Principles of Mathematics 12

April 2002 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		Sub-Organizers
1. Problem Solving	A	Problem Set
2. Patterns and Relations	B	Geometric Sequences and Series
	C/D	Logarithms and Exponents
	C/D	Trigonometry
3. Shape and Space	E	Conics
	F	Transformations
4. Statistics and Probability	G	Combinatorics
	G	Probability
	G	Statistics

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	C	K	1.5	2	D6	23.	B	K	1.5	3	E2
2.	D	U	1.5	2	C3	24.	B	U	1.5	3	E2
3.	A	U	1.5	2	C4	25.	D	U	1.5	3	E2
4.	A	U	1.5	2	C6	26.	D	H	1.5	3	E2
5.	D	U	1.5	2	C8	27.	A	K	1.5	3	F3
6.	C	U	1.5	2	C5	28.	A	U	1.5	3	F2
7.	C	U	1.5	2	D6	29.	A	U	1.5	3	F3
8.	B	U	1.5	2	C7	30.	A	H	1.5	3	F5
9.	A	H	1.5	2	C8	31.	B	H	1.5	3	F6
10.	D	H	1.5	2	D7	32.	A	K	1.5	4	G6
11.	C	K	1.5	2	B1	33.	A	U	1.5	4	G8
12.	C	U	1.5	2	B1	34.	C	K	1.5	4	G11
13.	B	U	1.5	2	B1	35.	B	U	1.5	4	G13
14.	C	U	1.5	2	B1	36.	B	H	1.5	4	G12
15.	B	H	1.5	2	B1	37.	A	K	1.5	4	G1
16.	D	K	1.5	2	D2	38.	C	U	1.5	4	G1
17.	C	U	1.5	2	D3	39.	C	U	1.5	4	G2
18.	A	U	1.5	2	D4	40.	C	U	1.5	4	G2
19.	D	H	1.5	2	D1	41.	A	H	1.5	4	G3
20.	D	U	1.5	2	C2	42.	C	U	1.5	1	A1
21.	C	U	1.5	2	D2, D3	43.	A	U	1.5	1	A1
22.	D	H	1.5	2	C2, B1	44.	D	H	1.5	1	A1

Multiple Choice = 66 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1.	1	U	5	3	E3
2.	2	U	5	2	C2
3.	3	U	5	1	A1
4.	4	H	5	2	C7, C8
5.	5	U	4	4	G6, G7
6a.	6	U	2	3	F6
6b.	7	U	3	3	F6
7a.	8	U	2	4	G11, G12
7b.	9	U	3	4	G11, G12

Written Response = 34 marks

Multiple Choice = 66 (44 questions)
Written Response = 34 (7 questions)
EXAMINATION TOTAL = 100 marks

LEGEND:

Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
CO = Curriculum Organizer
PLO = Prescribed Learning Outcome

1. Change to standard form. **(5 marks)**

$$4x^2 - 9y^2 + 32x + 18y + 91 = 0$$

solution

$$4x^2 - 9y^2 + 32x + 18y + 91 = 0$$

$$\left(4x^2 + 32x\right) - \left(9y^2 - 18y\right) = -91$$

$\frac{1}{2}$ mark **$\frac{1}{2}$ mark each** **$\frac{1}{2}$ mark** **$\frac{1}{2}$ mark each**

$$4\left(x^2 + 8x + 16\right) - 9\left(y^2 - 2y + 1\right) = -91 + 64 - 9$$

$\frac{1}{2}$ mark **$\frac{1}{2}$ mark**

$$4(x+4)^2 - 9(y-1)^2 = -36$$

$$\frac{(x+4)^2}{9} - \frac{(y-1)^2}{4} = -1 \quad \leftarrow \textbf{1 mark}$$

2. Solve algebraically. **(5 marks)**

$$\log_2(2-2x)+\log_2(1-x)=5$$

solution

$\log_2(2-2x)+\log_2(1-x)=5$

$\log_2(2-2x)(1-x)=5$ ← **1 mark**

$(2-2x)(1-x)=32$ ← **1 mark**

$2-4x+2x^2=32$

$2x^2-4x-30=0$ ← **1 mark**

$x^2-2x-15=0$

$(x-5)(x+3)=0$

$x=5,\ -3$ ← **1 mark**

↑

reject

↑

1 mark

3. A biologist determines that a particular type of bacteria grows continuously according to the formula $P = P_0 e^{kt}$. Determine the value of the continuous growth rate if the population of the bacteria increases from 500 to 1500 in 8 days. **(5 marks)**

