

**Mathematics 12**  
June 1997 Provincial Examination  
**ANSWER KEY / SCORING GUIDE**

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- Topics:**
1. Trigonometry
  2. Quadratic Relations
  3. Exponential and Logarithmic Functions
  4. Polynomial Functions
  5. Sequences and Series
  6. Introduction to Calculus
  7. Geometry
  8. Problem Solving

**Part A: Multiple Choice**

**Q C T K S ILO**

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1.	K	2	D	1	18	26.	K	4	B	1	38
2.	U	2	B	1	14	27.	U	4	C	1	37
3.	K	2	B	1	17	28.	U	4	B	1	35
4.	U	2	A	1	17	29.	U	4	D	1	43
5.	U	2	B	1	10	30.	H	4	A	1	41
6.	U	2	A	1	22	31.	K	5	A	1	46
7.	U	2	A	1	21	32.	U	5	B	1	46
8.	U	2	A	1	17	33.	U	5	A	1	45
9.	H	2	A	1	15	34.	U	5	C	1	46
10.	H	2	D	1	20	35.	U	5	D	1	46
11.	K	1	B	1	01	36.	U	5	C	1	46
12.	U	1	C	1	05	37.	U	5	B	1	47
13.	U	1	B	1	02	38.	H	5	A	1	46
14.	U	1	B	1	08	39.	K	6	B	1	57
15.	U	1	B	1	03	40.	U	6	C	1	50
16.	H	1	C	1	03	41.	U	6	C	1	51
17.	H	1	D	1	08	42.	U	6	C	1	52
18.	H	1	C	1	06	43.	U	6	C	1	61
19.	K	3	A	1	28	44.	U	6	D	1	54
20.	U	3	C	1	26	45.	H	6	D	1	53
21.	U	3	C	1	29	46.	H	7	C	1	63
22.	U	3	A	1	24	47.	H	7	B	1	63
23.	H	3	B	1	32	48.	U	8	D	1	64
24.	U	3	B	1	31	49.	H	8	D	1	64
25.	H	3	D	1	31	50.	H	8	B	1	64

**Part B: Written Response**

<b>Q</b>	<b>B</b>	<b>C</b>	<b>T</b>	<b>S</b>	<b>ILO</b>	<b>Q</b>	<b>B</b>	<b>C</b>	<b>T</b>	<b>S</b>	<b>ILO</b>
1.	1	U	1	2	06	4.	5	U	2	3	19
2.	2	U	3	3	32	5.	6	U	4	3	40
3a.	3	U	6	2	60	6.	7	U	8	2	64
3b.	4	U	6	1	60	7.	8	H	7	4	63

Multiple Choice = 50 (50 questions)

Written Response = 20 (7 questions)

