JANUARY 1994 MATHEMATICS 12 PROVINCIAL EXAMINATION
ANSWER KEY / SCORING GUIDE

ITEM CLASSIFICATION

TOPICS:
1. Trigonometry
2. Quadratic Relations
3. Exponential and Logarithmic Functions
4. Polynomial Functions
5. Sequences and Series
6. Introduction to Calculus
7. Geometry
8. Problem Solving

PART A: MULTIPLE-CHOICE

Q	C	T	K	S	ILO
1.	K	2	A	1	12.17
2.	U	2	C	1	12.17
3.	U	2	C	1	12.13
4.	U	2	A	1	12.17
5.	U	2	C	1	12.18
6.	U	2	A	1	12.15
7.	U	2	C	1	12.11
8.	U	2	B	1	12.16
9.	H	2	D	1	12.16
10.	H	2	A	1	12.14
11.	K	1	B	1	12.05
12.	U	1	C	1	12.02
13.	K	1	B	1	12.04
14.	U	1	D	1	12.01
15.	U	1	A	1	12.07
16.	U	1	A	1	12.06
17.	H	1	D	1	12.07
18.	K	3	D	1	12.28
19.	U	3	B	1	12.29
20.	U	3	D	1	12.31
21.	U	3	B	1	12.26
22.	U	3	A	1	12.31
23.	U	3	A	1	12.24
24.	H	3	C	1	12.28
25.	K	4	C	1	12.38
26.	U	4	A	1	12.39
27.	U	4	D	1	12.37
28.	U	4	D	1	12.41
29.	U	4	D	1	12.36
30.	H	4	B	1	12.43
31.	K	5	D	1	12.46
32.	U	5	C	1	12.46
33.	U	5	B	1	12.46
34.	U	5	C	1	12.46
35.	U	5	C	1	12.47
36.	H	5	B	1	12.46
37.	U	5	A	1	12.46
38.	H	5	B	1	12.47
39.	K	6	C	1	12.57
40.	U	6	C	1	12.51
41.	K	6	D	1	12.59
42.	U	6	B	1	12.50
43.	U	6	B	1	12.55
44.	U	6	C	1	12.53
45.	H	6	D	1	12.60
46.	H	7	B	1	12.63
47.	U	7	A	1	12.63
48.	U	8	C	1	12.64
49.	H	8	D	1	12.64
50.	H	8	B	1	12.64

PART B: WRITTEN-RESPONSE

Q	B	C	T	S	ILO
1.	1	U	4	2	12.40
2.	2	U	1	3	12.03
3.	3	U	2	3	12.21
4.	4	H	7	4	12.63
5.	5	U	3	3	12.33
6.	6	U	8	3	12.64
7.	7	U	6	2	12.56

KEY:
Q = Question
B = Score box number
C = Cognitive level
T = Topic
S = Score
ILO = Intended Learning Outcome
K = Keyed response

1. Solve: $2x^3 - x^2 - 8x + 4 = 0$ **(2 marks)**

Solution:

$$2x^3 - x^2 - 8x + 4 = 0$$

$$\begin{array}{c|cccc} & 2 & -1 & -8 & +4 \\ \hline 2 & 2 & +3 & -2 & 0 \end{array}$$

← first root **1 mark**

if student recognizes a root **½ mark**

if they verify that root another **½ mark**

$$2x^2 + 3x - 2 = 0$$

$$(2x-1)(x+2) = 0$$

$x = \frac{1}{2}$ and $x = -2$ ← other two roots

↑ ↑

½ mark **½ mark**

∴ The roots are: $-2, \ \frac{1}{2}, \ 2$

AlternateSolution:

$$2x^3 - x^2 - 8x + 4 = 0$$

$x^2(2x-1) - 4(2x-1) = 0$ ← **½ mark**

$$\left.\begin{array}{l} (x^2-4)(2x-1) = 0 \\ (x+2)(x-2)(2x-1) = 0 \end{array}\right\}$$ **½ mark** for either step

$$(x^2-4)(2x-1) = 0$$

$$(x+2)(x-2)(2x-1) = 0$$

$x = -2, \quad 2, \quad \frac{1}{2}$ **1 mark for all 3 roots**

deduct **½ mark** for each missing or incorrect root

(to a maximum of **1 mark**)

2. Solve for x: $2\tan^2 x - 5\tan x - 3 = 0$, where $0 \le x < 2\pi$ (accurate to 2 decimal places)
(3 marks)

Solution:

