Principles of Mathematics 12

April 2003 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		Sub-Organizers
1. Problem Solving	A	Problem Solving and Cross Topic Problems
2. Patterns and Relations	B	Geometric Sequences and Series
	C/D	Logarithms and Exponents
	C/D	Trigonometry
3. Shape and Space	E	Conics
	F	Transformations
4. Statistics and Probability	G	Combinatorics
	G	Probability
	G	Statistics

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	D	U	1.5	2	C4	23.	B	H	1.5	2	D3, C6, A2
2.	D	U	1.5	2	C8	24.	C	K	1.5	3	E2
3.	C	U	1.5	2	C3	25.	D	U	1.5	3	E2
4.	A	U	1.5	2	D6	26.	D	U	1.5	3	E3
5.	D	U	1.5	2	C5, A9	27.	D	H	1.5	3	E2
6.	D	U	1.5	2	C8	28.	B	K	1.5	3	F3
7.	D	U	1.5	2	C5	29.	A	U	1.5	3	F3
8.	C	U	1.5	2	D5	30.	A	U	1.5	3	F2
9.	B	H	1.5	2	D6	31.	A	U	1.5	3	F4
10.	B	H	1.5	2	C4, C6	32.	C	U	1.5	3	F5
11.	C	U	1.5	2	B1	33.	A	H	1.5	3	F6
12.	C	U	1.5	2	B1	34.	B	K	1.5	4	G7
13.	C	U	1.5	2	B1	35.	B	U	1.5	4	G5
14.	D	U	1.5	2	B1	36.	B	U	1.5	4	G5
15.	A	H	1.5	2	B3	37.	A	U	1.5	4	G11
16.	C	K	1.5	2	C2	38.	D	K	1.5	4	G8
17.	B	K	1.5	2	D3	39.	C	U	1.5	4	G13
18.	C	U	1.5	2	C2	40.	A	H	1.5	4	G12
19.	B	U	1.5	2	D1	41.	B	K	1.5	4	G8
20.	A	H	1.5	2	C1	42.	A	U	1.5	4	G2
21.	D	U	1.5	2	D1	43.	D	U	1.5	4	G1
22.	B	U	1.5	2	D4	44.	C	H	1.5	4	G3

Multiple Choice = 66 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1a.	1	U	2	3	F1
1b.	2	U	2	3	F2, F3
2.	3	U	4	3	E3
3.	4	U	5	2	C2
4.	5	U	4	4	G8
5.	6	U	4	4	G12
6a.	7	U	2	4	G2
6b.	8	U	2	4	G2
7a.	9	U	3	2	D7
7b.	10	U	1	2	D7
8.	11	H	5	2	C7

