

**JUNE 1995 MATHEMATICS 12 PROVINCIAL EXAMINATION
ANSWER KEY / SCORING GUIDE**

ITEM CLASSIFICATION

- TOPICS**
1. Trigonometry
 2. Quadratic Relations
 3. Exponential and Logarithmic Functions
 4. Polynomial Functions
 5. Sequences and Series
 6. Introduction to Calculus
 7. Geometry
 8. Problem Solving

PART A: MULTIPLE-CHOICE

Q	C	T	K	S	ILO	Q	C	T	K	S	ILO
1.	K	2	D	1	12.14	26.	K	4	A	1	12.40
2.	K	2	A	1	12.17	27.	K	4	C	1	12.40
3.	U	2	B	1	12.11	28.	U	4	C	1	12.38
4.	U	2	C	1	12.18	29.	U	4	D	1	12.37
5.	U	2	D	1	12.17	30.	U	4	D	1	12.35
6.	U	2	D	1	12.15	31.	U	4	A	1	12.40
7.	U	2	A	1	12.15	32.	U	4	C	1	12.43
8.	U	2	C	1	12.16	33.	H	4	A	1	12.36
9.	H	2	A	1	12.16	34.	K	5	C	1	12.46
10.	H	2	C	1	12.20	35.	U	5	C	1	12.46
11.	U	1	C	1	12.01	36.	U	5	B	1	12.47
12.	K	1	B	1	12.02	37.	U	5	A	1	12.46
13.	U	1	C	1	12.05	38.	H	5	C	1	12.46
14.	U	1	D	1	12.08	39.	K	6	C	1	12.57
15.	U	1	C	1	12.09	40.	U	6	B	1	12.51
16.	U	1	C	1	12.09	41.	U	6	B	1	12.59
17.	H	1	D	1	12.06	42.	U	6	D	1	12.58
18.	H	1	B	1	12.07	43.	U	6	A	1	12.53
19.	K	3	D	1	12.28	44.	U	6	C	1	12.51
20.	U	3	A	1	12.31	45.	H	6	D	1	12.56
21.	U	3	B	1	12.30	46.	H	7	B	1	12.63/12.64
22.	U	3	D	1	12.24	47.	U	7	B	1	12.63/12.64
23.	U	3	D	1	12.29	48.	U	8	A	1	12.64
24.	H	3	A	1	12.31	49.	U	8	B	1	12.64
25.	H	3	B	1	12.30	50.	U	8	B	1	12.64

PART B: WRITTEN-RESPONSE

Q	B	C	T	S	ILO	Q	B	C	T	S	ILO
1.	1	U	2	3	12.19	5.	5	U	1	2	12.08
2.	2	U	3	3	12.32	6.	6	U	6	3	12.60
3.	3	H	7	4	12.63	7.	7	U	8	2	12.64
4.	4	U	5	3	12.48						

Multiple-choice = 50 (50 questions)

Written-response = 20 (7 questions)

Total = 70 marks

LEGEND:

Q = Question

K = Keyed response

B = Score box number

C = Cognitive level

S = Score

T = Topic

ILO = Intended Learning Outcome

PART B: WRITTEN-RESPONSE

1. The sum of the squares of two positive numbers is 914. Find the two numbers if the difference of their squares is 336. **(3 marks)**

Response:

$$\frac{1}{2} \text{ mark} \rightarrow x^2 + y^2 = 914 \qquad y^2 - x^2 = 336 \leftarrow \frac{1}{2} \text{ mark}$$

$$2y^2 = 1250 \leftarrow \mathbf{1 \text{ mark}}$$

$$y^2 = 625$$

$$y = 25 \leftarrow \frac{1}{2} \text{ mark}$$

$$x = 17 \leftarrow \frac{1}{2} \text{ mark}$$

\therefore The numbers are 25 and 17

2. Solve: $\log_4(7 - 3x) + \log_4(x + 4) = 2$

(3 marks)