$2\tan^2 x - 5\tan x - 3 = 0$

$(2\tan x + 1)(\tan x - 3) = 0 \leftarrow$ **½ mark**

$\tan x = -\frac{1}{2}$ $\tan x = 3 \leftarrow$ **½ mark**

$\text{ref} = 0.46$ $\text{ref} = 1.25$

$x = \{2.68, \quad 5.82, \quad 1.25, \quad 4.39\}$

½ mark for each of 4 values of *x*

•if substitution for tan x is not stated or is unclear

e.g. $(2x+1)(x-3) = 0 \quad \rightarrow \quad x = -\frac{1}{2}, \quad 3$

– ½ mark deduction

•if student takes tan .5 and tan 3 instead of inverse tan

– cap of 1 mark

or solving quadratic using quadratic formula

$$\tan x = \frac{5 \pm \sqrt{25 - 4(2)(-3)}}{4}$$ **½ mark**

$$= \frac{5 \pm \sqrt{49}}{4}$$

$$= \frac{5 \pm 7}{4}$$

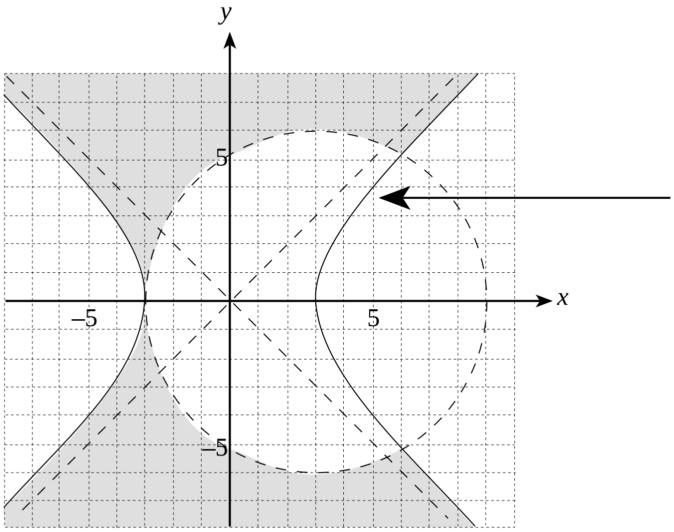
$$= -\frac{1}{2}, \quad 3$$ **½ mark**

3. Graph the following system of inequalities: **(3 marks)**

$$(x-3)^2 + y^2 > 36$$

$$x^2 - y^2 \le 9$$

Solution:

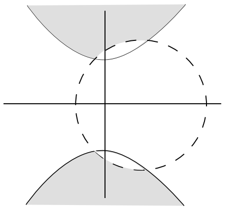


no deduction for solid boundary here (should be dotted, but this was ignored)

no deduction for solid asymptotes, nor for missing asymptotes provided the behaviour of the curve is correct – e.g. passes through $(5, \pm 4)$ $(-5, \pm 4)$

circle:	shift	½ **mark** (centre **and** radius)
	dotted	½ **mark**
	shading	½ **mark**
hyperbola:	shape	½ **mark**
	solid	½ **mark**
	shading	½ **mark**

Note: Deduct ½ **mark** if intersection of regions is not clear (cross-hatching of two regions is allowed).
Deduct ½ **mark** if dotted/solid is reversed.
Deduct ½ **mark** if asymptotic behaviour of hyperbola is poorly presented.
Deduct ½ **mark** if hyperbola opens vertically but shading (as determined by test point) is correct.
i.e.



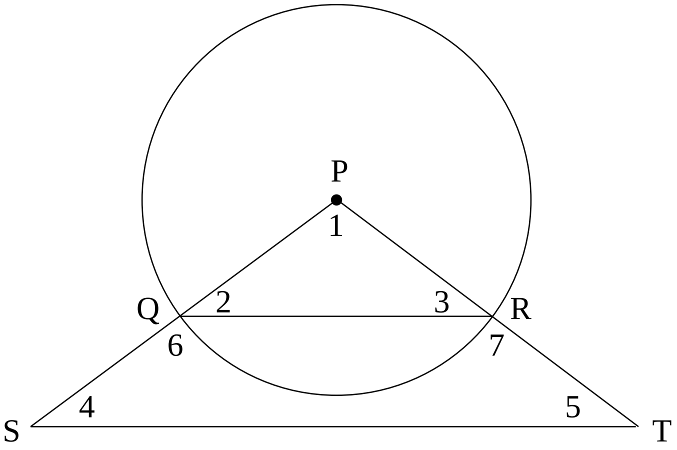
If either conic is incorrect but has been shaded appropriately, then cap of ½ **mark** for this conic.
e.g. $x^2 + y^2 \le 9$ instead of $x^2 - y^2 \le 9$

