Mathematics 12

April 1998 Provincial Examination

ANSWER KEY / SCORING GUIDE

Topics:
1. Trigonometry
2. Quadratic Relations
3. Exponential and Logarithmic Functions
4. Polynomial Functions
5. Sequences and Series
6. Introduction to Calculus
7. Geometry
8. Problem Solving

Part A: Multiple Choice

Q	K	C	T	ILO	Q	K	C	T	ILO
1.	D	K	2	17	26.	A	K	4	36
2.	C	K	2	17	27.	D	U	4	37
3.	A	U	2	12	28.	A	U	4	43
4.	D	U	2	17	29.	A	U	4	40
5.	C	U	2	13, 15	30.	C	H	4	40, 41
6.	D	U	2	17, 18	31.	C	U	5	46
7.	A	U	2	15	32.	A	U	5	46
8.	D	U	2	21	33.	B	U	5	45
9.	D	H	2	20	34.	D	K	5	46
10.	D	H	2	17	35.	B	U	5	46
11.	A	K	1	04	36.	C	U	5	46
12.	B	U	1	01	37.	B	U	5	47
13.	C	U	1	06	38.	C	H	5	46
14.	C	U	1	06	39.	B	K	6	57
15.	A	U	1	02	40.	A	U	6	53
16.	D	U	1	08	41.	C	U	6	50
17.	A	H	1	08	42.	C	U	6	51
18.	D	H	1	09, 07	43.	B	U	6	60
19.	C	K	3	29	44.	A	U	6	52
20.	B	U	3	31	45.	D	H	6	58
21.	B	U	3	32	46.	C	U	7	63
22.	B	U	3	30	47.	B	H	7	63
23.	B	U	3	24	48.	B	U	8	64
24.	A	H	3	31	49.	B	U	8	64
25.	D	H	3	32	50.	D	H	8	64

Multiple Choice = 50 marks

Part B: Written Response

Q	B	C	S	T	ILO
1.	1	U	2	1	09
2.	2	U	3	3	33
3.	3	U	3	4	39
4.	4	U	3	2	22
5.	5	U	3	6	62
6.	6	H	4	7	63
7.	7	U	2	8	64

Written Response = 20 marks

Multiple Choice = 50 (50 questions)
Written Response = 20 (7 questions)
EXAMINATION TOTAL = 70 marks

LEGEND:

Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
T = Topic
ILO = Intended Learning Outcome

PART B: WRITTEN RESPONSE

Value: 20 marks **Suggested Time: 45 minutes**

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

Full marks will NOT be given for the final answer only.

1. Solve for x: $3\sin^2 x - 8\sin x + 4 = 0$, where $0 \le x < 2\pi$.
(Accurate to at least 2 decimal places.) **(2 marks)**

SOLUTION:

$$3\sin^2 x - 8\sin x + 4 = 0$$

$$(3\sin x - 2)(\sin x - 2) = 0 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$3\sin x - 2 = 0 \qquad\qquad \sin x - 2 = 0$$

$$\sin x = \frac{2}{3} \qquad\qquad \sin x = 2 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\underset{\substack{\uparrow \\ \frac{1}{2}\ \textbf{mark}}}{x = 0.73}\ ,\ \underset{\substack{\uparrow \\ \frac{1}{2}\ \textbf{mark}}}{2.41} \qquad\qquad x = \varnothing$$

2. A colony of ants has a population of 250 000. It is growing at a rate of 3.7% per annum. How long will it take for the population to reach 1 000 000? (Accurate to the nearest year.) **(3 marks)**

SOLUTION:

$$1\,000\,000 = \underset{\substack{\uparrow \\ \frac{1}{2}\ \textbf{mark}}}{250\,000(1.037)^t} \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$4 = 1.037^t$$

$$\log 4 = \log 1.037^t \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\log 4 = t \log 1.037 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\frac{\log 4}{\log 1.037} = t \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$t = 38.15 \text{ years}$$

$$t = 38 \text{ years} \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

and $t = 39$ years was accepted

3. Determine the polynomial function of degree 3, with zeros of -2, 0, and 3, that passes through the point $(2,\ 5)$. Answer may be left in factored form. **(3 marks)**