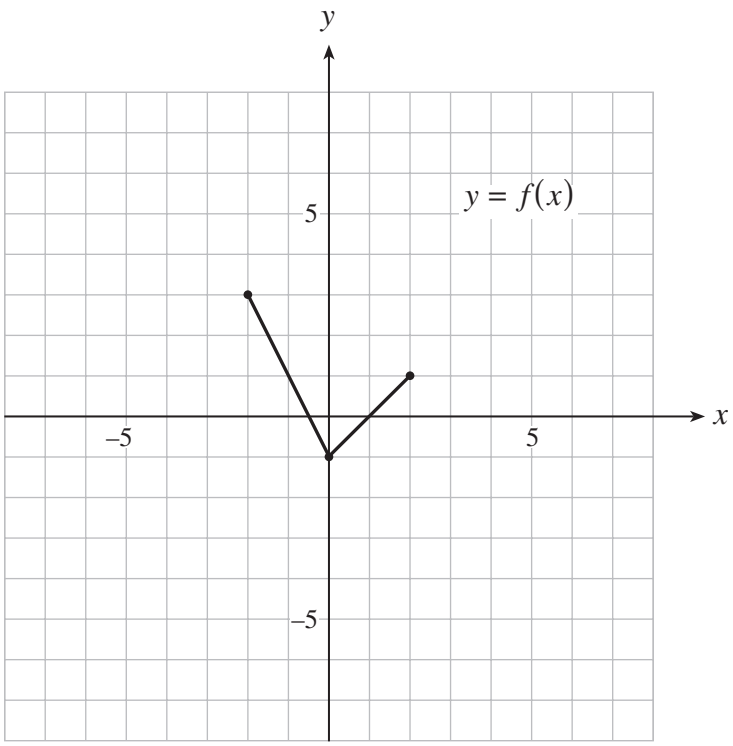
Written Response = 34 marks

Multiple Choice = 66 (44 questions)
Written Response = 34 (8 questions)
EXAMINATION TOTAL = 100 marks

LEGEND:

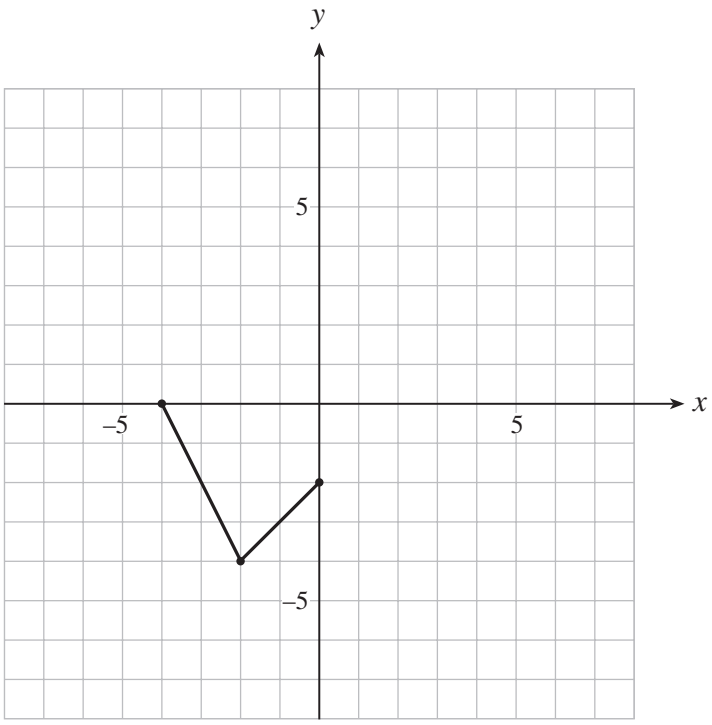
Q = Question Number
B = Score Box Number
PLO = Prescribed Learning Outcome
K = Keyed Response
S = Score
C = Cognitive Level
CO = Curriculum Organizer