4. Complete the proof. **(4 marks)**

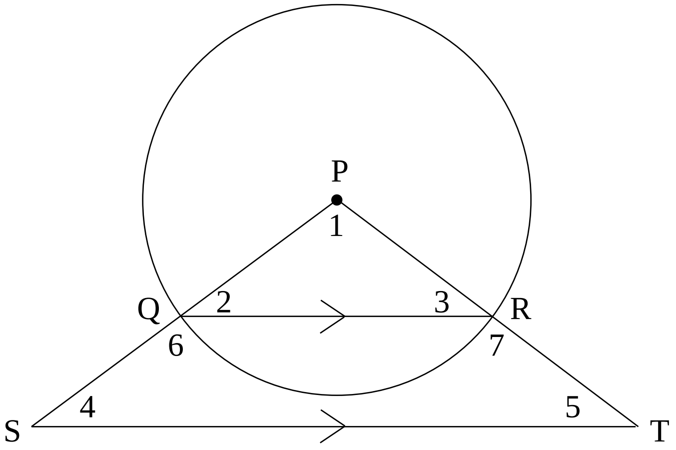
Given: P is the centre

QR || ST

Prove: PS = PT



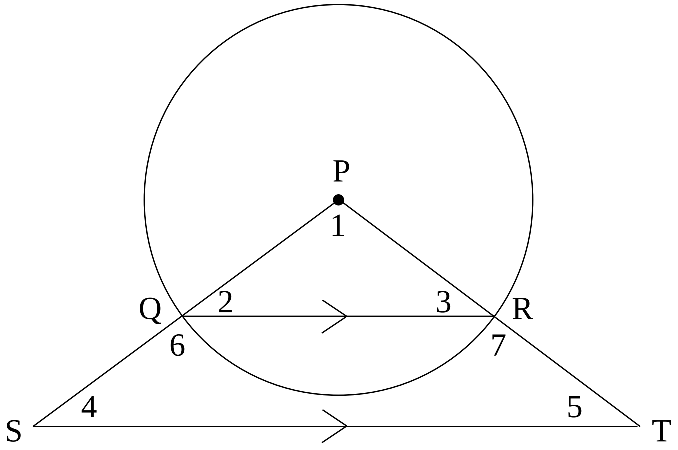
Solution:



Proof

Statement	Reason
P is the centre	given
QR || ST	given
$\angle 2 = \angle 4$	corr $\angle$s are = ← ½ **mark**
$\angle 3 = \angle 5$	corr $\angle$s are = ← ½ **mark**
PQ = PR	radii ← ½ **mark**
$\angle 2 = \angle 3$	$\angle$s opp = sides are = ← ½ **mark**
$\angle 4 = \angle 5$	$\angle$s = to = $\angle$s ← **1 mark**
PS = PT	sides opp = $\angle$s are = ← **1 mark**

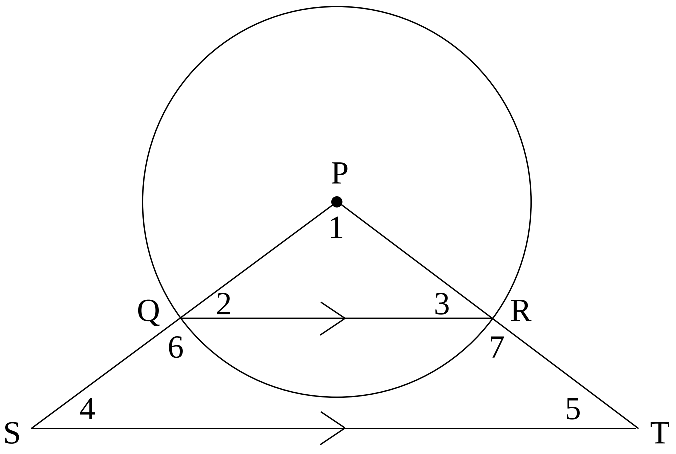
AlternateSolution1:



Proof

Statement	Reason	
P is the centre	given	
PQ = PR	radii	← ½ **mark**
$\angle 2 = \angle 3$	$\angle$s opp = sides are =	← ½ **mark**
$\angle 6 = \angle 7$	supplements of = $\angle$s are =	← ½ **mark**
QR ∣∣ ST	given	
$\angle 4$ and $\angle 6$ are supplementary	interior $\angle$s, same side of transversal	← ½ **mark**
$\angle 5$ and $\angle 7$ are supplementary	interior $\angle$s, same side of transversal	
$\angle 4 = \angle 5$	supplements of = $\angle$s are =	← **1 mark**
PS = PT	sides opp = $\angle$s are =	← **1 mark**

