Mathematics 12

June 1996 Provincial Examination

ANSWER KEY / SCORING GUIDE

Topics:
1. Trigonometry
2. Quadratic Relations
3. Exponential and Logarithmic Functions
4. Polynomial Functions
5. Sequences and Series
6. Introduction to Calculus
7. Geometry
8. Problem Solving

Part A: Multiple Choice

Q	C	T	K	S	ILO	Q	C	T	K	S	ILO
1.	K	2	C	1	12.14	26.	K	4	A	1	12.41
2.	K	2	D	1	12.17	27.	U	4	B	1	12.41
3.	U	2	A	1	12.11	28.	U	4	B	1	12.37
4.	U	2	B	1	12.17	29.	U	4	C	1	12.40
5.	U	2	C	1	12.16	30.	U	4	C	1	12.43
6.	U	2	C	1	12.20	31.	U	4	B	1	12.35
7.	U	2	A	1	12.18	32.	U	4	C	1	12.39
8.	U	2	A	1	12.15	33.	H	4	B	1	12.36
9.	H	2	A	1	12.17	34.	U	5	B	1	12.46
10.	H	2	B	1	12.13	35.	K	5	B	1	12.46
11.	K	1	D	1	12.01	36.	U	5	C	1	12.45
12.	U	1	D	1	12.02	37.	U	5	B	1	12.46
13.	U	1	D	1	12.07	38.	H	5	D	1	12.47
14.	U	1	D	1	12.05	39.	K	6	A	1	12.57
15.	U	1	A	1	12.08	40.	U	6	C	1	12.50
16.	U	1	C	1	12.09	41.	U	6	C	1	12.53
17.	H	1	C	1	12.08	42.	U	6	B	1	12.51
18.	H	1	D	1	12.09	43.	U	6	C	1	12.58
19.	K	3	D	1	12.28	44.	U	6	B	1	12.61
20.	U	3	D	1	12.26	45.	H	6	D	1	12.57
21.	U	3	A	1	12.31	46.	H	7	B	1	12.63
22.	U	3	A	1	12.24	47.	H	7	A	1	12.63
23.	U	3	B	1	12.31	48.	U	8	A	1	12.64
24.	U	3	D	1	12.27	49.	U	8	D	1	12.64
25.	H	3	C	1	12.30	50.	U	8	B	1	12.64

Part B: Written Response

Q	B	C	T	S	ILO
1.	1	U	1	2	12.06
2.	2	U	5	3	12.46
3.	3	U	3	3	12.32
4.	4	U	2	3	12.22

Q	B	C	T	S	ILO
5.	5	H	7	4	12.63
6.	6	U	6	3	12.56
7.	7	H	8	2	12.64

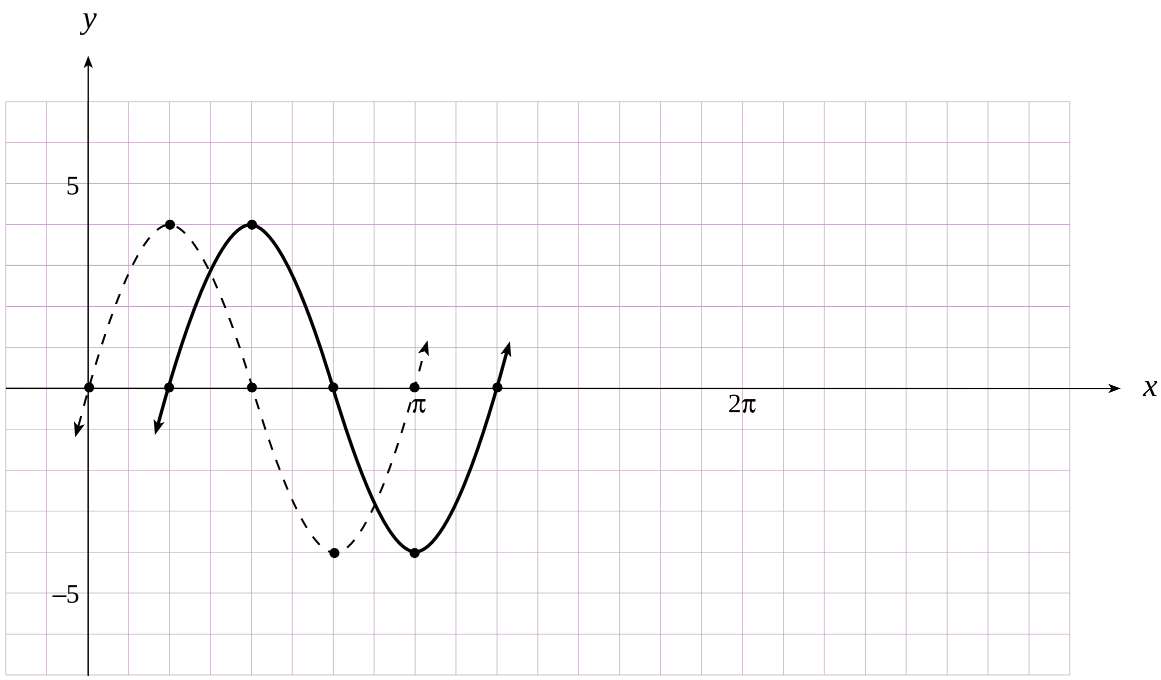
Multiple Choice = 50 (50 questions)
Written Response = 20 (7 questions)
Total = 70 marks

