AUGUST 1995 MATHEMATICS 12 PROVINCIAL EXAMINATION
ANSWER KEY / SCORING GUIDE

ITEM CLASSIFICATION

TOPICS
1. Trigonometry
2. Quadratic Relations
3. Exponential and Logarithmic Functions
4. Polynomial Functions
5. Sequences and Series
6. Introduction to Calculus
7. Geometry
8. Problem Solving

PART A: MULTIPLE-CHOICE

Q	C	T	K	S	ILO
1.	K	2	C	1	12.13
2.	K	2	C	1	12.17
3.	U	2	D	1	12.17
4.	U	2	D	1	12.17
5.	U	2	A	1	12.14
6.	U	2	C	1	12.18
7.	U	2	D	1	12.15
8.	U	2	D	1	12.19
9.	H	2	A	1	12.20
10.	H	2	C	1	12.18
11.	K	1	C	1	12.05
12.	K	1	B	1	12.05
13.	U	1	D	1	12.02
14.	U	1	B	1	12.03
15.	U	1	C	1	12.07
16.	U	1	D	1	12.07
17.	H	1	C	1	12.04
18.	H	1	A	1	12.02
19.	K	3	D	1	12.28
20.	U	3	B	1	12.32
21.	K	3	B	1	12.26
22.	U	3	A	1	12.25
23.	U	3	B	1	12.30
24.	H	3	B	1	12.31
25.	H	3	B	1	12.31

Q	C	T	K	S	ILO
26.	K	4	D	1	12.38
27.	U	4	A	1	12.34
28.	U	4	B	1	12.42
29.	U	4	A	1	12.39
30.	U	4	C	1	12.40
31.	U	4	B	1	12.40
32.	U	4	D	1	12.37
33.	H	4	A	1	12.43/12.12
34.	K	5	B	1	12.46
35.	U	5	D	1	12.46
36.	U	5	D	1	12.46
37.	U	5	A	1	12.47
38.	H	5	A	1	12.45
39.	U	6	C	1	12.50
40.	U	6	A	1	12.52
41.	U	6	C	1	12.51
42.	K	6	B	1	12.56
43.	U	6	B	1	12.49
44.	H	6	A	1	12.58
45.	H	6	C	1	12.61
46.	H	7	B	1	12.63
47.	H	7	D	1	12.63
48.	H	8	C	1	12.64
49.	H	8	C	1	12.64
50.	U	8	A	1	12.64

PART B: WRITTEN-RESPONSE

Q	B	C	T	S	ILO
1.	1	U	5	2	12.46
2.	2	U	3	3	12.32
3a.	3	U	6	2	12.61
3b.	4	U	6	1	12.58

Q	B	C	T	S	ILO
4.	5	U	2	3	12.22
5.	6	U	1	3	12.08
6.	7	U	8	2	12.64
7.	8	H	7	4	12.63

Multiple-choice = 50 (50 questions)
Written-response = 20 (7 questions)
Total = 70 marks

LEGEND:
Q = Question
K = Keyed response
B = Score box number
C = Cognitive level
S = Score
T = Topic
ILO = Intended Learning Outcome

PART B: WRITTEN-RESPONSE

1. Determine the sum of the arithmetic series $-6-3+0+\ldots+621$. **(2 marks)**

Response:

$t_n = a+(n-1)d$ $\qquad$ $S_n = \frac{n}{2}(a+\ell)$

½ mark → $621 = -6+(n-1)3$

$627 = (n-1)3$ $\qquad$ $S_n = \frac{210}{2}(-6+621)$ **← ½ mark**

$209 = n-1$

½ mark → $210 = n$ $\qquad$ $S_n = 64\ 575$ **← ½ mark**

2. Solve for x: $\log_2(3x+2)+\log_2(x-1)=1$	**(3 marks)**

Response:

$\log_2(3x+2)(x-1)=1$ ← $\frac{1}{2}$ **mark**

$(3x+2)(x-1)=2$ ← **1 mark**

$3x^2-x-2=2$

$3x^2-x-4=0$ ← $\frac{1}{2}$ **mark**

$(3x-4)(x+1)=0$

$x=\frac{4}{3}$ soln. **or** ~~$x=-1$~~ reject

↑ $\frac{1}{2}$ **mark** ↑ $\frac{1}{2}$ **mark**

3. Given the function $f(x) = 2x^3 - 3x^2 - 12x + 4$,
a) determine the coordinates of the critical points of $f(x)$. **(2 marks)**

Response:

$2x^3 - 3x^2 - 12x + 4$

$6x^2 - 6x - 12 = 0$ ← **1 mark**

$x^2 - x - 2 = 0$

$(x-2)(x-1) = 0$

$x = 2$ **or** $x = -1$ ← **$\frac{1}{2}$ mark**

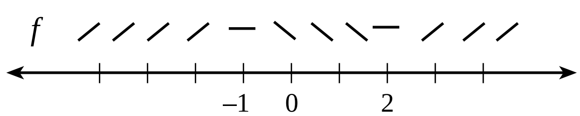
$(2,\ -16)$ $(-1,\ 11)$ ← **$\frac{1}{2}$ mark**

b) determine where $f(x)$ is increasing. **(1 mark)**

Response:

increasing

$x^2 - x - 2 > 0$



← **$\frac{1}{2}$ mark**

f' + + + + + + + 0 – – – – – – 0 + + + + + + +

$x > 2$ **or** $x < -1$ ← **$\frac{1}{2}$ mark**

4. Determine all real ordered pairs that satisfy the following system: **(3 marks)**

$$y^2 - x^2 = 16$$

$$y = \frac{6}{x}$$

(Give answers that are exact **or** accurate to 2 decimal places.)

Response:

$\boxed{1}$ $y^2 - x^2 = 16$

$\boxed{2}$ $y = \frac{6}{x}$

$\boxed{1}$ $\left(\frac{6}{x}\right)^2 - x^2 = 16$ ← **½ mark**

$$\frac{36}{x^2} - x^2 = 16$$

$$x^2\left[\frac{36}{x^2} - x^2\right] = [16]x^2$$

$$36 - x^4 = 16x^2$$

$x^4 + 16x^2 - 36 = 0$ ← **½ mark**

$(x^2 - 2)(x^2 + 18) = 0$ ← **½ mark**

$x^2 = 2$ **or** $\underbrace{x^2 = -18}_{\text{no real roots}}$

$x = \pm\sqrt{2}$ ← **1 mark**

$y = \frac{6}{\sqrt{2}}$ **or** $3\sqrt{2}$

or $y = \frac{6}{-\sqrt{2}}$ **or** $-3\sqrt{2}$ ← **½ mark**

Answer: $\left(\sqrt{2},\ 3\sqrt{2}\right)$ $\left(-\sqrt{2},\ -3\sqrt{2}\right)$

or

$\left(\sqrt{2},\ \frac{6}{\sqrt{2}}\right)$ $\left(-\sqrt{2},\ \frac{-6}{\sqrt{2}}\right)$ **or** $(1.41,\ 4.24)$ $(-1.41,\ -4.24)$

5. Prove the identity. **(3 marks)**

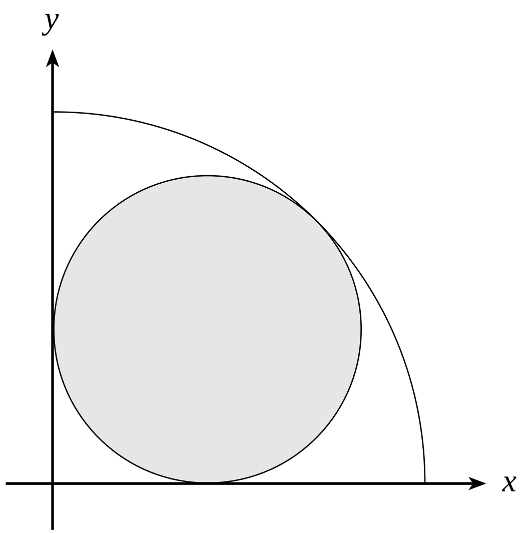
$$\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$$

Response:

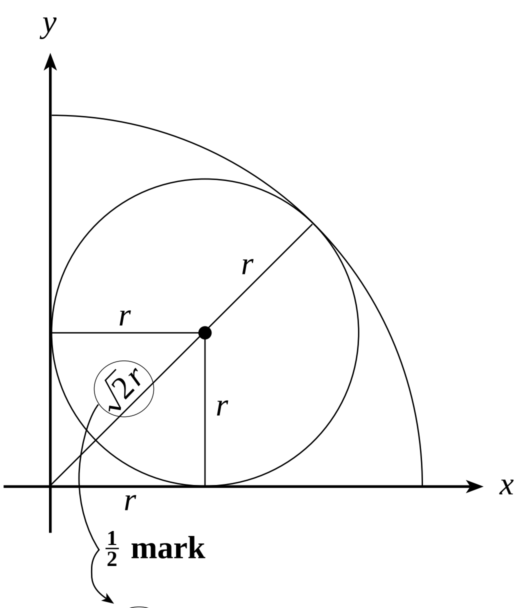
Left side	Right side
$\tan^2\theta - \sin^2\theta$	$\tan^2\theta\sin^2\theta$
$\frac{1}{2}$ **mark** $\rightarrow$ $\left\{ \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta \right.$	
$\frac{1}{2}$ **mark** $\rightarrow$ $\left\{ \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta\,\frac{\cos^2\theta}{\cos^2\theta} \right.$	
$\frac{1}{2}$ **mark** $\rightarrow$ $\left\{ \frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta} \right.$	
$\frac{1}{2}$ **mark** $\rightarrow$ $\left\{ \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta} \right.$	
$\frac{1}{2}$ **mark** $\rightarrow$ $\left\{ \frac{\sin^2\theta\sin^2\theta}{\cos^2\theta} \right.$	
$\frac{1}{2}$ **mark** $\rightarrow$ $\tan^2\theta\sin^2\theta$	

