JANUARY 1995 MATHEMATICS 12 PROVINCIAL EXAMINATION
ANSWER KEY / SCORING GUIDE

ITEM CLASSIFICATION

TOPICS

1. Trigonometry
2. Quadratic Relations
3. Exponential and Logarithmic Functions
4. Polynomial Functions
5. Sequences and Series
6. Introduction to Calculus
7. Geometry
8. Problem Solving

PART A: MULTIPLE-CHOICE

Q	C	T	K	S	ILO
1.	U	2	D	1	12.13
2.	U	2	B	1	12.14
3.	K	2	A	1	12.17
4.	U	2	D	1	12.11
5.	U	2	A	1	12.17
6.	U	2	D	1	12.15
7.	U	2	C	1	12.18
8.	U	2	A	1	12.22
9.	H	2	C	1	12.20
10.	H	2	D	1	12.16
11.	U	1	B	1	12.01
12.	U	1	A	1	12.02
13.	K	1	C	1	12.06
14.	U	1	D	1	12.03
15.	U	1	A	1	12.06
16.	U	1	D	1	12.03
17.	H	1	A	1	12.08
18.	H	1	B	1	12.06
19.	K	3	D	1	12.29
20.	U	3	C	1	12.26
21.	U	3	D	1	12.30
22.	U	3	B	1	12.25
23.	U	3	B	1	12.31
24.	H	3	C	1	12.27
25.	H	3	C	1	12.28/12.31
26.	U	4	D	1	12.35
27.	U	4	B	1	12.37
28.	U	4	B	1	12.40
29.	K	4	D	1	12.38
30.	U	4	A	1	12.37
31.	U	4	A	1	12.37
32.	U	4	C	1	12.40
33.	U	4	A	1	12.43
34.	U	5	C	1	12.46
35.	U	5	D	1	12.46
36.	K	5	B	1	12.46
37.	U	5	D	1	12.45
38.	H	5	B	1	12.47
39.	K	6	D	1	12.56
40.	U	6	A	1	12.57
41.	U	6	C	1	12.50
42.	U	6	D	1	12.51
43.	U	6	B	1	12.61
44.	H	6	B	1	12.60
45.	H	6	A	1	12.58
46.	H	7	C	1	12.63
47.	U	7	C	1	12.63
48.	U	8	C	1	12.64
49.	U	8	C	1	12.64
50.	H	8	C	1	12.64

PART B: WRITTEN-RESPONSE

Q	B	C	T	S	ILO
1.	1	U	3	3	12.32
2.	2	U	5	3	12.46
3.	3	U	2	3	12.19
4.	4	U	1	2	12.08
5.	5	U	8	2	12.64
6.	6	H	7	4	12.63
7.	7	U	6	3	12.62

Multiple-choice = 50 (50 questions)
Written-response = 20 (7 questions)
Total = 70 marks

LEGEND:

Q = Question
K = Keyed response
B = Score box number
C = Cognitive level
S = Score
T = Topic
ILO = Intended Learning Outcome

PART B: WRITTEN-RESPONSE

1. Solve: $\log(10-3x) - 2\log x = 0$ **(3 marks)**

Response:

$\log(10-3x) - 2\log x = 0$

$\log(10-3x) - \log x^2 = 0 \quad \leftarrow \frac{1}{2}$ **mark**

$\log\dfrac{(10-3x)}{x^2} = 0 \quad \leftarrow$ **1 mark**

$\dfrac{10-3x}{x^2} = 10^0 \quad \leftarrow \frac{1}{2}$ **mark**

$$\left.\begin{array}{l} 10-3x=x^2 \\ x^2+3x-10=0 \\ (x+5)(x-2)=0 \end{array}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark (any line)}$$

$\log(10-3x) = 2\log x$

$\log(10-3x) = \log x^2 \quad \leftarrow$ **1 mark**

$10-3x = x^2 \quad \leftarrow$ **1 mark**

$$\left.\begin{array}{l} x^2+3x-10=0 \\ (x+5)(x-2)=0 \end{array}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$x = -5,\quad 2$ * $\quad \leftarrow \frac{1}{2}$ **mark**

* $x = -5$ reject **or** $x = 2$ soln. $\leftarrow \frac{1}{2}$ **mark** (for solve)

***Note:** $-\frac{1}{2}$ **mark** if no rejection

Generally:

1) if an equation **only** (with no log concepts demonstrated) is shown:
   - a) = 0 → Cap: 1 mark
   - b) = 1 → Cap: $1\frac{1}{2}$ marks

2) if at least one correct log concept is demonstrated and equation is shown:
   - a) = 0 → Cap: $1\frac{1}{2}$ marks
   - b) = 1 → Cap: 2 marks

- **Note:** In both 1) and 2), must solve and reject correctly to receive max.