**Total = 70 marks**

**LEGEND:**

**Q** = Question Number

**C** = Cognitive Level

**T** = Topic

**K** = Keyed Response

**S** = Score

**ILO** = Intended Learning Outcome

**B** = Score Box Number

**PART B: WRITTEN RESPONSE**

**Value: 20 marks**

**Suggested Time: 45 minutes**

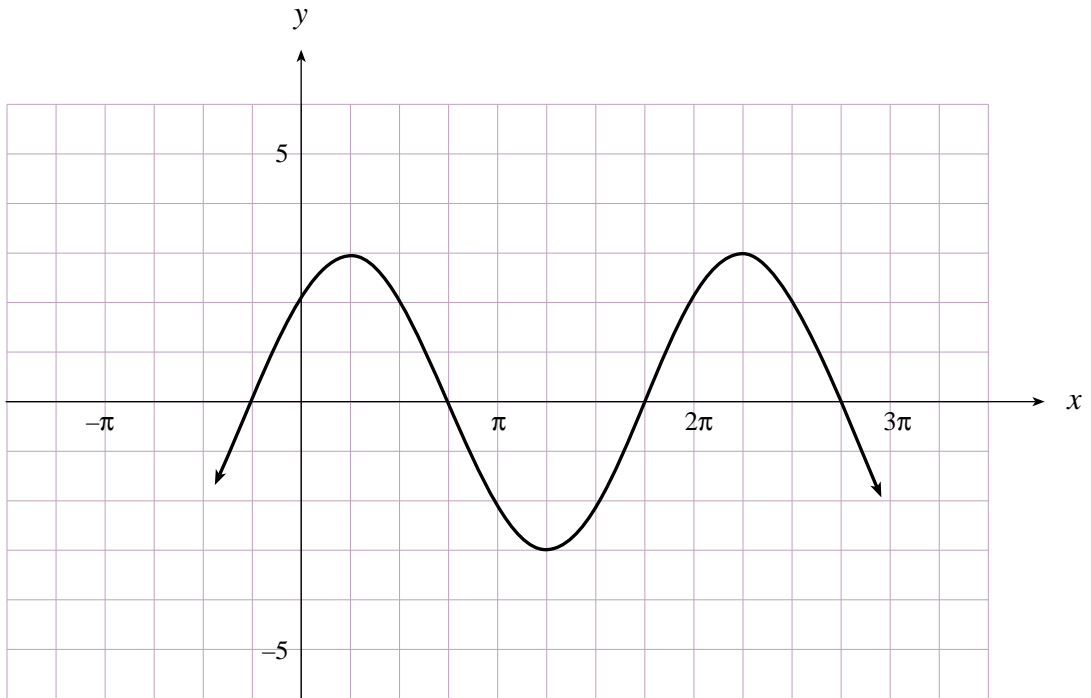
**INSTRUCTIONS:** Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

**Full marks will NOT be given for the final answer only.**

1. Graph at least one period of  $y = -3 \sin\left(x - \frac{3\pi}{4}\right)$ .

**(2 marks)**

**Solution:**



- Sine curve/shape and period ←  $\frac{1}{2}$  **mark**
- Reflection ←  $\frac{1}{2}$  **mark**
- Amplitude ←  $\frac{1}{2}$  **mark**
- Phase shift ←  $\frac{1}{2}$  **mark**

$$2. \text{ Solve for } x: 2 \log(4-x) - \log 3 = \log(10-x)$$

**(3 marks)**

**Solution:**

$$\log \frac{(4-x)^2}{3} = \log(10-x) \quad \leftarrow \mathbf{1 \text{ mark}}$$

$$\frac{(4-x)^2}{3} = 10-x \quad \leftarrow \frac{1}{2} \mathbf{mark}$$

$$16 - 8x + x^2 = 30 - 3x$$

$$x^2 - 5x - 14 = 0 \quad \leftarrow \frac{1}{2} \mathbf{mark}$$

$$(x-7)(x+2) = 0$$

$$x = -2, \cancel{7}$$

$\frac{1}{2} \mathbf{mark}$   $\leftarrow$   $\uparrow$   
**reject**  
 $\uparrow$   
 $\frac{1}{2} \mathbf{mark}$

$$\therefore x = -2$$

3. A particle moves along the  $x$ -axis so that its position at time  $t$  is  $x = 4t^3 - 21t^2 + 30t$ , where  $t$  is measured in seconds, and  $x$  is measured in metres.

a) Determine the time(s) when the particle is stopped.

(2 marks)

**Solution:**

$$v = x' = 12t^2 - 42t + 30 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$12t^2 - 42t + 30 = 0 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$6(2t^2 - 7t + 5) = 0$$

$$6(2t - 5)(t - 1) = 0$$

$$t = \frac{5}{2} \text{ s} \quad \text{or} \quad t = 1 \text{ s}$$

↑

$\frac{1}{2}$  mark

↑

$\frac{1}{2}$  mark

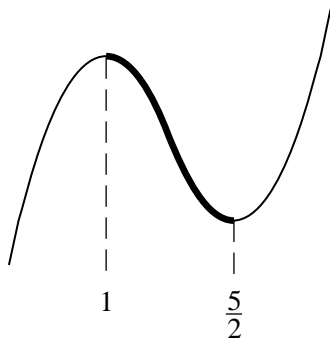
b) Determine when the particle is moving to the left.