1. The graph of $y = f(x)$ is shown below.



a) On the grid provided, sketch the graph of $y = f(x+2) - 3$. **(2 marks)**

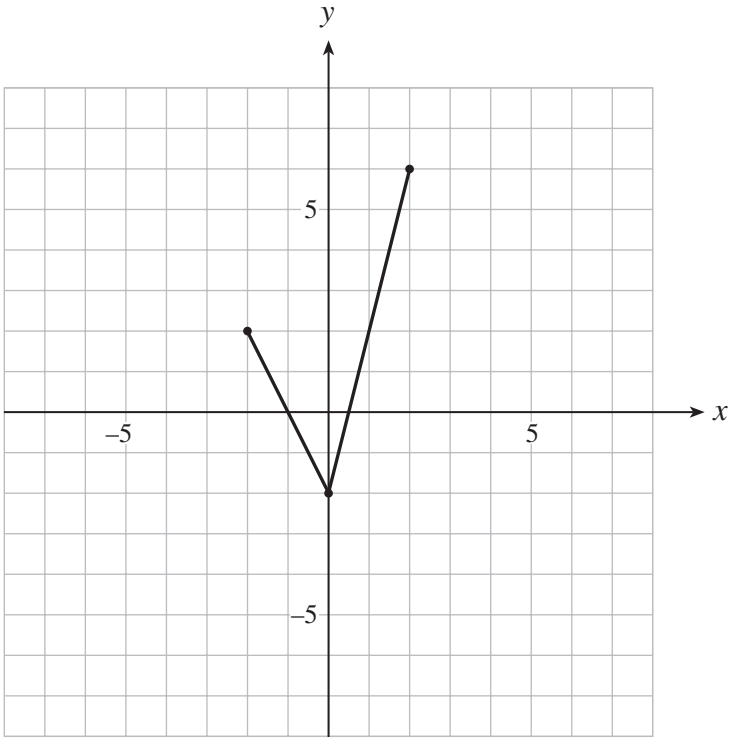
solution



1 mark for horizontal translation
1 mark for vertical translation

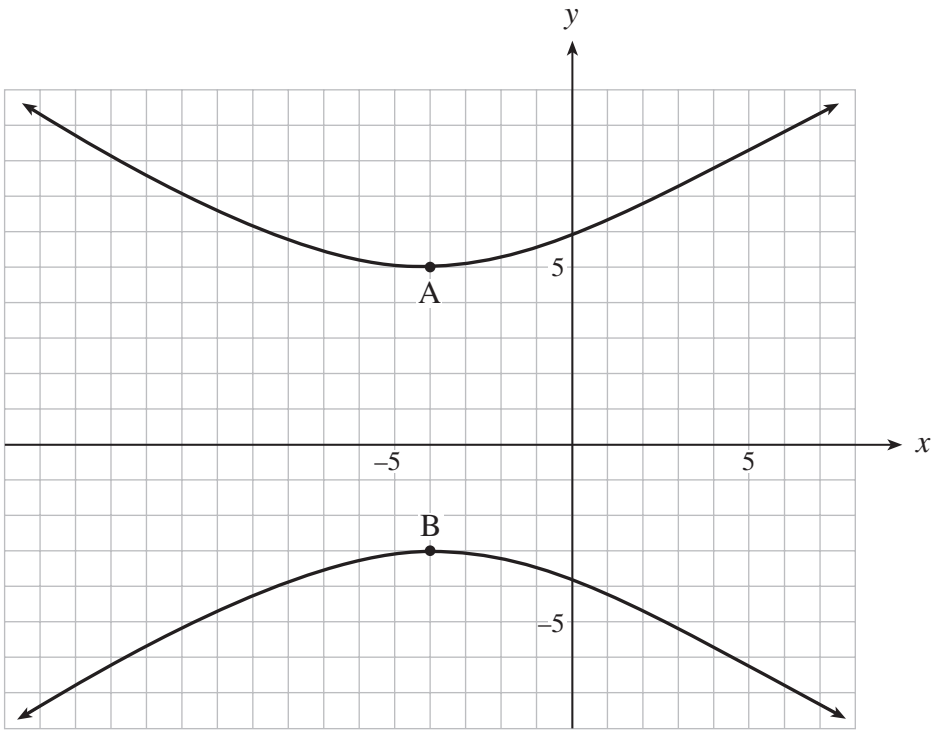
b) On the grid provided, sketch the graph of $y = 2f(-x)$. **(2 marks)**