Response:

$$\log_4(7 - 3x) + \log_4(x + 4) = 2$$

$$\log_4(7 - 3x)(x + 4) = 2 \quad \leftarrow \mathbf{1 \text{ mark}}$$

$$(7 - 3x)(x + 4) = 4^2 \quad \leftarrow \mathbf{1 \text{ mark}}$$

$$7x + 28 - 3x^2 - 12x = 16$$

$$3x^2 + 5x - 12 = 0$$

$$(3x - 4)(x + 3) = 0$$

$$x = \frac{4}{3} \quad \mathbf{or} \quad x = -3$$

↑

$\frac{1}{2}$ mark

↑

$\frac{1}{2}$ mark

$\frac{1}{2}$ mark deduction if values are rejected

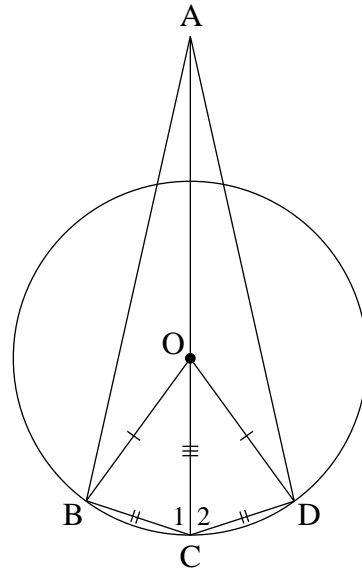
3. Complete the proof.

(4 marks)

Given: Circle with centre at O
 $BC = CD$

Prove: $AB = AD$

Note: Students are encouraged to number angles.



Response:

Statement	Proof	Reason
Circle with centre at O		given
$BO = DO$		= radii
$BC = DC$		given
$OC = OC$		same side
$\triangle BOC \cong \triangle DOC$		SSS
$\angle 1 = \angle 2$		CPCTC
$AC = AC$		same side
$\triangle ABC \cong \triangle ADC$		SAS
$AB = AD$		CPCTC

← 2 marks

← 2 marks

3. Complete the proof.

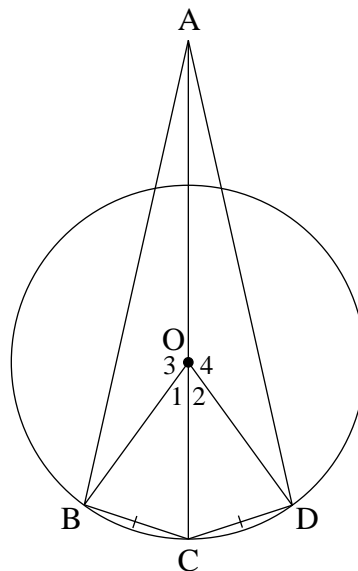
(4 marks)

Given: Circle with centre at O

$$BC = CD$$

Prove: $AB = AD$

Note: Students are encouraged to number angles.



Alternate Response:

Statement	Proof	Reason
Circle with centre at O	given	} ← 2 marks
$BC = CD$	given	
$\angle 1 = \angle 2$	central \angle s on = chords are =	
$\angle 3 = \angle 4$	supplements of = \angle s are =	
$BO = DO$	= radii	} ← 2 marks
$AO = AO$	same side	
$\triangle AOB \cong \triangle AOD$	SAS	
$AB = AD$	CPCTC	

4. Find the sum of all multiples of 6 between 100 and 1 000.

(3 marks)

Response:

$$102 + 108 + 114 + \dots + 996 \quad \leftarrow \begin{cases} \frac{1}{2} \text{ mark for } 102 \\ \frac{1}{2} \text{ mark for } 996 \end{cases}$$

$$t_n = a + (n-1)d$$

$$102 + (n-1)6 = 996$$

$$n = 150$$

← 1 mark

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_{150} = \frac{150}{2}(102 + 996) = 82\,350$$

$\underbrace{\hspace{1.5cm}}_{\frac{1}{2} \text{ mark}} \quad \underbrace{\hspace{1.5cm}}_{\frac{1}{2} \text{ mark}}$

5. Prove the identity.

(2 marks)

$$\frac{1}{1 + \sin \theta} = \sec^2 \theta - \frac{\tan \theta}{\cos \theta}$$

Response:

Left Side	Right Side
$\frac{1}{1 + \sin \theta}$	$\sec^2 \theta - \frac{\tan \theta}{\cos \theta}$
$= \frac{1}{1 + \sin \theta} \frac{(1 - \sin \theta)}{(1 - \sin \theta)}$	$= \frac{1}{\cos^2 \theta} - \frac{\frac{\sin \theta}{\cos \theta}}{\cos \theta}$
$\frac{1}{2} \text{ mark} \rightarrow = \frac{1 - \sin \theta}{1 - \sin^2 \theta}$	$= \frac{1}{\cos^2 \theta} - \frac{\sin \theta}{\cos^2 \theta}$ ← $\frac{1}{2} \text{ mark}$ $\left\{ \begin{array}{l} \text{for 2 correct substitutions} \\ \text{leading to a useful result} \end{array} \right.$
$\frac{1}{2} \text{ mark} \rightarrow = \frac{1 - \sin \theta}{\cos^2 \theta}$	$= \frac{1 - \sin \theta}{\cos^2 \theta}$ ← $\frac{1}{2} \text{ mark}$ $\left\{ \begin{array}{l} \text{if correct with a} \\ \text{common denominator} \end{array} \right.$
LS = RS	

5. Prove the identity.

(2 marks)

$$\frac{1}{1 + \sin \theta} = \sec^2 \theta - \frac{\tan \theta}{\cos \theta}$$

Alternate Response:

Left Side	Right Side
$\frac{1}{1 + \sin \theta}$	$\sec^2 \theta - \frac{\tan \theta}{\cos \theta}$
	$= \frac{1}{\cos^2 \theta} - \frac{\sin \theta}{\cos \theta}$
	$= \frac{1}{\cos^2 \theta} - \frac{\sin \theta}{\cos^2 \theta}$ ← $\frac{1}{2}$ mark { for correct substitution leading to a useful result
	$= \frac{1 - \sin \theta}{\cos^2 \theta}$ ← $\frac{1}{2}$ mark { if correct with a common denominator
	$= \frac{1 - \sin \theta}{1 - \sin^2 \theta}$ ← $\frac{1}{2}$ mark
	$= \frac{\cancel{(1 - \sin \theta)}}{(1 + \sin \theta)\cancel{(1 - \sin \theta)}}$ ← $\frac{1}{2}$ mark
	$= \frac{1}{1 + \sin \theta}$

LS = RS

Factoring step not required, but if shown must reduce correctly

6. The distance D travelled by an aircraft along a runway before takeoff is given by $D = \frac{3}{5}t^2 + 8t$, where D is measured in metres from the start of the takeoff run, and t is measured in seconds. If the aircraft becomes airborne when its velocity reaches 44 m/s, what distance (in metres) will it travel along the runway before becoming airborne? **(3 marks)**

Response:

$$D = \frac{3}{5}t^2 + 8t$$

$$V = \frac{6}{5}t + 8 \quad \leftarrow \mathbf{1 \text{ mark}} \left(\frac{1}{2} \text{ for velocity} = D', \frac{1}{2} \text{ for correct derivative} \right)$$

$$\frac{6}{5}t + 8 = 44 \quad \leftarrow \mathbf{\frac{1}{2} \text{ mark}}$$

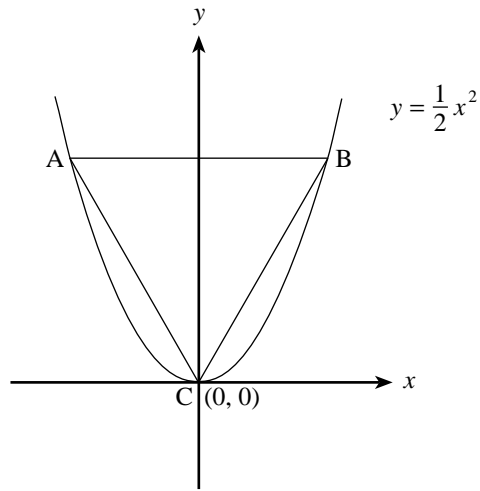
$$\frac{6}{5}t = 36$$

$$t = 30 \text{ s} \quad \leftarrow \mathbf{\frac{1}{2} \text{ mark}}$$

$$D = \frac{3}{5}(30)^2 + 8(30) \quad \leftarrow \mathbf{\frac{1}{2} \text{ mark}}$$