(1 mark)

**Solution:**

Consider graph of position function  $x$

$$x = 4t^3 - 21t^2 + 30t$$

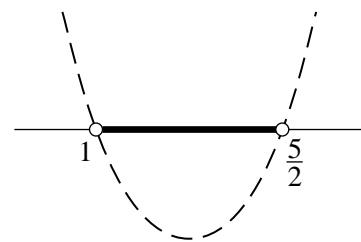


decreasing  $\Rightarrow$  moving left

or

Consider sign of  $x'$

$$x' = 6(2t - 5)(t - 1) < 0 \quad \leftarrow \frac{1}{2} \text{ mark}$$

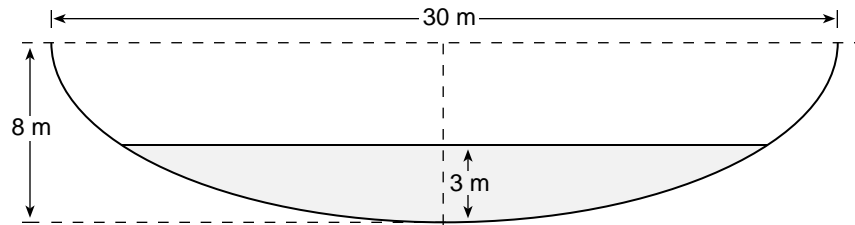


$x' < 0 \Rightarrow$  moving left

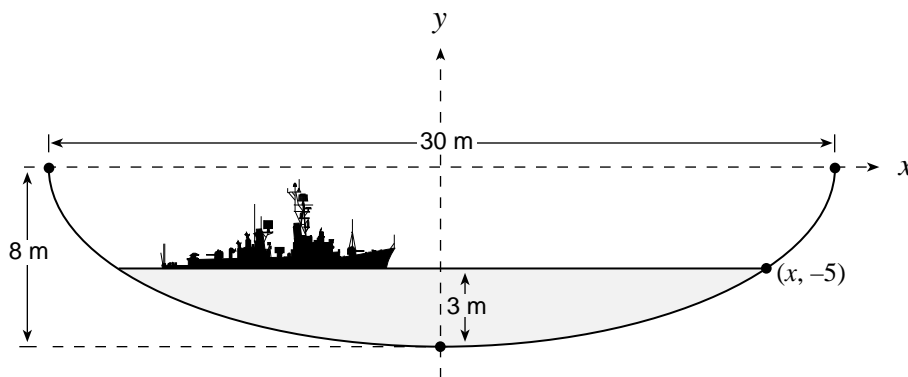
$\therefore$  particle is moving left when

$$1 < t < \frac{5}{2} \quad \leftarrow \frac{1}{2} \text{ mark}$$

4. The cross section of a drainage canal has the shape of a semi-ellipse and measures 30 m across and 8 m deep at its deepest point. Presently the maximum depth of water in the canal is 3 m. Calculate the width of the water surface. (Accurate to at least 2 decimal places.) **(3 marks)**



**Solution:**



$$\frac{x^2}{15^2} + \frac{y^2}{8^2} = 1 \quad \leftarrow \frac{1}{2} \text{ mark (form of equation)}$$

$$\begin{array}{cc} \uparrow & \uparrow \\ \frac{1}{2} \text{ mark} & \frac{1}{2} \text{ mark} \end{array}$$

$$\frac{x^2}{15^2} + \frac{(-5)^2}{8^2} = 1 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$x^2 = \left(1 - \frac{5^2}{8^2}\right) 15^2$$

$$x = 11.709 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\text{Width} = 23.42 \text{ m} \quad \leftarrow \frac{1}{2} \text{ mark}$$

5. If  $-2$  is a root of  $2x^3 + kx^2 - 11x + 6 = 0$ , determine the other two roots.

**(3 marks)**

**Solution:**

$$2(-2)^3 + k(-2)^2 - 11(-2) + 6 = 0 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$4k = -12$$

$$k = -3 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$2x^3 - 3x^2 - 11x + 6 = 0$$

$$\begin{array}{r|rrrr} -2 & 2 & -3 & -11 & 6 \\ & & -4 & 14 & -6 \\ \hline & 2 & -7 & 3 & 0 \end{array} \quad \left. \vphantom{\begin{array}{r|rrrr} -2 & 2 & -3 & -11 & 6 \\ & & -4 & 14 & -6 \\ \hline & 2 & -7 & 3 & 0 \end{array}} \right\} \leftarrow \frac{1}{2} \text{ mark}$$

$$2x^2 - 7x + 3 = 0 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$(2x - 1)(x - 3) = 0 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$x = \frac{1}{2} \quad \text{or} \quad 3$$