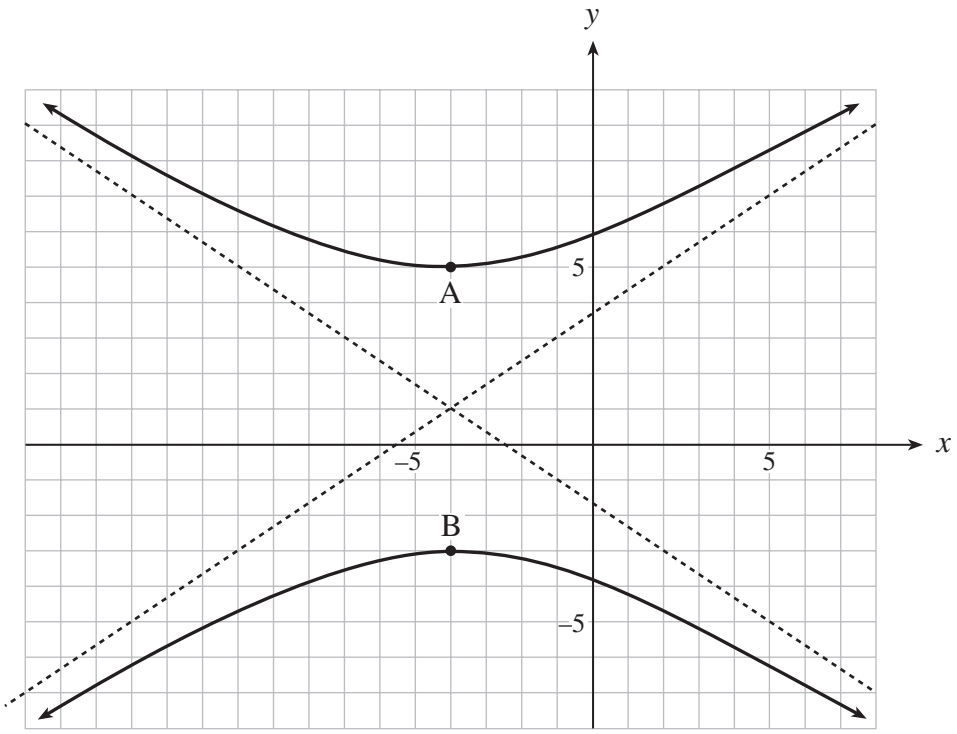
solution



1 mark for reflection in y-axis
1 mark for vertical expansion

2. Determine an equation of the hyperbola graphed below if the slopes of the asymptotes are $\pm\frac{2}{3}$ and the vertices are points A and B. **(4 marks)**





$\frac{1}{2}$ **mark** $\frac{1}{2}$ **mark**

$$\frac{(x+4)^2}{36} - \frac{(y-1)^2}{16} = -1$$ ← **1 mark** for equation format

1 mark **1 mark**

3. Solve algebraically: $\log_2 x = 3 - \log_2(x+2)$ **(5 marks)**

solution

$$\log_2 x = 3 - \log_2(x+2)$$

$$\log_2 x + \log_2(x+2) = 3 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$\log_2\left(x^2+2x\right) = 3 \qquad \leftarrow \textbf{1 mark}$$

$$x^2+2x = 8 \qquad \leftarrow \textbf{1 mark}$$

$$x^2+2x-8 = 0 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$(x+4)(x-2) = 0$$

$\tfrac{1}{2}$ **mark** $\rightarrow x = -4, \quad x = 2$

$\Downarrow$ $\qquad$ $\uparrow$

$\varnothing$ $\qquad$ $\tfrac{1}{2}$ **mark**

reject

$\uparrow$

1 mark

$\therefore\ x = 2$

alternate solution

$$\log_2 x = 3 - \log_2(x+2)$$

$$\log_2 x = \log_2 8 - \log_2(x+2) \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$\log_2 x = \log_2 \frac{8}{x+2} \qquad \leftarrow \textbf{1 mark}$$

$$x = \frac{8}{x+2} \qquad \leftarrow \textbf{1 mark}$$

$$x^2+2x = 8$$

$$x^2+2x-8 = 0 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$(x+4)(x-2) = 0$$

$\tfrac{1}{2}$ **mark** $\rightarrow x = -4, \quad x = 2$

$\Downarrow$ $\qquad$ $\uparrow$

$\varnothing$ $\qquad$ $\tfrac{1}{2}$ **mark**

reject

$\uparrow$

1 mark

$\therefore\ x = 2$

4. Determine the first 3 terms of the expansion: $(x-2y)^7$ **(4 marks)**

solution

$$(x-2y)^7 = {}_7C_0\, x^7(-2y)^0 + {}_7C_1\, x^6(-2y)^1 + {}_7C_2\, x^5(-2y)^2 + \ldots$$