3) Guess and check: if no explanation why only 2 is the answer → $1\frac{1}{2}$ marks

4) Answer only with check → 1 mark

5) Answer only → $\frac{1}{2}$ mark

6) Answer (correct) with erroneous work → 0 (eg. $10-3x-2x=0 \Rightarrow x=2$)

2. The first three terms of an arithmetic sequence are $x+4$, x^2+5 and $x+30$. Determine the values of the first three terms of all such sequences. **(3 marks)**

Response:

$\frac{(x+4)+(x+30)}{2}=x^2+5$ ← **1 mark**

$\frac{2x+34}{2}=x^2+5$

$x+17=x^2+5$

$0=x^2-x-12$ ← **½ mark**

or

$x+30-\left(x^2+5\right)=x^2+5-(x+4)$ ← **1 mark**

$x+30-x^2-5=x^2+5-x-4$

$0=2x^2-2x-24$

$0=x^2-x-12$ ← **½ mark**

$(x-4)(x+3)=0$

$x=4$ **or** $x=-3$ ← **½ mark** for both

sequences are: $x=4$: 8, 21, 34 ← **½ mark**

$x=-3$: 1, 14, 27 ← **½ mark**

Also:

$t_n=a+(n-1)d$

$x+30=(x+4)+2\left[x^2-x+1\right]$ ← **1 mark**

$x+30=x+4+2x^2-2x+2$

$$\left.\begin{aligned}&2x^2-2x-24=0\\&x^2-x-12=0\end{aligned}\right\}\leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$(x-4)(x+3)=0$

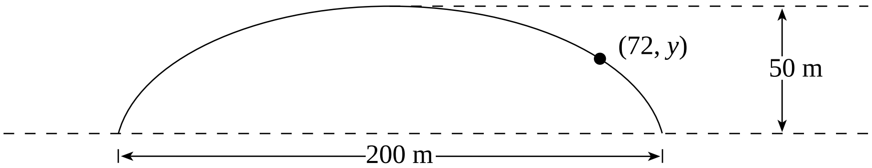
$x=4,\ -3$ ← **½ mark**

$x=4\ \rightarrow\ 8,\ 21,\ 34$ ← **½ mark**

$x=-3\ \rightarrow\ 1,\ 14,\ 27$ ← **½ mark**

3. A sports stadium has a semi-elliptical dome for its roof. If its maximum height is 50 m and its span is 200 m, how high is the dome at a point 72 m from the centre? (Accurate to 1 decimal place.)

(3 marks)



Response:

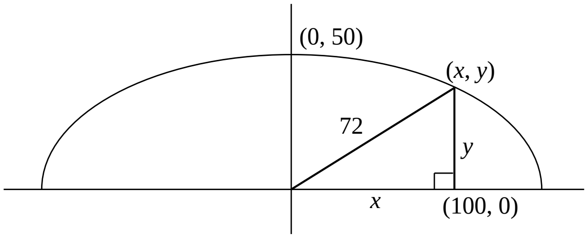
$$\frac{x^2}{100^2} + \frac{y^2}{50^2} = 1 \left\{ \begin{array}{l} \tfrac{1}{2}\ \textbf{mark}\text{ for standard form equation} \\ \qquad \left(\text{i.e. } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\right) \\ \tfrac{1}{2}\ \textbf{mark}\text{ for 100 plugged into a standard form} \\ \tfrac{1}{2}\ \textbf{mark}\text{ for 50 plugged into a standard form} \end{array}\right.$$

$$\frac{72^2}{100^2} + \frac{y^2}{50^2} = 1 \leftarrow \tfrac{1}{2}\ \textbf{mark}\text{ for } (72,\ y)$$

$$\left.\begin{aligned} y^2 &= \left(1 - \frac{72^2}{100^2}\right)50^2 \\ y^2 &= 1\,204 \end{aligned}\right\} \tfrac{1}{2}\ \textbf{mark}\text{ for solving for } y^2$$

$$y = \boxed{34.7\text{ m}} \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

Alternate Interpretation:



$$\frac{x^2}{100^2} + \frac{y^2}{50^2} = 1 \left\{ \begin{array}{l} \tfrac{1}{2}\ \textbf{mark} \text{ for standard form equation} \\ \tfrac{1}{2}\ \textbf{mark} \text{ for 100 plugged into a standard form} \\ \tfrac{1}{2}\ \textbf{mark} \text{ for 50 plugged into a standard form} \end{array} \right.$$

$$\left. \begin{array}{l} x^2 + y^2 = 72^2 \\ x^2 = 72^2 - y^2 \\ \dfrac{72^2 - y^2}{100^2} + \dfrac{y^2}{50^2} = 1 \end{array} \right\} \tfrac{1}{2}\ \textbf{mark} \text{ for linking } x,\ y \text{ and 72 correctly}$$

$$y^2 = \frac{4\,816}{3} \leftarrow \tfrac{1}{2}\ \textbf{mark} \text{ for solving for } y^2$$

$$y = \boxed{40.1\text{ m}} \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