LS = RS

6. A circle is inscribed in the quadrant I sector of $x^2 + y^2 = 121$. Determine the area of the inscribed circle that is shaded. (Accurate to 1 decimal place.) **(2 marks)**



Response:



$\sqrt{2}r + r = 11$ ← **½ mark**

$r = \dfrac{11}{\sqrt{2}+1} \doteq 4.556$ ← **½ mark**

$A = \pi r^2$

$= \pi\left(\dfrac{11}{\sqrt{2}+1}\right)^2$

$= 65.2 \text{ units}^2$ ← **½ mark**

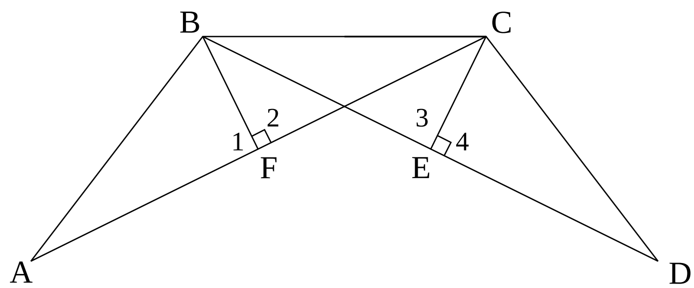
7. Complete the proof. **(4 marks)**

Given: BF is the perpendicular bisector of AC

CE is the perpendicular bisector of BD

Prove: AB = CD

Note: Students are encouraged to number angles.



Response:

Proof

	Statement	Reason
$1\frac{1}{2}$ marks →	BF is $\perp$ bisector of AC	given
	AF = CF	definition of bisect
	$\angle 1 = 90°$, $\angle 2 = 90°$	definition of $\perp$
	$\angle 1 = \angle 2$	substitution
	BF = BF	same side
	$\Delta AFB \cong \Delta CFB$	SAS
$\frac{1}{2}$ mark →	AB = CB	CPCTC
$1\frac{1}{2}$ marks →	CE is $\perp$ bisector of BD	given
	EB = ED	definition of bisect
	$\angle 3 = 90°$, $\angle 4 = 90°$	definition of $\perp$
	$\angle 3 = \angle 4$	substitution
	CE = CE	same side
	$\Delta BCE \cong \Delta DCE$	SAS
$\frac{1}{2}$ mark →	BC = CD	CPCTC
	AB = CD	substitution (both = CB)

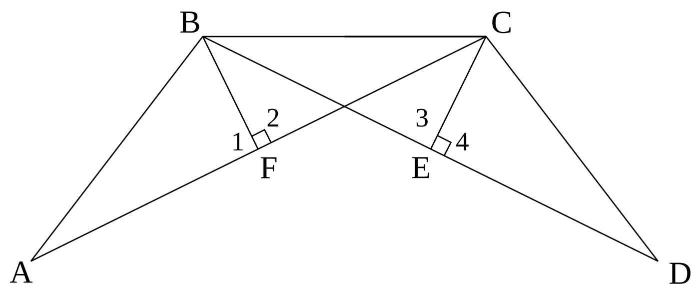
7. Complete the proof. **(4 marks)**

Given: BF is the perpendicular bisector of AC

CE is the perpendicular bisector of BD

Prove: AB = CD

Note: Students are encouraged to number angles.



Note: Students who are familiar with the theorem "Any point on the ⊥ bisector of a segment is equidistant from the endpoints of the segment" could do the following proof.

Alternate Response:

Proof

	Statement	Reason
3 marks →	BF is ⊥ bisector of AC	given
	CE is ⊥ bisector of BD	given
	AB = BC	point on the ⊥ bisector is = distance from endpoints
	BC = CD	point on the ⊥ bisector is = distance from endpoints
1 mark →	AB = CD	substitution (transitivity)

END OF KEY