SOLUTION:

$$y = ax(x+2)(x-3) \quad \left.\right\} \leftarrow$$

$\frac{1}{2}$ mark zeros to factors

1 mark polynomial function with '*a*' $\left(\frac{1}{2}\text{ mark}\right)$ and = *y* $\left(\frac{1}{2}\text{ mark}\right)$

$$5 = a(2)(2+2)(2-3) \qquad \leftarrow \tfrac{1}{2} \textbf{ mark substitution}$$

$$5 = -8a$$

$$a = -\frac{5}{8} \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$y = -\frac{5}{8}x(x+2)(x-3) \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

4. Determine all ordered pairs that satisfy the following system.
(Answer exact or accurate to at least 2 decimal places.) **(3 marks)**

$$x - y^2 = -3$$

$$2x^2 + 3y^2 = 18$$

SOLUTION:

$$\begin{aligned} 3x - 3y^2 &= -9 && \leftarrow \tfrac{1}{2}\ \textbf{mark} \\ 2x^2 + 3y^2 &= 18 \\ \hline 2x^2 + 3x &= 9 && \leftarrow \tfrac{1}{2}\ \textbf{mark} \end{aligned}$$

$$2x^2 + 3x - 9 = 0$$

$$(2x - 3)(x + 3) = 0$$

$$\begin{aligned} 2x - 3 &= 0 \\ x &= \frac{3}{2} \quad \leftarrow \tfrac{1}{2}\ \textbf{mark} \\ \frac{3}{2} &= y^2 - 3 \\ \frac{9}{2} &= y^2 \\ y &= \pm\frac{3\sqrt{2}}{2} \text{ or } \pm\frac{3}{\sqrt{2}} \end{aligned}$$

$$\left.\begin{aligned} &\therefore \left(\frac{3}{2},\ \frac{3\sqrt{2}}{2}\right),\ \left(\frac{3}{2},\ \frac{-3\sqrt{2}}{2}\right) \\ &\textbf{or } (1.5,\ 2.12),\ (1.5,\ -2.12) \end{aligned}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\begin{aligned} x + 3 &= 0 \\ x &= -3 \quad \leftarrow \tfrac{1}{2}\ \textbf{mark} \\ -3 &= y^2 - 3 \\ 0 &= y^2 \\ y &= 0 \end{aligned}$$

$$\therefore (-3,\ 0) \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

Note: Deduct $\frac{1}{2}$ **mark** only **once** if not given as ordered pairs.

4. Determine all ordered pairs that satisfy the following system.
(Answer exact or accurate to at least 2 decimal places.) **(3 marks)**

$$\begin{aligned} x - y^2 &= -3 \\ 2x^2 + 3y^2 &= 18 \end{aligned}$$

ALTERNATE SOLUTION:

$$y^2 = x + 3 \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$2x^2 + 3(x+3) = 18 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$2x^2 + 3x + 9 = 18$$

$$2x^2 + 3x - 9 = 0$$

$$(2x-3)(x+3) = 0$$

$$2x - 3 = 0$$

$$x = \frac{3}{2} \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\frac{3}{2} = y^2 - 3$$

$$\frac{9}{2} = y^2$$

$$y = \pm\frac{3\sqrt{2}}{2} \ \text{ or } \ \pm\frac{3}{\sqrt{2}}$$

$$\left.\begin{array}{l} \therefore \left(\dfrac{3}{2},\ \dfrac{3\sqrt{2}}{2}\right),\ \left(\dfrac{3}{2},\ \dfrac{-3\sqrt{2}}{2}\right) \\[2ex] \textbf{or}\ \ (1.5,\ 2.12),\ (1.5,\ -2.12) \end{array}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$x + 3 = 0$$

$$x = -3 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

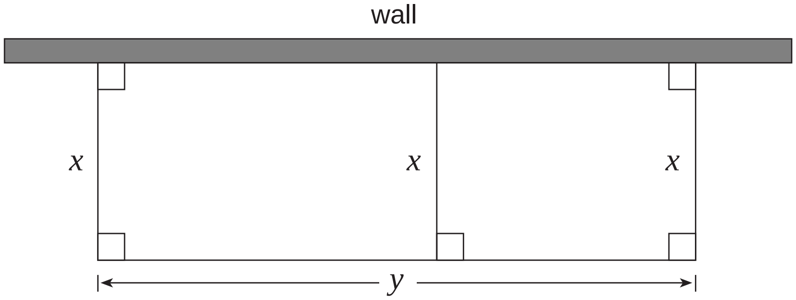
$$-3 = y^2 - 3$$

$$0 = y^2$$

$$y = 0$$

$$\therefore (-3,\ 0) \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

5. A rectangular pigpen is to be constructed having one side along an existing wall. The pigpen is also to be divided into two parts as shown in the diagram.