$\frac{1}{2}$  mark

**Alternate Solution:**

$$\begin{array}{r|rrrr} -2 & 2 & k & -11 & 6 \\ & & -4 & 14 & -6 \\ \hline & 2 & -7 & 3 & 0 \end{array} \quad \left. \vphantom{\begin{array}{r|rrrr} -2 & 2 & k & -11 & 6 \\ & & -4 & 14 & -6 \\ \hline & 2 & -7 & 3 & 0 \end{array}} \right\} \leftarrow 1 \frac{1}{2} \text{ marks} \quad (\text{Note: it is not necessary to find } k.)$$

$$2x^2 - 7x + 3 = 0 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$(2x - 1)(x - 3) = 0 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$x = \frac{1}{2} \quad \text{or} \quad 3$$

$\frac{1}{2}$  mark



6. Two functions are defined by the equations  $f(t) = t^2 - 4t - 6$  and  $g(t) = t^2 + 2t - 5$ .  
Graph the region defined by the following inequality. **(2 marks)**

$$f(x) + g(y) \leq 0$$

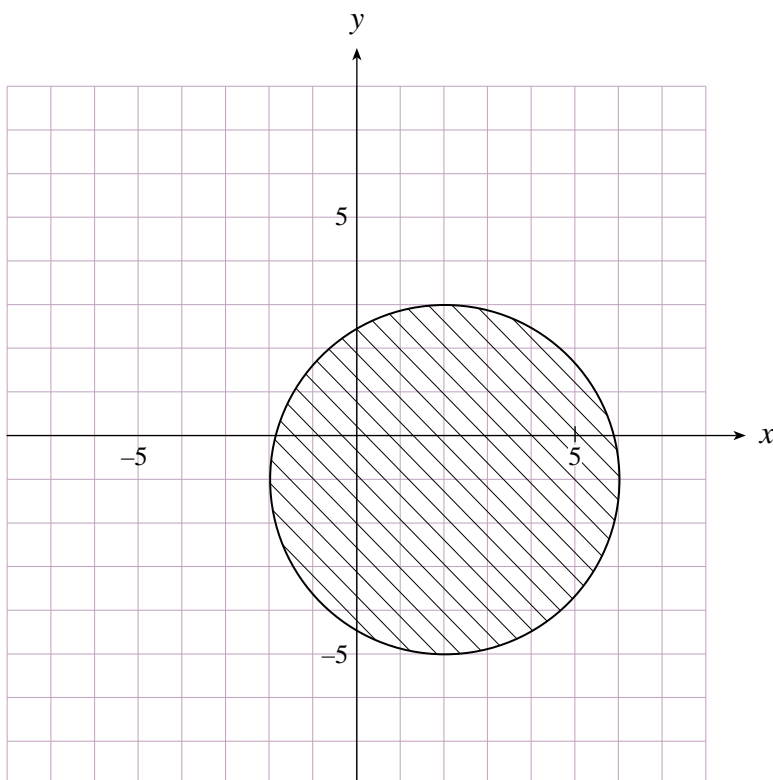
**Solution:**

$$\left. \begin{array}{l} f(x) = x^2 - 4x - 6 \\ g(y) = y^2 + 2y - 5 \end{array} \right\} \leftarrow \frac{1}{2} \text{ mark}$$

$$x^2 - 4x - 6 + y^2 + 2y - 5 \leq 0 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$x^2 - 4x + 4 + y^2 + 2y + 1 \leq 11 + 4 + 1$$

$$(x - 2)^2 + (y + 1)^2 \leq 16 \quad \leftarrow \frac{1}{2} \text{ mark}$$



$\leftarrow \frac{1}{2}$  mark for graph

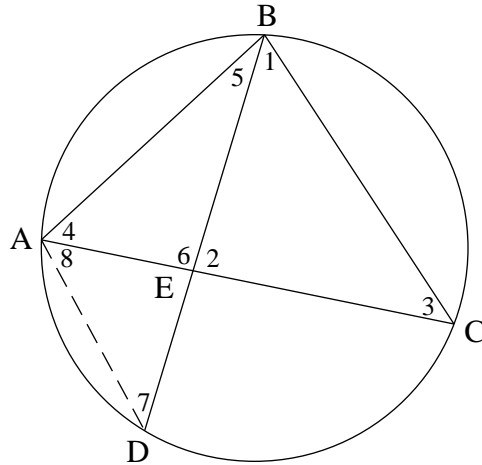
7. Complete the proof.