solution

$$1\,500 = 500e^{8k} \quad \leftarrow \textbf{1 mark}$$

$$3 = e^{8k} \quad \leftarrow \textbf{1 mark}$$

$$\ln 3 = \ln e^{8k} \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\ln 3 = 8k \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\frac{\ln 3}{8} = k \quad \leftarrow \textbf{1 mark}$$

$$\left.\begin{array}{l} k = 0.1373 \text{ or } k = 13.73\% \\ k = 0.14 \text{ or } k = 14\% \end{array}\right\} \leftarrow \textbf{1 mark}$$

alternate solution 1

$$1\,500 = 500e^{8k} \quad \leftarrow \textbf{1 mark}$$

$$3 = e^{8k} \quad \leftarrow \textbf{1 mark}$$

$$\log 3 = \log e^{8k} \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\log 3 = 8k \log e \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\frac{\log 3}{8 \log e} = k \quad \leftarrow \textbf{1 mark}$$

$$\left.\begin{array}{l} k = 0.1373 \text{ or } k = 13.73\% \\ k = 0.14 \text{ or } k = 14\% \end{array}\right\} \leftarrow \textbf{1 mark}$$

3. A biologist determines that a particular type of bacteria grows continuously according to the formula $P = P_0 e^{kt}$. Determine the value of the continuous growth rate if the population of the bacteria increases from 500 to 1500 in 8 days. **(5 marks)**

alternate solution 2

$1\,500 = 500e^{8k}$ ← **1 mark**

$3 = e^{8k}$ ← **1 mark**

$\ln 3 = 8k$ ← **1 mark**

$k = \frac{\ln 3}{8}$ ← **1 mark**

$k = 0.1373$ or $k = 13.73\%$
$k = 0.14$ or $k = 14\%$
} ← **1 mark**

4. Prove: **(5 marks)**

$$\frac{\sin 2x}{1+\cos 2x} = \frac{\sec^2 x - 1}{\tan x}$$

solution

LEFT SIDE	RIGHT SIDE
$\dfrac{\sin 2x}{1+\cos 2x}$	$\dfrac{\sec^2 x - 1}{\tan x}$
1 mark $\rightarrow$ $= \dfrac{\sin 2x}{1+\left(2\cos^2 x - 1\right)}$	$= \dfrac{\tan^2 x}{\tan x}$ $\leftarrow$ **1 mark**
1 mark $\rightarrow$ $= \dfrac{2\sin x \cos x}{2\cos^2 x}$	$= \tan x$ $\leftarrow$ $\frac{1}{2}$ **mark**
$\frac{1}{2}$ **mark** $\rightarrow$ $= \dfrac{\sin x}{\cos x}$	
$\frac{1}{2}$ **mark** $\rightarrow$ $= \tan x$	