If a total of 300 metres of fencing is used, determine the maximum area that the pigpen can have. **(3 marks)**

SOLUTION:

$A = xy$ ← $\frac{1}{2}$ **mark**

$y + 3x = 300$ ← $\frac{1}{2}$ **mark**

$y = 300 - 3x$

$A = x(300 - 3x)$ ← $\frac{1}{2}$ **mark**

$A = -3x^2 + 300x$

$\frac{dA}{dx} = -6x + 300$ ← $\frac{1}{2}$ **mark**

$-6x + 300 = 0$ ← $\frac{1}{2}$ **mark**

$x = 50$

$A = 50[300 - 3(50)]$

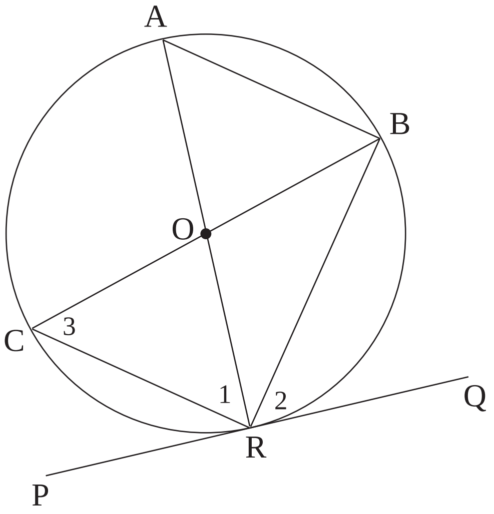
$A = 7\,500 \text{ m}^2$ ← $\frac{1}{2}$ **mark**

6. Complete the proof. **(4 marks)**

Given: AR and BC are diameters.
PQ is tangent to the circle at R

Prove: $\angle 1 = \angle 2$

Note: Students are encouraged to number angles.



SOLUTION:

PROOF

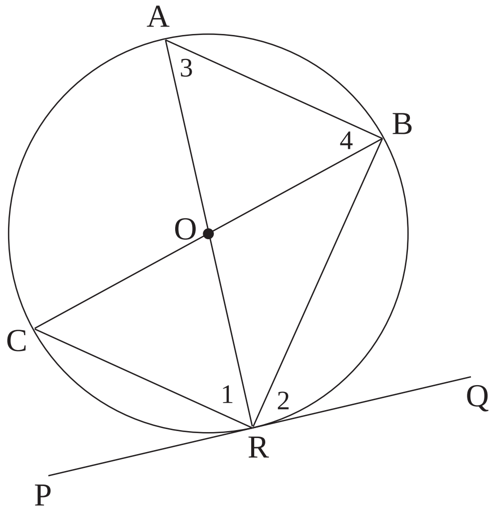
	Statement	Reason
	PQ is tangent to circle at R	given
1 mark →	$\angle 2 = \angle 3$	$\angle$ between tangent and chord
	AR and BC are diameters	given
1 mark →	CO = RO	radii =
$1\frac{1}{2}$ marks →	$\angle 1 = \angle 3$	$\angle$s opposite = sides are =
$\frac{1}{2}$ mark →	$\angle 1 = \angle 2$	both = $\angle 3$

6. Complete the proof. **(4 marks)**

Given: AR and BC are diameters.
PQ is tangent to the circle at R

Prove: $\angle 1 = \angle 2$

Note: Students are encouraged to number angles.



ALTERNATE SOLUTION #1:

PROOF

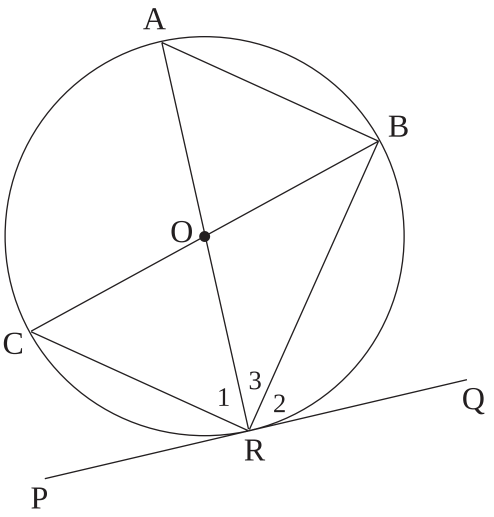
	Statement	Reason
	PQ is tangent to circle at R	given
1 mark →	$\angle 2 = \angle 3$	∠ between tangent and chord
	AR and BC are diameters	given
$\frac{1}{2}$ **mark** →	AO = BO	radii =
$\frac{1}{2}$ **mark** →	$\angle 3 = \angle 4$	∠s opposite = sides are =
$\frac{1}{2}$ **mark** →	$\angle 2 = \angle 4$	both = ∠3
1 mark →	$\angle 1 = \angle 4$	inscribed ∠s on same arc
$\frac{1}{2}$ **mark** →	$\angle 1 = \angle 2$	both = ∠4

