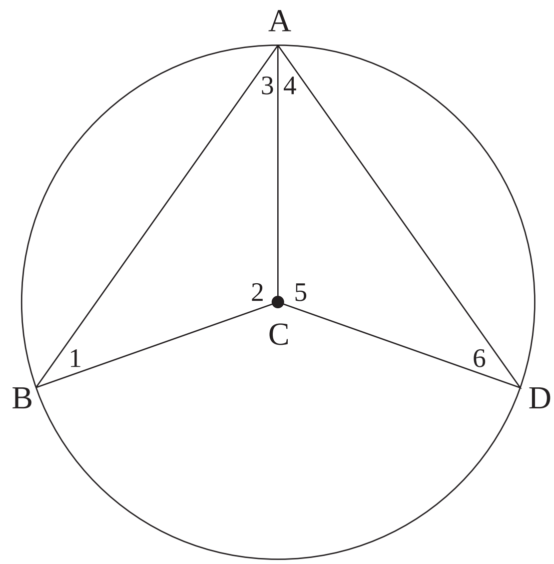
↑ ↑

$\frac{1}{2}$ **mark each**

8. Complete the proof. **(4 marks)**

Given: Circle with centre C
$\angle 1 = \angle 6$

Prove: AB = AD



Solution

Method 1:

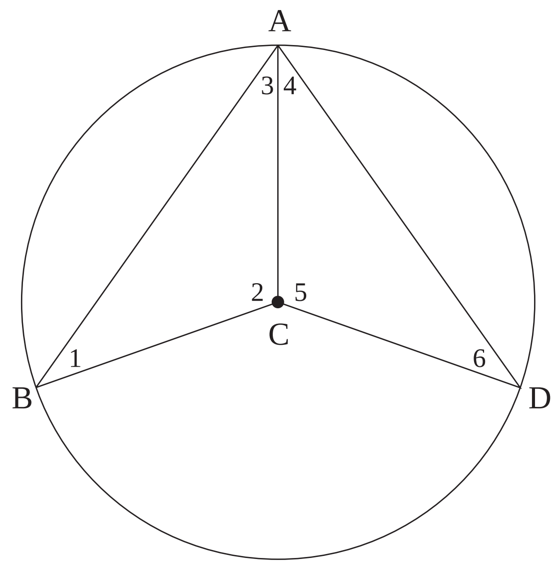
PROOF

Statement	Reason	
C is the centre	given	
AC = BC	radii =	← $\frac{1}{2}$ **mark**
$\angle 1 = \angle 3$	$\angle$s opposite = sides are =	← **1 mark**
AC = DC	radii =	
$\angle 4 = \angle 6$	$\angle$s opposite = sides are =	
$\angle 1 = \angle 6$	given	
$\angle 3 = \angle 4$	substitution	← $\frac{1}{2}$ **mark**
$\frac{1}{2}$ **mark** → $\angle 2 = \angle 5$	3rd $\angle$s of Δs are =	← $\frac{1}{2}$ **mark**
AB = AD	chords opposite = central $\angle$s are =	← **1 mark**

8. Complete the proof. **(4 marks)**

Given: Circle with centre C
$\angle 1 = \angle 6$

Prove: AB = AD



Alternate Solution 1

Method 1:

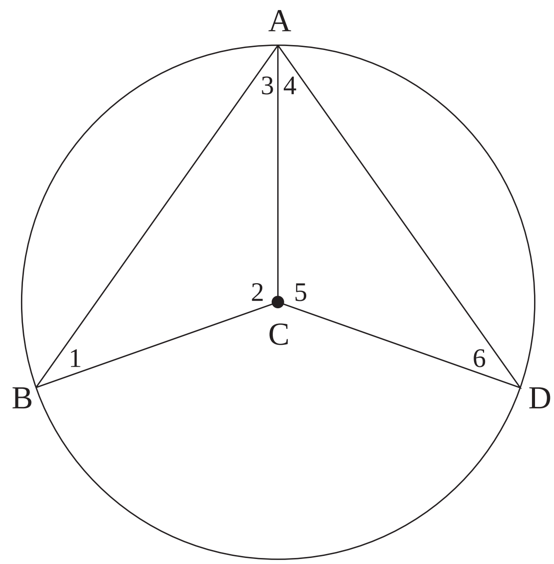
PROOF

Statement	Reason	
C is the centre	given	
AC = BC	radii =	← $\frac{1}{2}$ **mark**
$\angle 1 = \angle 3$	$\angle$s opposite = sides are =	← **1 mark**
AC = DC	radii =	
$\angle 4 = \angle 6$	$\angle$s opposite = sides are =	
$\angle 1 = \angle 6$	given	
$\angle 3 = \angle 4$	substitution	← $\frac{1}{2}$ **mark**
AC = AC	same side	← $\frac{1}{2}$ **mark**
$\frac{1}{2}$ **mark** → $\Delta ABC \cong \Delta ADC$	AAS	← $\frac{1}{2}$ **mark**
AB = AD	CPCTC	← $\frac{1}{2}$ **mark**

8. Complete the proof. **(4 marks)**

Given: Circle with centre C
$\angle 1 = \angle 6$

Prove: AB = AD



Alternate Solution 2

Method 1:

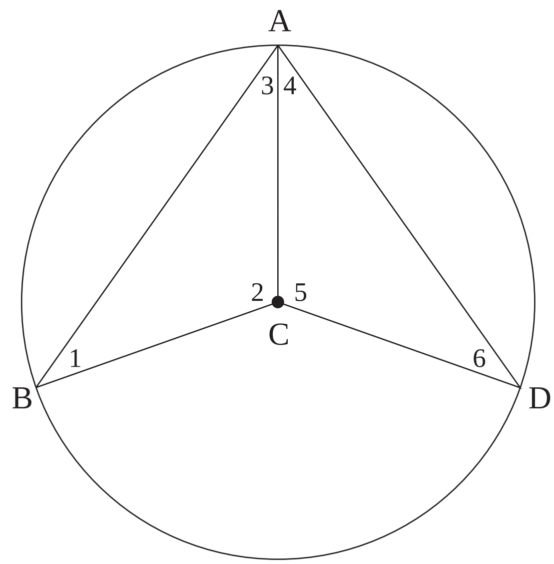
PROOF

Statement	Reason
C is the centre	given
AC = BC	radii = ← **$\frac{1}{2}$ mark**
$\angle 1 = \angle 3$	$\angle$s opposite = sides are = ← **1 mark**
AC = DC	radii =
$\angle 4 = \angle 6$	$\angle$s opposite = sides are =
$\angle 1 = \angle 6$	given
$\angle 3 = \angle 4$	substitution ← **$\frac{1}{2}$ mark**
BC = DC	radii = ← **$\frac{1}{2}$ mark**
$\frac{1}{2}$ mark → $\Delta ABC \cong \Delta ADC$	AAS ← **$\frac{1}{2}$ mark**
AB = AD	CPCTC ← **$\frac{1}{2}$ mark**

8. Complete the proof. **(4 marks)**

Given: Circle with centre C
$\angle 1 = \angle 6$

Prove: AB = AD



Alternate Solution 3

Method 1:

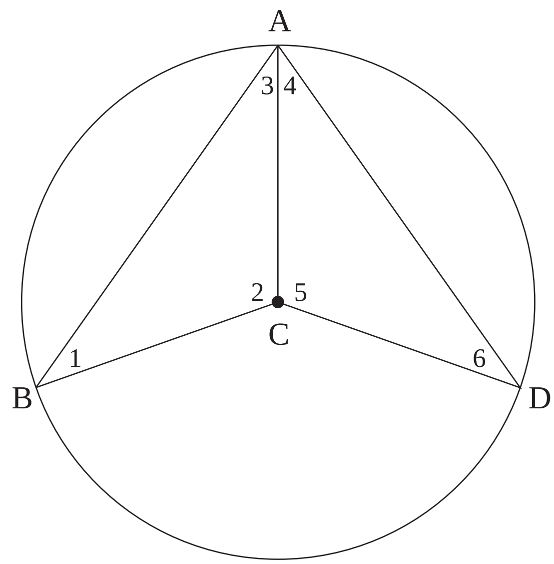
PROOF

Statement	Reason	
C is the centre	given	
AC = BC	radii =	← $\frac{1}{2}$ **mark**
$\angle 1 = \angle 3$	$\angle$s opposite = sides are =	← **1 mark**
AC = DC	radii =	
$\angle 4 = \angle 6$	$\angle$s opposite = sides are =	
$\angle 1 = \angle 6$	given	
$\angle 3 = \angle 4$	substitution	← $\frac{1}{2}$ **mark**
BC = DC	radii =	
AC = AC	same side	
$\angle 2 = \angle 5$	3rd $\angle$s of Δs are =	← $\frac{1}{2}$ **mark**
$\frac{1}{2}$ **mark** → $\Delta ABC \cong \Delta ADC$	SAS	← $\frac{1}{2}$ **mark**
AB = AD	CPCTC	← $\frac{1}{2}$ **mark**

8. Complete the proof. **(4 marks)**

Given: Circle with centre C
$\angle 1 = \angle 6$

Prove: AB = AD



Solution

Method 2:

Since all radii are = $\left(\frac{1}{2}\ \textbf{mark}\right)$, the two triangles are isosceles $\left(\frac{1}{2}\ \textbf{mark}\right)$

$\therefore$ $\angle 1 = \angle 3$, $\angle 4 = \angle 6$ because $\angle$s opposite = sides are = $\leftarrow \frac{1}{2}$ **mark**

Since $\angle 1 = \angle 6 \Rightarrow \angle 3 = \angle 4$ $\leftarrow \frac{1}{2}$ **mark**

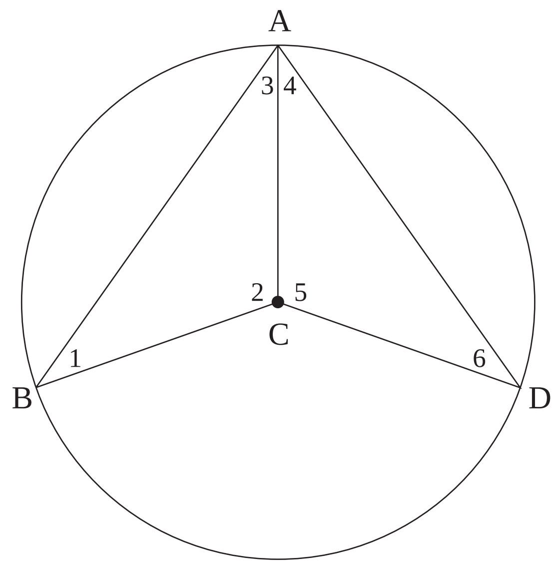
$\therefore$ $\angle 2 = \angle 5$ $\left(\frac{1}{2}\ \textbf{mark}\right)$ because 3rd angles of Δs are = $\left(\frac{1}{2}\ \textbf{mark}\right)$

So AB = AD since chords opposite = central $\angle$s are = $\leftarrow$ **1 mark**

8. Complete the proof. **(4 marks)**

Given: Circle with centre C
$\angle 1 = \angle 6$

Prove: AB = AD



Alternate Solution

Method 2:

Since all radii are = $\left(\frac{1}{2}\right.$ **mark**$\left.\right)$, the two triangles are isosceles $\left(\frac{1}{2}\right.$ **mark**$\left.\right)$

$\therefore$ $\angle 1 = \angle 3$, $\angle 4 = \angle 6$ because $\angle$s opposite = sides are = $\leftarrow \frac{1}{2}$ **mark**

Since $\angle 1 = \angle 6 \Rightarrow \angle 3 = \angle 4$ $\leftarrow \frac{1}{2}$ **mark**

$\therefore$ since AC = AC by same side

$\Delta ABC \cong \Delta ADC$ $\left(\frac{1}{2}\right.$ **mark**$\left.\right)$ by AAS $\left(\frac{1}{2}\right.$ **mark**$\left.\right)$

$\therefore$ AB = AD by CPCTC $\leftarrow$ **1 mark**

END OF KEY