Mathematics 12

August 1996 Provincial Examination

ANSWER KEY / SCORING GUIDE

Topics:
1. Trigonometry
2. Quadratic Relations
3. Exponential and Logarithmic Functions
4. Polynomial Functions
5. Sequences and Series
6. Introduction to Calculus
7. Geometry
8. Problem Solving

Part A: Multiple Choice

Q	C	T	K	S	ILO	Q	C	T	K	S	ILO
1.	K	2	C	1	12.20	26.	U	4	A	1	12.35
2.	K	2	B	1	12.17	27.	U	4	C	1	12.41
3.	U	2	A	1	12.14	28.	U	4	A	1	12.40
4.	U	2	C	1	12.21	29.	U	4	B	1	12.43
5.	U	2	B	1	12.16	30.	H	4	D	1	12.37
6.	U	2	A	1	12.22	31.	K	5	D	1	12.46
7.	U	2	C	1	12.17	32.	U	5	B	1	12.46
8.	U	2	A	1	12.15	33.	U	5	A	1	12.46
9.	H	2	A	1	12.12	34.	U	5	B	1	12.46
10.	H	2	D	1	12.18	35.	U	5	C	1	12.48
11.	U	1	B	1	12.01	36.	U	5	D	1	12.46
12.	U	1	A	1	12.02	37.	H	5	D	1	12.47
13.	U	1	C	1	12.03	38.	H	5	A	1	12.45
14.	K	1	C	1	12.05	39.	U	6	C	1	12.57
15.	U	1	D	1	12.06	40.	K	6	C	1	12.56
16.	U	1	A	1	12.08	41.	U	6	A	1	12.53
17.	H	1	D	1	12.07	42.	U	6	C	1	12.55
18.	K	3	B	1	12.28	43.	U	6	D	1	12.60
19.	U	3	A	1	12.32	44.	U	6	B	1	12.51
20.	U	3	B	1	12.31	45.	H	6	B	1	12.58
21.	U	3	D	1	12.32	46.	U	7	B	1	12.63
22.	U	3	B	1	12.25	47.	U	7	D	1	12.63
23.	H	3	C	1	12.31	48.	U	8	C	1	12.64
24.	H	3	D	1	12.64, 12.30	49.	H	8	D	1	12.64
25.	K	4	C	1	12.38	50.	H	8	C	1	12.64

Part B: Written Response

Q	B	C	T	S	ILO	Q	B	C	T	S	ILO
1.	1	U	1	3	12.03	5a.	5	U	6	1	12.62
2.	2	U	3	3	12.26	5b.	6	U	6	2	12.62
3.	3	U	4	2	12.39	6.	7	U	8	2	12.64
4.	4	U	2	3	12.23	7.	8	H	7	4	12.63

Multiple Choice = 50 (50 questions)
Written Response = 20 (7 questions)
Total = 70 marks

LEGEND:

Q = Question Number **C** = Cognitive Level **T** = Topic
K = Keyed Response **S** = Score **ILO** = Intended Learning Outcome
B = Score Box Number

PART B: WRITTEN RESPONSE

Value: 20 marks **Suggested Time: 45 minutes**

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

Full marks will NOT be given for the final answer only.

1. Solve: $3\sin^2\theta - 7\sin\theta + 2 = 0,\ 0 \le \theta < 2\pi$ (Accurate to at least 2 decimal places.) **(3 marks)**

Solution:

$$3\sin^2\theta - 7\sin\theta + 2 = 0$$

½ mark $\rightarrow$ $(3\sin\theta - 1)(\sin\theta - 2) = 0$

½ mark $\rightarrow$ $\sin\theta = \frac{1}{3}$ $\qquad$ $\sin\theta = 2$ $\leftarrow$ **½ mark**