4. Prove the identity. **(2 marks)**

$$\frac{\sec\theta-\cos\theta}{\tan\theta}=\sin\theta$$

Response:

Left side	Right side
$\frac{\sec\theta-\cos\theta}{\tan\theta}$	$\sin\theta$
$\frac{1}{2}$ **mark** → $\left\{ = \frac{\frac{1}{\cos\theta}-\cos\theta}{\frac{\sin\theta}{\cos\theta}} \right.$	In general: $\frac{1}{2}$ **mark** → replacing $\sec\theta+\tan\theta$ $\frac{1}{2}$ **mark** → **process** of eliminating complex fraction
$\frac{1}{2}$ **mark** → $\left\{ = \frac{\left(\frac{1}{\cos\theta}-\cos\theta\right)\cos\theta}{\left(\frac{\sin\theta}{\cos\theta}\right)\cos\theta} \right.$	$\frac{1}{2}$ **mark** → simplifying complex fraction $\frac{1}{2}$ **mark** → final substitution and simplification
$\frac{1}{2}$ **mark** → $= \frac{1-\cos^2\theta}{\sin\theta}$	
$\frac{1}{2}$ **mark** → $\left\{ \begin{array}{l} = \frac{\sin^2\theta}{\sin\theta} \\ = \sin\theta \end{array} \right.$	

LS = RS

4. Prove the identity. **(2 marks)**

$$\frac{\sec\theta - \cos\theta}{\tan\theta} = \sin\theta$$

Alternate Response:

	Left side	Right side
	$\dfrac{\sec\theta - \cos\theta}{\tan\theta}$	$\sin\theta$
$\frac{1}{2}$ **mark** →	$\left\{ = \dfrac{\dfrac{1}{\cos\theta} - \cos\theta}{\dfrac{\sin\theta}{\cos\theta}} \right.$	
$\frac{1}{2}$ **mark** →	$= \left(\dfrac{1}{\cos\theta} - \cos\theta\right)\left(\dfrac{\cos\theta}{\sin\theta}\right)$	
$\frac{1}{2}$ **mark** →	$= \dfrac{1}{\sin\theta} - \dfrac{\cos^2\theta}{\sin\theta}$	
	$= \dfrac{1 - \cos^2\theta}{\sin\theta}$	
$\frac{1}{2}$ **mark** →	$\left\{ \begin{array}{l} = \dfrac{\sin^2\theta}{\sin\theta} \\ = \sin\theta \end{array} \right.$	

LS = RS

5. Three numbers a, b and c exist such that $a+b=-4$, $a+c=25$ and $b+c=5$. Determine the value of 'a'. **(2 marks)**

Response:

$$a+b=-4, \quad a+c=25, \quad b+c=5$$

1 mark for process → $\{ \quad b=-4-a \qquad c=25-a$

$$\begin{aligned} b \quad + \quad c &= 5 \\ -4-a+25-a &= 5 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}\ \text{(equation in 1 variable)} \\ -2a+21 &= 5 \\ -2a &= -16 \\ a &= 8 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark} \end{aligned}$$

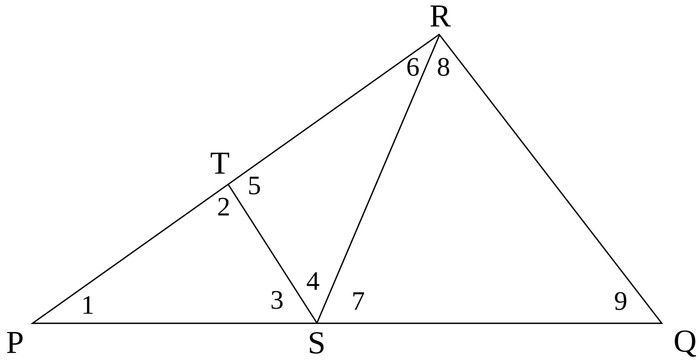
Alternate Response:

$$\begin{array}{c} \textbf{1 mark} \\ \uparrow \\ \text{process} \end{array} \left\{ \begin{array}{r} a+b=-4 \\ a+c=25 \\ \hline 2a+b+c=21 \end{array} \right. \qquad \text{since } b+c=5$$

$$\begin{aligned} \Rightarrow 2a+5 &= 21 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}\ \text{(equation in 1 variable)} \\ 2a &= 16 \\ a &= 8 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark} \end{aligned}$$

Note to markers: $1\frac{1}{2}$ marks to arrive at a solved equation in 1 variable (even if this variable is b or c).