$$= x^7 + 7x^6(-2y) + 21x^5(4y^2) + \ldots$$

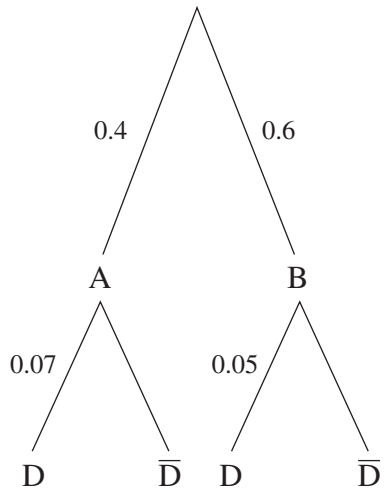
$$= x^7 - 14x^6y + 84x^5y^2 + \ldots$$

$\frac{1}{2}$ mark, $\frac{1}{2}$ mark, $\frac{1}{2}$ mark, $\frac{1}{2}$ mark, $\frac{1}{2}$ mark, $\frac{1}{2}$ mark, 1 mark

Note to markers: it is not necessary for students to write $+\ldots$

5. A building supply store buys 40% of its pine boards from sawmill A and 60% from sawmill B. Due to pine beetle infestation, 7% of the boards from sawmill A and 5% from sawmill B have a blue discoloration. If a randomly picked board is discoloured, what is the probability that it came from sawmill A? **(4 marks)**

solution



$P(\text{discoloured from A}) = (0.4)(0.07) = 0.028$ ← **1 mark**

$P(\text{discoloured from B}) = (0.6)(0.05) = 0.030$ ← **1 mark**

$$P(\text{discoloured}) = (0.4)(0.07) + (0.6)(0.05)$$
$$= 0.028 + 0.030$$
$$= 0.058$$ ← **1 mark**

$$P(\text{A}\,|\,\text{D}) = \frac{0.028}{0.058}$$ ← **1 mark**
$$= 0.4827586207$$
$$\approx 0.48$$

6. The masses of bags of tortilla chips produced in a factory are normally distributed with a mean mass of 340 g and a standard deviation of 5 g.

a) What proportion of the bags will contain between 334 g and 347 g? **(2 marks)**

solution

normalcdf(334, 347, 340, 5) ← **1 mark**

= 80.42%
or 0.8042
0.80
80%

← **1 mark**

b) The factory has an acceptable minimum mass per bag. All bags with a smaller mass are rejected. If 8% of the bags are rejected, calculate the acceptable minimum mass. **(2 marks)**

solution

invNorm(0.08, 340, 5) = 332.97 g

$\frac{1}{2}$ **mark** — $\frac{1}{2}$ **mark** — **1 mark**

7. A Ferris wheel has a radius of 25 m and its centre is 27 m above the ground. It rotates once every 40 seconds. Sandy gets on the Ferris wheel at its lowest point and then the wheel starts to rotate.

   a) Determine a sinusoidal equation that gives Sandy's height, h, above the ground as a function of the elapsed time, t, where h is in metres and t is in seconds. **(3 marks)**

solution

$\frac{1}{2}$ **mark** **1 mark** $\frac{1}{2}$ **mark**

$$h = -25\cos\frac{2\pi}{40}t + 27$$

1 mark

b) Determine the first time, t (in seconds), when Sandy will be 35 m above the ground. **(1 mark)**

solution

$$35 = -25\cos\frac{2\pi}{40}t + 27 \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$t = 12.07 \text{ s} \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

Note to markers: answer only is acceptable for full marks. No deduction for units missing.

8. Prove the identity: **(5 marks)**

$$\frac{\sin x}{1-\sin x} - \frac{\sin x}{1+\sin x} = 2\tan^2 x$$

solution

	LEFT SIDE	RIGHT SIDE
	$\frac{\sin x}{1-\sin x} - \frac{\sin x}{1+\sin x}$	$2\tan^2 x$
$\frac{1}{2}$ mark → **1 mark →**	$= \frac{\sin x(1+\sin x) - \sin x(1-\sin x)}{1-\sin^2 x}$	
1 mark → **1 mark →**	$= \frac{\sin x + \sin^2 x - \sin x + \sin^2 x}{\cos^2 x}$	
$\frac{1}{2}$ mark →	$= \frac{2\sin^2 x}{\cos^2 x}$	
1 mark →	$= 2\tan^2 x$	

LS = RS

END OF KEY