$\Downarrow$ $\qquad$ $\Downarrow$

reference $\angle = 0.34$ $\qquad$ $\varnothing$ $\leftarrow$ **½ mark**

$\therefore\ \theta = 0.34\ ,\ 2.80$

$\uparrow$ $\qquad$ $\uparrow$

½ mark **½ mark**

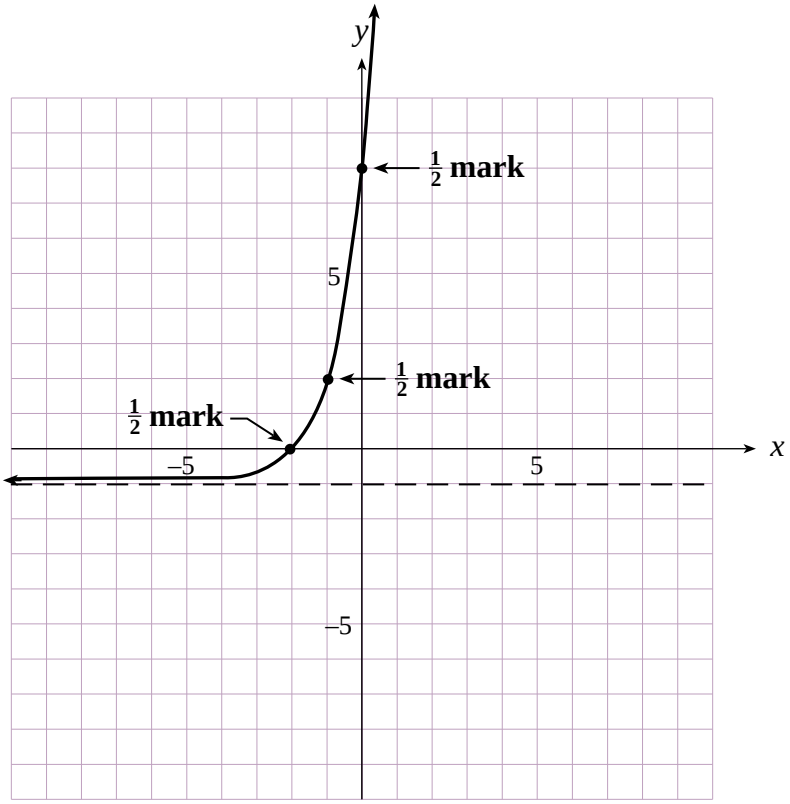
2. Graph $y = 3^{x+2} - 1$. Indicate the asymptote with a dotted or broken line and clearly show at least three points on the graph. **(3 marks)**

Solution:

$y = 3^{x+2} - 1$

asymptote $y = -1$

x	y
0	8
−1	2
−2	0



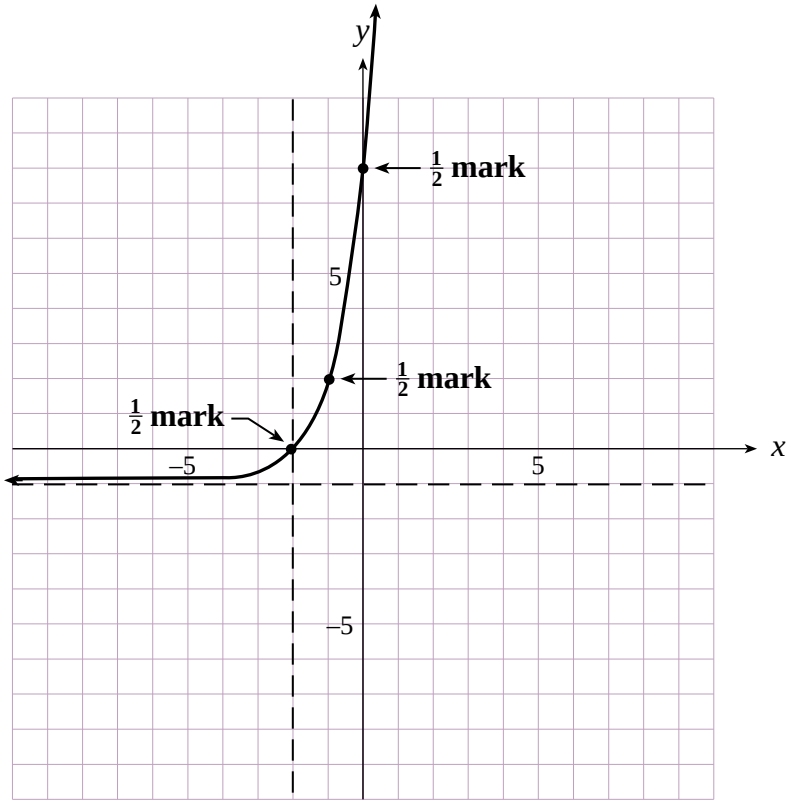
1 mark for shape

← **½ mark** asymptote

2. Graph $y = 3^{x+2} - 1$. Indicate the asymptote with a dotted or broken line and clearly show at least three points on the graph. **(3 marks)**

Alternate Solution: (using translation)

$y = 3^{x+2} - 1$



1 mark for shape

← **½ mark** asymptote

shift $(-2\,,\ -1)$

base eqn

↓

$y = 3^x$

x	y
2	9
1	3
0	1
–1	$\frac{1}{3}$

3. A polynomial function of degree 3 has zeros $-2,\ 2,\ 4,$ and passes through the point $(3,\ -25)$. Determine an equation of the function. (Answer may be left in factored form.)

(2 marks)

Solution:

Zeros: $\pm 2,\ 4$ through $(3,\ -25)$

$\frac{1}{2}$ mark
$\downarrow$

$f(x) = a(x-2)(x+2)(x-4)$ $\leftarrow$ **$\frac{1}{2}$ mark**

$f(3) = a(3-2)(3+2)(3-4) = -25$ $\leftarrow$ **$\frac{1}{2}$ mark**

$a(1)(5)(-1) = -25$

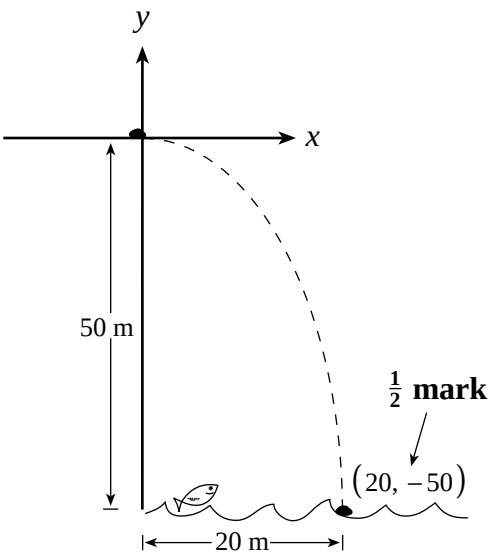
$-5a = -25$

$a = 5$

$\therefore\ f(x) = 5(x-2)(x+2)(x-4)$ $\leftarrow$ **$\frac{1}{2}$ mark**

4. A rock is kicked off a vertical cliff and falls in a parabolic path to the water below. The cliff is 50 m high and the rock hits the water 20 m from the base of the cliff. What is the horizontal distance of the rock from the cliff face when the rock is at a height of 10 m above the water? (Accurate to at least 2 decimal places.) **(3 marks)**

Solution:

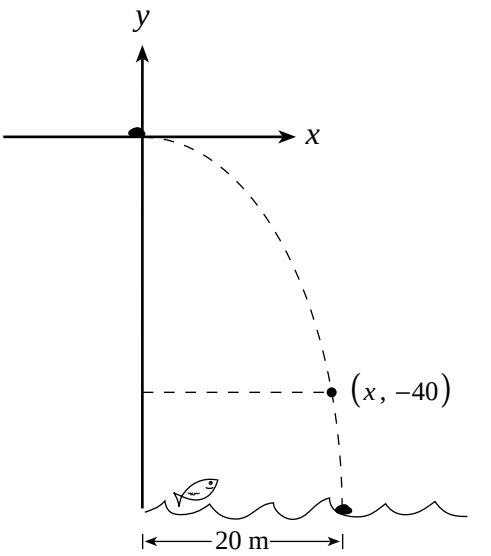


$y = ax^2$ ← ½ **mark**

$-50 = a(20)^2$ ← ½ **mark**

$-\frac{50}{400} = a$

Equation: $y = -\frac{1}{8}x^2$ ← ½ **mark**



½ **mark**

$-40 = -\frac{1}{8}x^2$

$x^2 = 320$

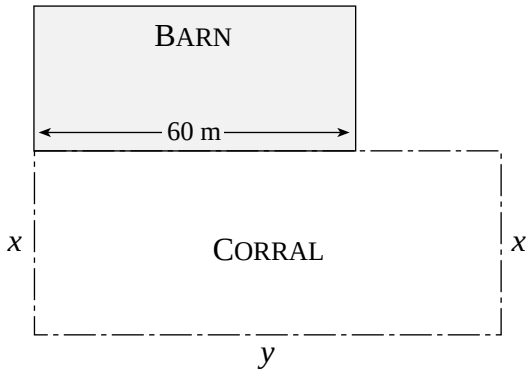
$x = 17.89$ m ← ½ **mark**

or $8\sqrt{5}$ m

Note: due to symmetry, this question can be done without negative *y*-coordinates, and thus without a negative "*a*".

5. A rancher wishes to build a rectangular corral, using the entire length of the barn as part of one side. The barn has a length of 60 m and the rancher has 220 m of fencing available.

   a) Show that an expression for the area of the corral is $A = 140x - x^2$. **(1 mark)**



Solution:

$A = xy$

$2x + 2y - 60 = 220$

$2x + 2y = 280$ } ← $\frac{1}{2}$ **mark**

$x + y = 140$

$y = 140 - x$

$A = x(140 - x)$

$A = 140x - x^2$ ← $\frac{1}{2}$ **mark**

b) Using $A = 140x - x^2$, determine the dimensions of the corral that will maximize its area.

(2 marks)

Solution:

$$A = 140x - x^2$$

$$A' = 140 - 2x = 0 \leftarrow \tfrac{1}{2}\textbf{ mark } (\text{for setting } A' = 0)$$

$\uparrow$

$\frac{1}{2}$ **mark** for taking derivative

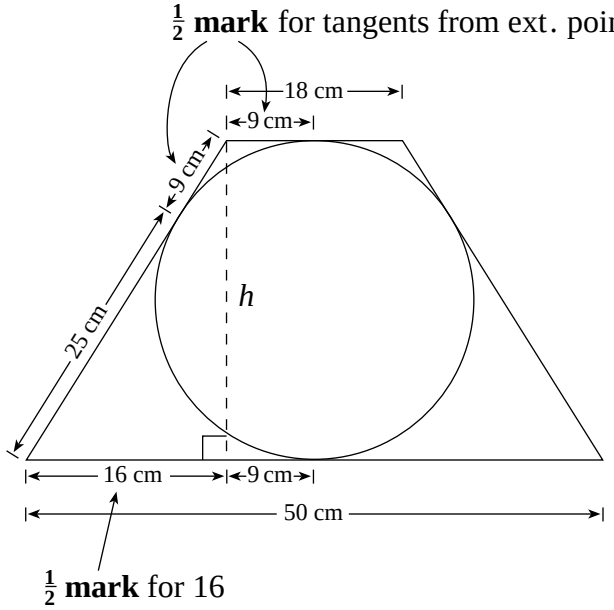
$$x = 70 \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$y = 140 - 70 = 70 \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$\therefore$ dimensions are $70\,\text{m}$ by $70\,\text{m}$