LS = RS

4. Prove: **(5 marks)**

$$\frac{\sin 2x}{1+\cos 2x} = \frac{\sec^2 x - 1}{\tan x}$$

alternate solution 1

LEFT SIDE	RIGHT SIDE
$\frac{\sin 2x}{1+\cos 2x}$	$\frac{\sec^2 x - 1}{\tan x}$
1 mark $\rightarrow$ $= \frac{\sin 2x}{1+\left(2\cos^2 x - 1\right)}$	$= \left(\frac{1}{\cos^2 x} - 1\right)\left(\frac{\cos x}{\sin x}\right)$ $\leftarrow \frac{1}{2}$ **mark** if either is correct
1 mark $\rightarrow$ $= \frac{2\sin x \cos x}{2\cos^2 x}$ $\frac{1}{2}$ **mark** $\rightarrow$	$= \left(\frac{1-\cos^2 x}{\cos^2 x}\right)\left(\frac{\cos x}{\sin x}\right)$ $\leftarrow \frac{1}{2}$ **mark**
$\frac{1}{2}$ **mark** $\rightarrow$ $= \frac{\sin x}{\cos x}$	$= \frac{\sin^2 x}{\sin x \cos x}$
$\frac{1}{2}$ **mark** $\rightarrow$ $= \tan x$	$= \tan x$ $\leftarrow \frac{1}{2}$ **mark**

LS = RS

4. Prove: **(5 marks)**

$$\frac{\sin 2x}{1+\cos 2x} = \frac{\sec^2 x - 1}{\tan x}$$

alternate solution 2

LEFT SIDE	RIGHT SIDE
$\dfrac{\sin 2x}{1+\cos 2x}$	$\dfrac{\sec^2 x - 1}{\tan x}$
1 mark $\rightarrow$ $= \dfrac{\sin 2x}{1+\left(2\cos^2 x - 1\right)}$	$= \dfrac{\dfrac{1}{\cos^2 x} - 1}{\dfrac{\sin x}{\cos x}}$ $\leftarrow$ **½ mark**
1 mark $\rightarrow$ $= \dfrac{2\sin x \cos x}{2\cos^2 x}$ **½ mark** $\rightarrow$	$= \dfrac{\left(\dfrac{1}{\cos^2 x} - 1\right)}{\left(\dfrac{\sin x}{\cos x}\right)} \dfrac{\cos^2 x}{\cos^2 x}$
½ mark $\rightarrow$ $= \dfrac{\sin x}{\cos x}$	
½ mark $\rightarrow$ $= \tan x$	$= \dfrac{1-\cos^2 x}{\sin x \cos x}$ $\leftarrow$ **½ mark**
	$= \dfrac{\sin^2 x}{\sin x \cos x}$
	$= \tan x$ $\leftarrow$ **½ mark**