AlternateSolution2:



Proof

Statement	Reason	
$QR \parallel ST$	given	
$\angle 2 = \angle 4$	corr $\angle$s are =	← **½ mark**
$\angle 3 = \angle 5$	corr $\angle$s are =	← **½ mark**
$\angle 1 = \angle 1$	same angle	
$\Delta QPR \sim \Delta SPT$	AAA	← **½ mark**
P is the centre	given	
$PQ = PR$	radii	← **½ mark**
$\frac{PQ}{PS} = \frac{PR}{PT}$	sides of similar Δs are proportional	← **1 mark**
$PS = PT$	multiplication property of equality	← **1 mark**

5. A city has a population of 15 000 and the population decreases by 8% per year. How many years will it take for the population to become 5 000? (accurate to 1 decimal place) **(3 marks)**

Solution:

$$\left.\begin{aligned} 5\,000 &= 15\,000(0.92)^t \\ \frac{1}{3} &= (0.92)^t \end{aligned}\right\}$$

← **1 mark** for correct equation if order of operations followed correctly (otherwise ½ **mark** for line 1)

$$\log\frac{1}{3} = t\log(0.92) \quad \leftarrow \left\{ \begin{array}{l} \text{½ mark for logs both sides} \\ \text{½ mark for applying power rule} \end{array}\right.$$

$$t = \frac{\log\left(\frac{1}{3}\right)}{\log(0.92)}$$

← ½ **mark** for division of logs to calculator ready answer

$$t \doteq 13.2$$

← ½ **mark** for correct solution

AlternateSolution:

$$t_m = ar^{m-1}$$

$$\left.\begin{aligned} 5\,000 &= 15\,000(0.92)^{m-1} \\ \frac{1}{3} &= (0.92)^{m-1} \end{aligned}\right\}$$

← **1 mark** for correct equation if order of operations correct

$$\left.\begin{aligned} \log\frac{1}{3} &= (m-1)\log(0.92) \\ m-1 &= \frac{\log\left(\frac{1}{3}\right)}{\log(0.92)} \end{aligned}\right\}$$

← ½ **mark** for logs both sides

½ **mark** for applying power rule

$$\left.\begin{aligned} m &= \frac{\log\left(\frac{1}{3}\right)}{\log(0.92)} + 1 \\ m &\doteq 14.2 \end{aligned}\right\}$$

← ½ **mark** for calculator ready solution to 14.2

But, number of terms m exceeds number of years by 1.

$\therefore 13.2$ years ← ½ **mark** for adjusting to 13.2

6. Determine all values of x for which $\sqrt{4-x^2}$ is an integer. **(3 marks)**

Solution:

Case 1. $4-x^2=0$ ← ½ **mark**

$x^2=4$

$x=\pm 2$ ← ½ **mark**

Case 2. $4-x^2=4$ ← ½ **mark**

$x^2=0$

$x=0$ ← ½ **mark**

Case 3. $4-x^2=1$ ← ½ **mark**

$x^2=3$

$x=\pm\sqrt{3}$ ← ½ **mark**

If no solution given, marks will be awarded for consideration of **domain restrictions**:

$$\left.\begin{array}{l} 4-x^2 \geq 0 \\ (2-x)(2+x) \geq 0 \end{array}\right\} \leftarrow \text{ either step } \tfrac{1}{2} \textbf{ mark}$$

$-2 \leq x \leq 2$ or $-2,\ -1,\ 0,\ 1,\ 2$

↑ ½ **mark** if it follows from above; **1 mark** if alone

↑ ½ **mark** if no work shown

note: $-2 < x < 2$ ← ½ **mark**

If both correct and incorrect values given, a deduction of ½ **mark** per incorrect value to a maximum of **1 mark** deduction.

If no work shown, and correct answers given, a maximum of **2 marks** awarded.

7. Given $f(x) = x^2 + 5x$, use the **definition of the derivative** to show that $f'(x) = 2x + 5$.

(2 marks)

Solution:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 5(x+h) - (x^2 + 5x)}{h} \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 5x + 5h - x^2 - 5x}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h + 5)}{h} \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$= \lim_{h \to 0} (2x + h + 5)$$

$$= 2x + 5 \leftarrow \tfrac{1}{2} \textbf{ mark}$$

END OF KEY