6. A circle is inscribed in an isosceles trapezoid having bases of 18 cm and 50 cm. Find the diameter of the circle. **(2 marks)**

Solution:



$h^2 + 16^2 = 34^2$ ← ½ **mark**

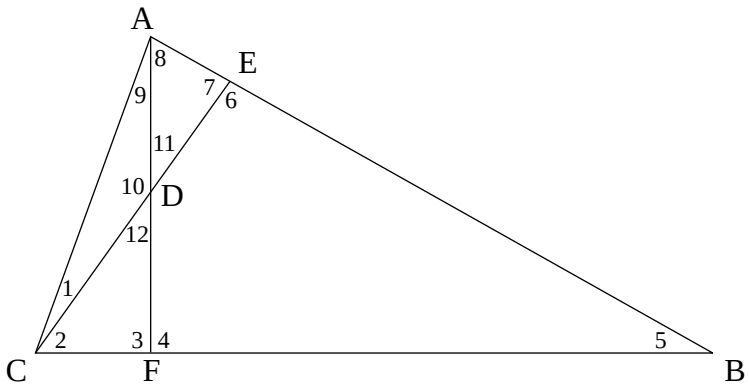
$h^2 = 900$

$h = 30$ cm ← ½ **mark**

7. Complete the proof. **(4 marks)**

Given: $CE \perp AB$
$AF \perp CB$
$DE = DF$

Prove: $\angle BCA = \angle BAC$



Solution:

Proof

Statement	Reason
1. $CE \perp AB$, $AF \perp CB$	given
2. $\angle 3 = 90°$, $\angle 7 = 90°$	definition of $\perp$ **($\frac{1}{2}$ mark)**
3. $\angle 3 = \angle 7$	substitution (both $= 90°$) **($\frac{1}{2}$ mark)**
4. $DF = DE$	given
5. $\angle 11 = \angle 12$	vertically opposite $\angle$s are = **($\frac{1}{2}$ mark)**
6. $\Delta DFC \cong \Delta DEA$	ASA **($\frac{1}{2}$ mark)**
7. $AD = CD$	CPCTC **$\frac{1}{2}$ mark**
8. $\angle 1 = \angle 9$	$\angle$s opposite = sides are = **$\frac{1}{2}$ mark**
9. $\angle 2 = \angle 8$	CPCTC
10. $\angle 1 + \angle 2 = \angle 8 + \angle 9$	equation property of addition **$\frac{1}{2}$ mark**
11. $\angle BCA = \angle BAC$	substitution **$\frac{1}{2}$ mark**

2 marks → lines 1–6 **2 marks** → lines 7–11

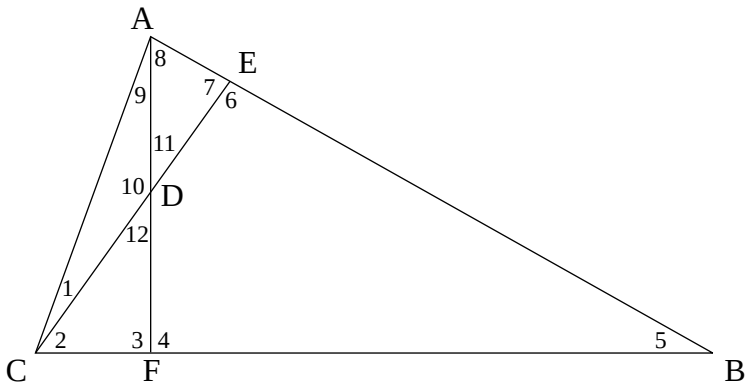
If statements 1. to 3. are given with the reason definition of $\perp$ or property of $\perp$ award 1 mark.

Deduct $\frac{1}{2}$ mark for not stating givens.