LEGEND:
Q = Question Number **C** = Cognitive Level **T** = Topic
K = Keyed Response **S** = Score **ILO** = Intended Learning Outcome
B = Score Box Number

PART B: WRITTEN RESPONSE

1. Graph at least one period of $y = 4\sin 2\left(x - \frac{\pi}{4}\right)$. **(2 marks)**

Solution:



Phase shift $\leftarrow \frac{1}{2}$ **mark**
Period $\leftarrow \frac{1}{2}$ **mark**
Amplitude $\leftarrow \frac{1}{2}$ **mark**
Shape $\leftarrow \frac{1}{2}$ **mark**

Amp = 4

Per = $\frac{2\pi}{2} = \pi$

Phase shift = $\frac{\pi}{4} \rightarrow$

Sine curve

2. Three consecutive terms in an arithmetic sequence are $8x+7$, $2x+5$ and $2x^2+x$. Determine the values of the three terms for all such sequences. **(3 marks)**

Solution:

$$t_2 - t_1 = t_3 - t_2 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$2x+5-(8x+7) = 2x^2+x-(2x+5) \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$-6x-2 = 2x^2-x-5$$

$$2x^2+5x-3=0 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$(2x-1)(x+3)=0$$

$$x=\frac{1}{2} \qquad x=-3 \quad \Big\} \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$\therefore$ terms are:

$11, \quad 6, \quad 1$ $\leftarrow \tfrac{1}{2}$ **mark**

or $-17, \ -1, \ 15$ $\leftarrow \tfrac{1}{2}$ **mark**

2. Three consecutive terms in an arithmetic sequence are $8x+7$, $2x+5$, and $2x^2+x$. Determine the values of the three terms for all such sequences. **(3 marks)**

Alternate Solution:

$$t_2 = \frac{t_1 + t_3}{2} \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$2x+5 = \frac{8x+7+2x^2+x}{2} \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$4x+10 = 9x+2x^2+7$$

$$0 = 2x^2+5x-3 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$0 = (2x-1)(x+3)$$

$$\left.x = \frac{1}{2} \qquad x = -3\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$\therefore$ terms are:

$11,\ 6,\ 1$ $\leftarrow \tfrac{1}{2}$ **mark**

or $-17,\ -1,\ 15$ $\leftarrow \tfrac{1}{2}$ **mark**

3. Solve: $\log_2(x+7)+\log_2(x+5)=3$	**(3 marks)**

Solution:

$$\log_2(x+7)+\log_2(x+5)=3$$

$$\log_2(x+7)(x+5)=3 \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$(x+7)(x+5)=8 \quad \leftarrow \textbf{1 mark}$$

$$x^2+12x+35=8$$

$$x^2+12x+27=0 \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$(x+3)(x+9)=0$$

$\tfrac{1}{2}$ **mark** $\rightarrow$ $x=-3$ or $x=-9$ $\leftarrow \tfrac{1}{2}$ **mark**

$x=-9$
$\uparrow$
reject

Deduct $\tfrac{1}{2}$ **mark** if –9 is **NOT** rejected.

$\therefore\ x=-3$

4. Solve the following system **for x only**. (Accurate to at least 2 decimal places.) **(3 marks)**

$$xy = 6$$

$$x^2 + y^2 = 15$$

Solution:

$$xy = 6$$

$$x^2 + y^2 = 15$$

$$y = \frac{6}{x} \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$x^2 + \left(\frac{6}{x}\right)^2 = 15 \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$x^2 + \frac{36}{x^2} = 15$$

$$x^4 + 36 = 15x^2 \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$x^4 - 15x^2 + 36 = 0$$

$$\left(x^2 - 12\right)\left(x^2 - 3\right) = 0 \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$x^2 = 12 \qquad\qquad x^2 = 3$$