$$D = 780 \text{ m} \quad \leftarrow \mathbf{\frac{1}{2} \text{ mark}}$$

7. Points A, B, and C are on the parabola $y = \frac{1}{2}x^2$ as shown in the diagram. If $\triangle ABC$ is equilateral, determine the x -coordinate of point B. (Give answers that are exact **or** accurate to 2 decimal places.) **(2 marks)**



Response:

Let the coordinates of point B be $\left(x, \frac{1}{2}x^2\right)$ $\leftarrow \frac{1}{2}$ mark

Length of each side of triangle = $2x$ $\leftarrow \frac{1}{2}$ mark

$$\sqrt{(x-0)^2 + \left(\frac{1}{2}x^2 - 0\right)^2} = 2x \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$x^2 + \frac{x^4}{4} = 4x^2$$

$$4x^2 + x^4 = 16x^2$$

$$x^4 - 12x^2 = 0$$

$$x^2(x^2 - 12) = 0$$

$$x = 0 \quad \text{or} \quad x = \sqrt{12} = 2\sqrt{3}$$

$$\therefore x\text{-coordinate of point B} = 2\sqrt{3} \quad (\text{or } \sqrt{12})$$

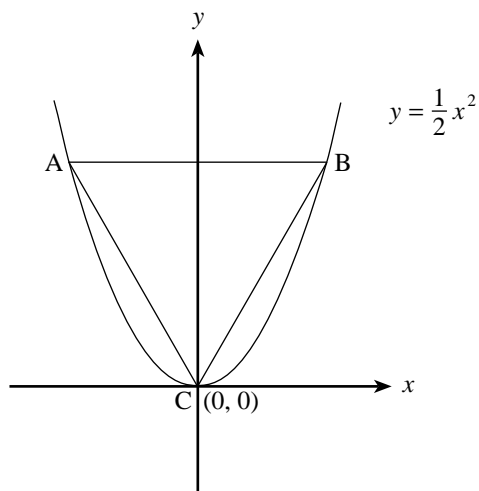
or

3.46

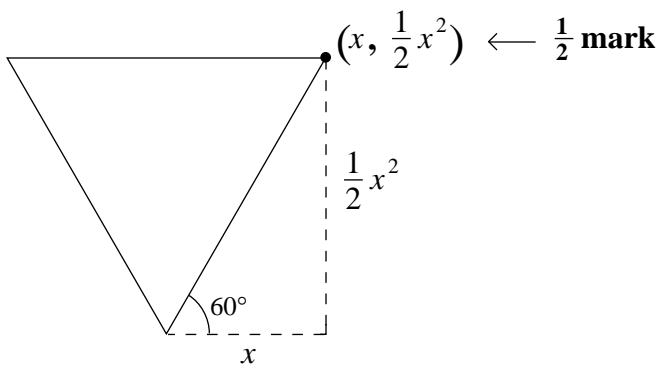
}

$\leftarrow \frac{1}{2}$ mark

7. Points A, B, and C are on the parabola $y = \frac{1}{2}x^2$ as shown in the diagram. If $\triangle ABC$ is equilateral, determine the x -coordinate of point B. (Give answers that are exact **or** accurate to 2 decimal places.) **(2 marks)**



Alternate Response:



$$\tan 60^\circ = \frac{\frac{1}{2}x^2}{x} = \frac{x}{2}$$

$$\begin{array}{cc} \uparrow & \uparrow \\ \frac{1}{2} \text{ mark} & \frac{1}{2} \text{ mark} \end{array}$$

$$\sqrt{3} = \frac{x}{2}$$

$$x = 2\sqrt{3} \quad \leftarrow \frac{1}{2} \text{ mark}$$

END OF KEY