7. Complete the proof. **(4 marks)**

Given: $CE \perp AB$
$AF \perp CB$
$DE = DF$

Prove: $\angle BCA = \angle BAC$



Alternate Solution #1:

Proof

Statement	Reason
$CE \perp AB$, $AF \perp CB$	given
$\angle 7 = 90°$, $\angle 3 = 90°$	definition of $\perp$ $\frac{1}{2}$ **mark**
$\angle 7 = \angle 3$	substitution (both $= 90°$) $\frac{1}{2}$ **mark**
$DE = DF$	given
$\angle 12 = \angle 11$	vertically opposite $\angle$s are = $\frac{1}{2}$ **mark**
$\Delta DFC \cong \Delta DEA$	ASA $\frac{1}{2}$ **mark**
$CF = AE$	CPCTC $\frac{1}{2}$ **mark**
$AD = CD$	CPCTC $\frac{1}{2}$ **mark**
$\angle 1 = \angle 9$	$\angle$s opposite = sides are = $\frac{1}{2}$ **mark**
$\Delta ACF \cong \Delta CAE$	AAS $\frac{1}{2}$ **mark**
$\angle BCA = \angle BAC$	CPCTC $\frac{1}{2}$ **mark**

2 marks → (CE ⊥ AB , AF ⊥ CB … $\Delta DFC \cong \Delta DEA$)

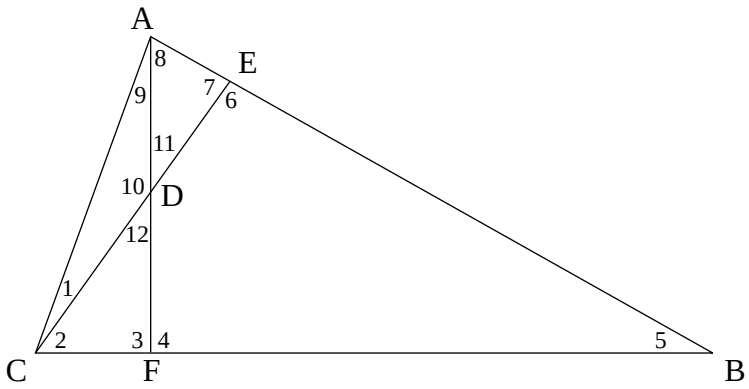
2 marks → ($CF = AE$ … $\angle BCA = \angle BAC$)

Deduct $\frac{1}{2}$ mark for not stating givens.

7. Complete the proof. **(4 marks)**

Given: $CE \perp AB$
$AF \perp CB$
$DE = DF$

Prove: $\angle BCA = \angle BAC$



Alternate Solution #2:

Proof

	Statement	Reason
2 marks →	$CE \perp AB$, $AF \perp CB$	given
	$\angle 3 = 90°$, $\angle 4 = 90°$, $\angle 6 = 90°$, $\angle 7 = 90°$	definition of $\perp$ $\frac{1}{2}$ **mark**
	$\angle 4 = \angle 6$	substitution (both = 90°)
	$\angle 3 = \angle 7$	substitution (both = 90°) } $\frac{1}{2}$ **mark**
	$DE = DF$	given
	$\angle 11 = \angle 12$	vertically opposite ∠s are = $\frac{1}{2}$ **mark**
	$\Delta DFC \cong \Delta DEA$	ASA $\frac{1}{2}$ **mark**
2 marks →	$AD = CD$	CPCTC
	$AD + DF = CD + DE$	equation property of addition } $\frac{1}{2}$ **mark**
	$AF = CE$	substitution $\frac{1}{2}$ **mark**
	$\angle 8 = \angle 2$	CPCTC
	$\Delta ABF \cong \Delta CBE$	**(Deduct $\frac{1}{2}$ mark if the above step is missing.)**
	$AB = CB$	ASA $\frac{1}{2}$ **mark**
	$\angle BCA = \angle BAC$	CPCTC ∠s opposite = sides are = } $\frac{1}{2}$ **mark**

Deduct $\frac{1}{2}$ mark for not stating givens.

END OF KEY