LS = RS

5. Solve algebraically: $\dfrac{(n-1)!}{(n-3)!} = 30$ **(4 marks)**

solution

$$\frac{(n-1)(n-2)(n-3)!}{(n-3)!} = 30 \qquad \leftarrow \textbf{1 mark}$$

$$n^2 - 3n + 2 = 30 \qquad \leftarrow \textbf{1 mark}$$

$$n^2 - 3n - 28 = 0$$

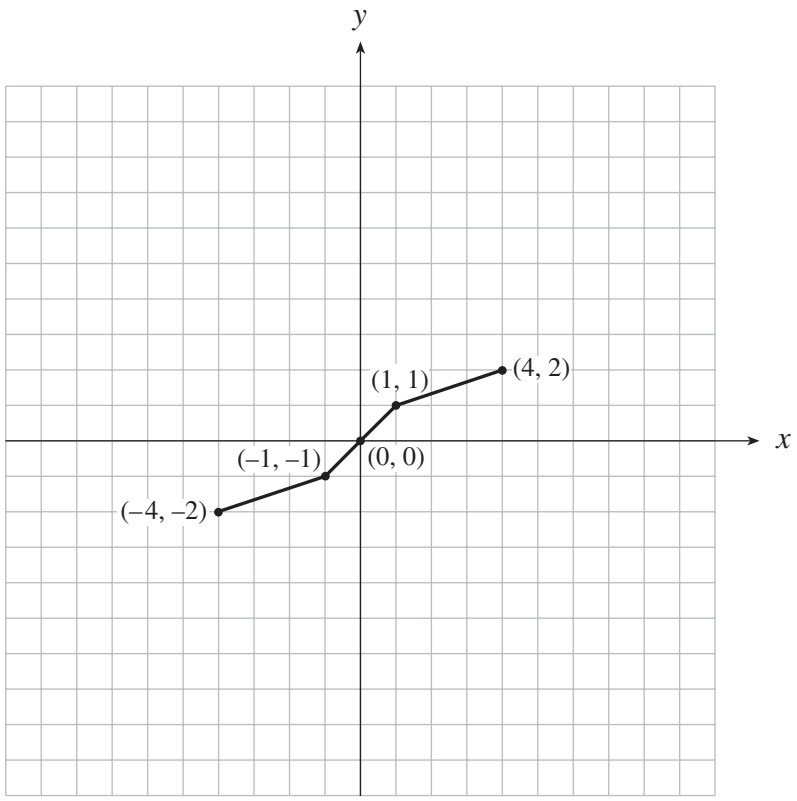
$$(n+4)(n-7) = 0$$

$$n = -4, \quad n = 7 \qquad \leftarrow \textbf{1 mark}$$

$$\downarrow$$

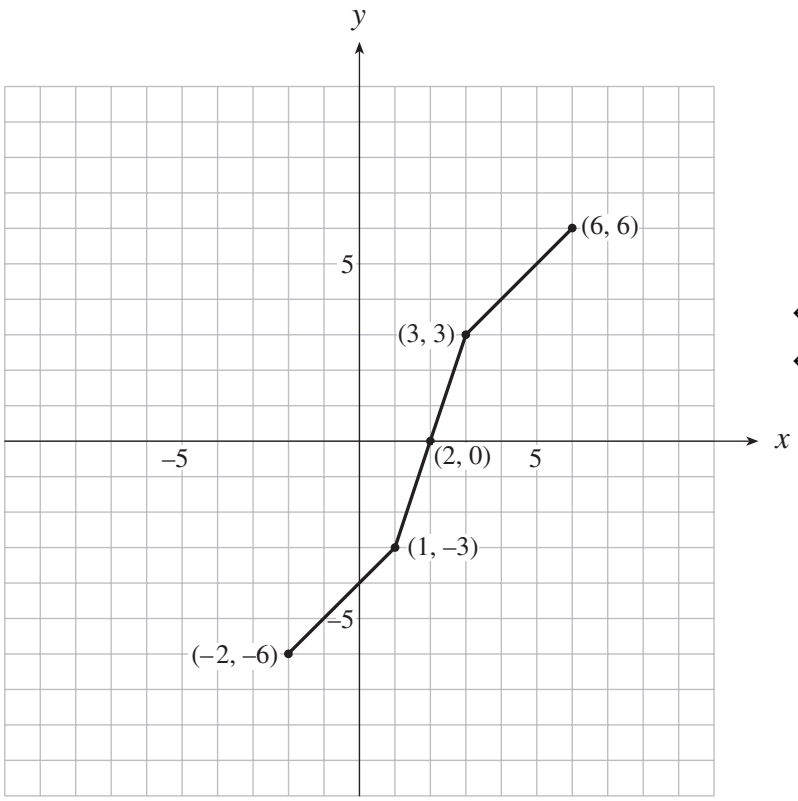
reject $\qquad \leftarrow \textbf{1 mark}$