6. Complete the proof. **(4 marks)**

Given: AR and BC are diameters.
PQ is tangent to the circle at R

Prove: $\angle 1 = \angle 2$

Note: Students are encouraged to number angles.



ALTERNATE SOLUTION #2:

PROOF

	Statement	Reason
	PQ is tangent to circle at R	given
	AR and BC are diameters	given
1 mark →	$\angle 3 + \angle 2 = 90^\circ$	tangent ⊥ radius
1 mark →	$\angle 1 + \angle 3 = 90^\circ$	inscribed ∠ on diameter = 90°
1 mark →	$\angle 3 + \angle 2 = \angle 1 + \angle 3$	both = 90° (substitution)
1 mark →	$\angle 2 = \angle 1$	equation property of subtraction

7. A function is defined by the equation:

$$f(t) = t^2 - 6t$$

Sketch the graph given by the solution of $f(x) - f(y) = 0$. **(2 marks)**

SOLUTION:

$$f(x) - f(y) = 0$$

$$x^2 - 6x - \left(y^2 - 6y\right) = 0 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$x^2 - 6x + 9 - \left(y^2 - 6y + 9\right) = 0$$

$$(x-3)^2 - (y-3)^2 = 0 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

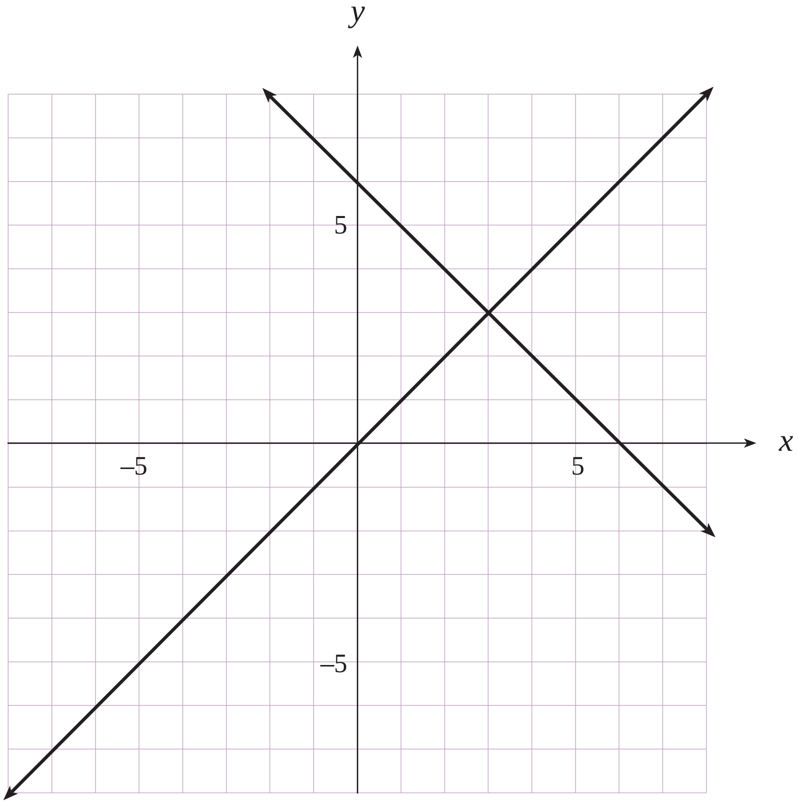
$$(y-3)^2 = (x-3)^2$$

$$y - 3 = \pm(x-3) \Rightarrow y = x \text{ or } y = -x + 6$$

OR

$$y - 3 = \pm(x-3) \Rightarrow y = \pm x$$

graph is shifted 3 units right and 3 units up



1 mark for graph

END OF KEY