(4 marks)

Given:  $BE = EC$

Prove:  $AE = DE$

Note: Students are encouraged to number angles.



**Solution:**

Statement	Proof Reason
join AD	construction
$BE = EC$	given
$\angle 1 = \angle 3$	$\angle$ s opposite = sides are = ← <b>1 mark</b>
$\angle 3 = \angle 7$	inscribed $\angle$ s on same chord are =
$\angle 1 = \angle 8$	inscribed $\angle$ s on same arc are = } ← <b>1 mark</b>
$\angle 7 = \angle 8$	both = to = $\angle$ s (substitution) ← <b>1 mark</b>
$AE = DE$	sides opposite = $\angle$ s are = ← <b>1 mark</b>

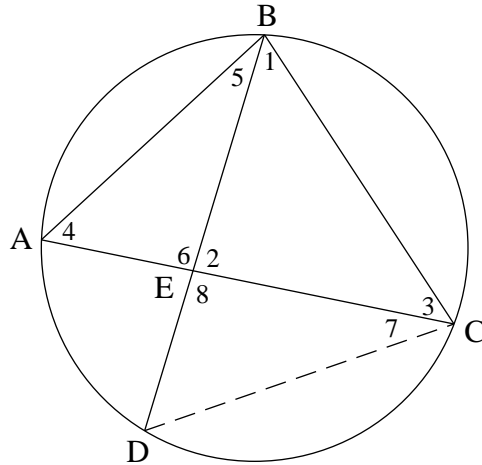
7. Complete the proof.

(4 marks)

Given:  $BE = EC$

Prove:  $AE = DE$

Note: Students are encouraged to number angles.



**Alternate Solution #1:**

Statement	Proof Reason
join DC	construction
$\angle 5 = \angle 7$	inscribed $\angle$ s on same arc are = ← <b>1 mark</b>
$BE = EC$	given
$\angle 6 = \angle 8$	vertically opposite $\angle$ s are = ← <b>1 mark</b>
$\triangle AEB \cong \triangle DEC$	ASA ← <b>1 mark</b>
$AE = DE$	CPCTC ← <b>1 mark</b>

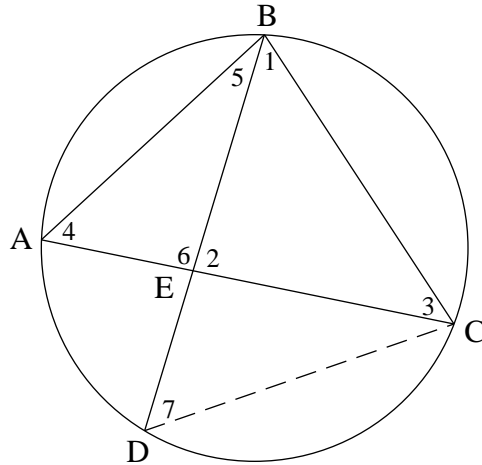
7. Complete the proof.

(4 marks)

Given:  $BE = EC$

Prove:  $AE = DE$

Note: Students are encouraged to number angles.



**Alternate Solution #2:**

Statement	Proof Reason
join DC	construction
$\angle 4 = \angle 7$	inscribed $\angle$ s on same chord are = ← <b>1 mark</b>
$BE = EC$	given
$\angle 1 = \angle 3$	$\angle$ s opposite = sides are = ← <b>1 mark</b>
$BC = BC$	same side
$\triangle ABC \cong \triangle DCB$	AAS ← <b>1 mark</b>
$AC = DB$	CPCTC ← $\frac{1}{2}$ <b>mark</b>
$AC - EC = DB - BE$	equation property of subtraction
$AE = DE$	substitution } ← $\frac{1}{2}$ <b>mark</b>

**END OF KEY**