6. The graph of the function $y = f(x)$ is shown below.



a) On the grid provided, sketch the graph of $y = 3f(x-2)$. **(2 marks)**

solution

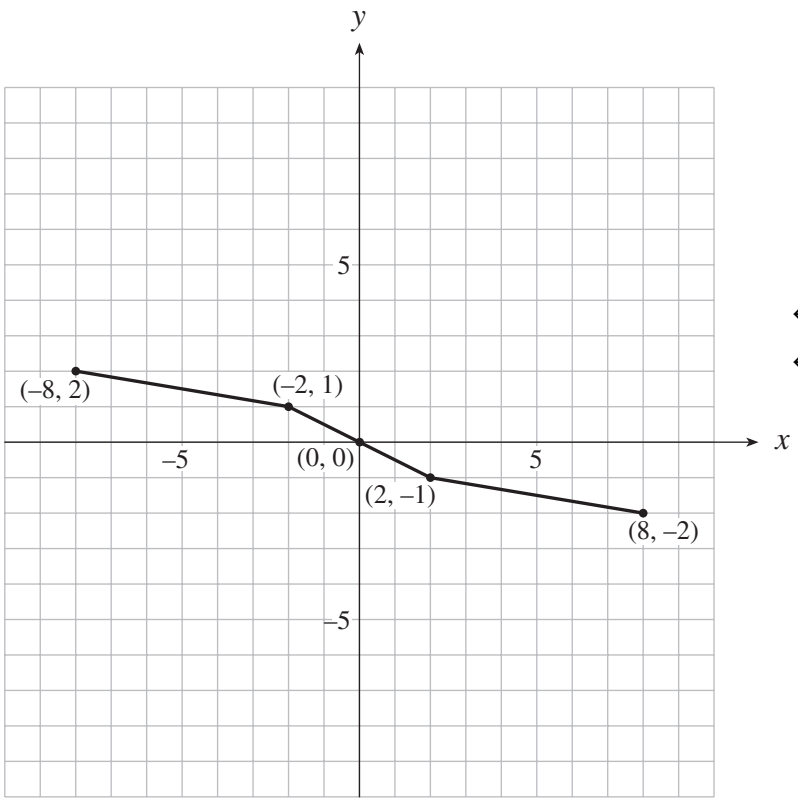


← **1 mark** for horizontal translation

← **1 mark** for vertical expansion

b) On the grid provided, sketch the graph of $y = -f\left(\frac{x}{2}\right)$. **(3 marks)**

solution



← **1 mark** for reflection

← **2 marks** for horizontal expansion

7. A hand of 5 cards is dealt from a standard deck of 52 cards.

   a) What is the probability that the hand contains 5 spades? (Answer accurate to at least 4 decimal places.) **(2 marks)**

solution

1 mark ↓

$$\frac{{}_{13}C_5}{{}_{52}C_5} = 0.0005$$

↑ $\frac{1}{2}$ **mark**

or

0.05%

or

4.95×10^{-4}

or

$$\frac{1\,287}{2\,598\,960}$$

} ← $\frac{1}{2}$ **mark**

alternate solution

$\frac{1}{2}$ **mark** → for first term

$$\underbrace{\frac{13}{52}\times\frac{12}{51}\times\frac{11}{50}\times\frac{10}{49}\times\frac{9}{48}}_{\text{1 mark}} = 0.0005$$

or

0.05%

or

4.95×10^{-4}

or

$$\frac{154\,440}{311\,875\,200}$$

} ← $\frac{1}{2}$ **mark**

b) What is the probability that the hand contains 2 hearts, 2 spades and 1 card that is not a heart or a spade? (Answer accurate to at least 4 decimal places.) **(3 marks)**

solution

$$\underbrace{\frac{\overbrace{\left({}_{13}C_2\right)\left({}_{13}C_2\right)\left({}_{26}C_1\right)}^{\frac{1}{2}\text{ mark each}}}{\left({}_{52}C_5\right)}}_{\frac{1}{2}\text{ mark}}$$

$$\underbrace{= 0.0609 \quad \textbf{or} \quad 6.09\% \quad \textbf{or} \quad \frac{158\,184}{2\,598\,960}}_{\textbf{1 mark}}$$

alternate solution

$$\underbrace{\left(\frac{13}{52}\times\frac{12}{51}\times\frac{13}{50}\times\frac{12}{49}\times\frac{26}{48}\right)}_{\textbf{1 mark}}\underbrace{\left(\frac{5!}{2!\ 2!\ 1!}\right)}_{\textbf{1 mark}}$$

$$\underbrace{= 0.0609 \quad \textbf{or} \quad 6.09\% \quad \textbf{or} \quad \frac{75\,928\,320}{1\,247\,500\,800}}_{\textbf{1 mark}}$$

END OF KEY