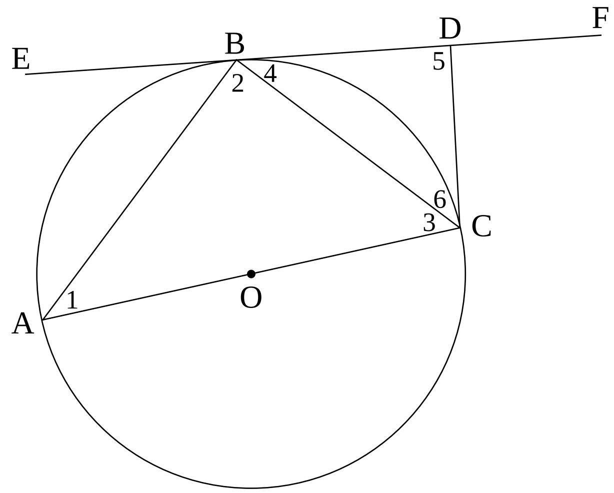
$$\left.\begin{aligned} x &= \pm\sqrt{12} \\ x &= \pm 2\sqrt{3} \\ x &= \pm 3.46 \end{aligned}\right\} \leftarrow \tfrac{1}{2}\textbf{ mark} \qquad \left.\begin{aligned} x &= \pm\sqrt{3} \\ x &= \pm 1.73 \end{aligned}\right\} \leftarrow \tfrac{1}{2}\textbf{ mark}$$

(Deduct $\tfrac{1}{2}$ **mark** if omit $\pm$)

5. Complete the proof. **(4 marks)**

Given: AC is a diameter of the circle with centre O
EF is a tangent at B
CD ⊥ EF

Prove: BC bisects ∠DCA



Solution:

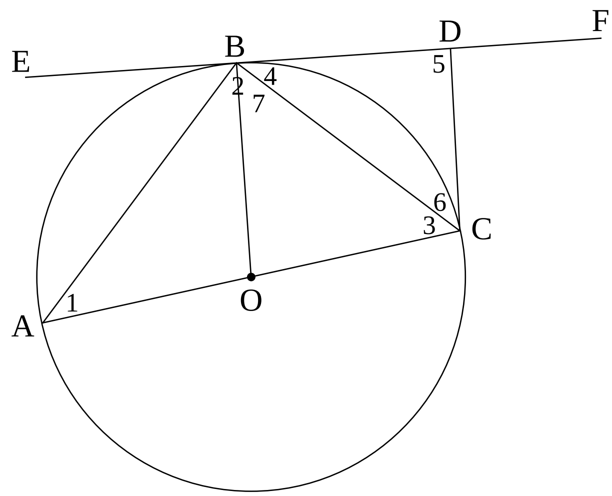
Proof

	Statement	Reason
2 marks →	EF is a tangent	given
	∠4 = ∠1	∠ between chord and tangent
	AC is diameter	given
	∠2 = 90°	inscribed ∠ in a semicircle
2 marks →	DC ⊥ EF	given
	∠5 = 90°	definition of ⊥
	∠2 = ∠5	both = 90° (substitution)
	∠3 = ∠6	3rd ∠s of Δs are =
	BC bisects ∠DCA	definition of ∠ bisector

5. Complete the proof. **(4 marks)**

Given: AC is a diameter of the circle with centre O
EF is a tangent at B
CD ⊥ EF

Prove: BC bisects ∠DCA



Alternate Solution:

Proof

	Statement	Reason
2 marks →	Join OB	construction
	EF is a tangent	given
	∠EBO = 90°	tangent ⊥ radius
	CD ⊥ EF	given
	∠5 = 90°	definition of ⊥
	∠5 = ∠EBO	substitution
	OB ∣∣ CD	corresponding ∠s are =
2 marks →	∠7 = ∠6	alternate interior ∠s are =
	OB = OC	radii are =
	∠7 = ∠3	∠s opposite = sides are =
	∠6 = ∠3	substitution
	BC bisects ∠DCA	definition of ∠ bisector

6. Given $f(x) = 3x^2$, use the definition of derivative to show that $f'(x) = 6x$. **(3 marks)**

Solution:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ ← **½ mark**

$$= \lim_{h \to 0} \frac{3(x+h)^2 - 3x^2}{h}$$ ← **1 mark**

$$= \lim_{h \to 0} \frac{3\left(x^2 + 2xh + h^2\right) - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{6xh + 3h^2}{h}$$ ← **½ mark**

$$= \lim_{h \to 0} \frac{h(6x + 3h)}{h}$$

$$= \lim_{h \to 0} (6x + 3h)$$

} ← **½ mark** (for the two lines above)

$$= 6x$$ ← **½ mark**

7. Determine the domain of the relation $\log_{x+4} y = \log_{x+4} x^2$.	**(2 marks)**

Solution:

$\log_{x+4} y = \log_{x+4} x^2$

Base restrictions		Argument restrictions
$x+4>0$	$x+4 \neq 1$	$x^2 > 0 \leftarrow \frac{1}{2}$ **mark**
$x>-4$	$x \neq -3$	$x \neq 0$
↑	↑	↑
$\frac{1}{2}$ **mark**	$\frac{1}{2}$ **mark**	$\frac{1}{2}$ **mark**

Three answers looked for:

$x>-4$ $\quad x \neq -3$ $\quad x \neq 0$ **or** $-4<x<-3$

1 mark for any one of these $\qquad -3<x<0$

$\frac{1}{2}$ **mark** for each of the other two $\qquad x>0$

END OF KEY