6. Complete the proof. **(4 marks)**



Given: TS || RQ
TS bisects $\angle PSR$

Prove: RS = QS

Response:

Proof

	Statement	Reason
2 marks →	TS || RS	given
	$\angle 4 = \angle 8$	alternate interior $\angle$s are =
	TS bisects $\angle PSR$	given
	$\angle 3 = \angle 4$	definition of $\angle$ bisector
	$\angle 3 = \angle 8$	both = $\angle 4$
	$\angle 3 = \angle 9$	corresponding $\angle$s are =
2 marks →	$\angle 8 = \angle 9$ $\frac{1}{2}$	both = $\angle 3$ 1
	RS = QS	sides opposite = $\angle$s are = $\frac{1}{2}$

In general:

- $\frac{1}{2}$ **mark** for first 2 statements (from givens)
- **1 mark** to get the 3rd (however they can)
- $1\frac{1}{2}$ **marks** for line $\angle 8 = \angle 9$ substitution
- $\frac{1}{2}$ **mark** for last line

If line $\angle 8 = \angle 9$, corr $\angle$s, **or** $\angle 8 = \angle 4$, alt. int. $\angle$s is missing, then no flow Cap: 2 marks

If line $\angle 3 = \angle 4$ missing, but bisecting given is stated Cap: 3 marks

$-\frac{1}{2}$ **mark**	wrong reason missing last line wrong info, not needed no given
−1 **mark**	reason for $\angle 8 = \angle 9$ missing or wrong ($-\frac{1}{2}$ **mark** if partial explained)

Deduction scale:

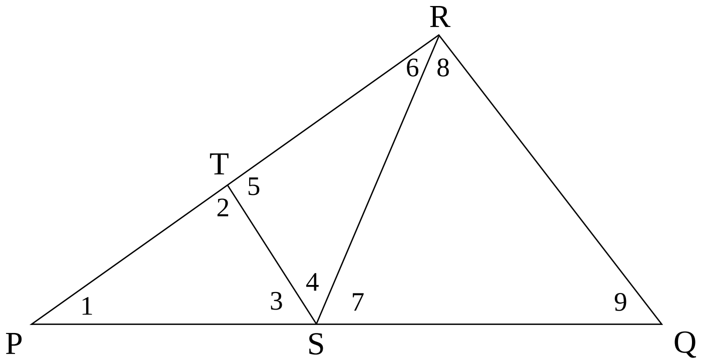
$\frac{1}{2}$ **mark**	→	deduct	0
1	→		$\frac{1}{2}$
$\mathbf{1\frac{1}{2} - 2}$	→		1
$\mathbf{2\frac{1}{2} - 4}$	→		$1\frac{1}{2}$

No deductions past point of giving marks

6. Complete the proof. **(4 marks)**

Given: TS || RQ
TS bisects ∠PSR

Prove: RS = QS



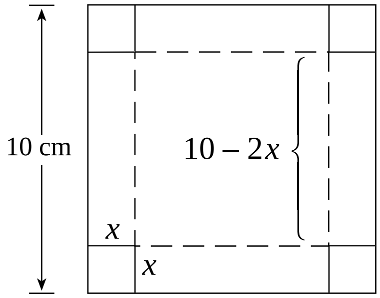
Alternate Response:

Proof

	Statement	Reason
	TS || RQ	given
	∠9 = ∠3	corresponding ∠s are =
2 marks →	∠8 = ∠4	alternate interior ∠s are =
	TS bisects ∠PSR	given
	∠3 = ∠4	definition of ∠ bisector
2 marks →	∠9 = ∠8 $\frac{1}{2}$	substitution 1
	PS = QS	sides opposite = ∠s are = $\frac{1}{2}$

7. A square piece of cardboard 10 cm by 10 cm will have equal squares with sides of length x cm cut from each corner. The sides will then be folded up to create a box with no top. Determine the value of x that will give the box a maximum volume. **(3 marks)**

Response:



$V = (10-2x)(10-2x)x$ ← **½ mark**

$V = \left(100-40x+4x^2\right)x$

$V = 4x^3-40x^2+100x$ ← **½ mark**

To maximize volume, $V' = 0$ ← **½ mark** for concept

$V' = 12x^2-80x+100 = 0$ ← **½ mark** for V'

$3x^2-20x+25 = 0$

$(3x-5)(x-5) = 0$

$x = 5$ reject **or** $x = \frac{5}{3}$ soln.

↑ ↑

½ mark **½ mark**

END OF KEY