Principles of Mathematics 12

January 2002 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		**Sub-Organizers**
1. Problem Solving	A	Problem Set
2. Patterns and Relations	B	Geometric Sequences and Series
	C/D	Logarithms and Exponents
	C/D	Trigonometry
3. Shape and Space	E	Conics
	F	Transformations
4. Statistics and Probability	G	Combinatorics
	G	Probability
	G	Statistics

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	D	K	1.5	2	C3	23.	A	K	1.5	3	E2
2.	C	K	1.5	2	D6	24.	D	U	1.5	3	E1
3.	B	U	1.5	2	C4	25.	D	U	1.5	3	E1
4.	B	U	1.5	2	D5	26.	D	H	1.5	3	E2
5.	C	U	1.5	2	D6	27.	B	K	1.5	3	F1
6.	D	U	1.5	2	C8	28.	C	U	1.5	3	F3
7.	C	U	1.5	2	C7	29.	B	U	1.5	3	F2, F3
8.	B	H	1.5	2	C8	30.	D	H	1.5	3	F4
9.	A	U	1.5	2	C5	31.	A	H	1.5	3	F3
10.	A	H	1.5	2	D7	32.	A	K	1.5	4	G7
11.	C	K	1.5	2	B1	33.	C	U	1.5	4	G8
12.	A	U	1.5	2	B1, B2	34.	A	U	1.5	4	G6
13.	C	U	1.5	2	B1	35.	B	H	1.5	4	G5
14.	D	U	1.5	2	B3	36.	C	K	1.5	4	G11
15.	C	H	1.5	2	B3	37.	C	K	1.5	4	G11
16.	B	K	1.5	2	D2	38.	B	U	1.5	4	G12
17.	D	U	1.5	2	D3	39.	D	H	1.5	4	G13
18.	A	U	1.5	2	D1	40.	A	U	1.5	4	G1
19.	B	U	1.5	2	C1	41.	C	U	1.5	4	G2
20.	C	U	1.5	2	D4	42.	C	U	1.5	1	A1
21.	D	U	1.5	2	C2	43.	B	U	1.5	1	A1, A4
22.	B	H	1.5	2	C2	44.	D	H	1.5	1	A1

Multiple Choice = 66 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1	1	U	5	3	E3
2a.	2	U	1	4	G9, G11
2b.	3	U	1	4	G9, G11
2c.	4	U	1	4	G9, G11
2d.	5	U	1	4	G9, G11
2e.	6	U	1	4	G9, G11
3.	7	U	5	2	D1, D3
4.	8	U	5	2	C6
5a.	9	U	2	3	F1, F2, F3, F5, F6
5b.	10	U	2	3	F1, F2, F3, F5, F6
5c.	11	U	1	3	F1, F2, F3, F5, F6
6.	12	H	5	1	A1
7.	13	U	4	4	G3

Written Response = 34 marks

Multiple Choice = 66 (44 questions)
Written Response = 34 (7 questions)
EXAMINATION TOTAL = 100 marks

LEGEND:

Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
CO = Curriculum Organizer
PLO = Prescribed Learning Outcome

1. Change to standard form: $9x^2 - 16y^2 - 36x - 96y - 252 = 0$ **(5 marks)**

solution

$\left(9x^2 - 36x\right) - \left(16y^2 + 96y\right) = 252$ ← $\frac{1}{2}$ **mark** for gathering terms

$9\left(x^2 - 4x + \quad\right) - 16\left(y^2 + 6y + \quad\right) = 252$ ← **1 mark**

($\frac{1}{2}$ **mark** each for factoring out coefficients)

$$\underset{\uparrow}{9}\left(x^2 - 4x + 4\right) - 16\left(y^2 + 6y + 9\right) = 252 + \underset{\uparrow}{36} - \underset{\uparrow}{144}$$

complete squares and balance

1 mark each → ($\frac{1}{2}$ **mark** for each correct numeral; $\frac{1}{2}$ **mark** for each correct sign)

$9(x-2)^2 - 16(y+3)^2 = 144$ ← $\frac{1}{2}$ **mark** for both correctly factored

($-\frac{1}{2}$ **mark** for incorrect addition from previous line)

$$\frac{9(x-2)^2}{144} - \frac{16(y+3)^2}{144} = \frac{144}{144}$$

$\dfrac{(x-2)^2}{16} - \dfrac{(y+3)^2}{9} = 1$ ← **1 mark** for correct standard form and correct vertex

Deduct $\frac{1}{2}$ mark for not being in standard form

Cap at 4 marks for wrong y-coordinate of vertex (i.e. $y - 3$) → usually an error from line 2

Deduct $\frac{1}{2}$ mark if $(x+2)$ was taken to be a transcription error

Cap at $3\frac{1}{2}$ marks for a "correct" ellipse (i.e. from $+16y^2$)

$$\frac{(x-2)^2}{16} + \frac{(y-3)^2}{9} = 1$$

2. A tetrahedral die has four sides numbered 1, 2, 3 and 4. Two tetrahedral dice are rolled. The sample space is shown below.

2nd die

1st die

	1	**2**	**3**	**4**
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)

Determine the probability that:

a) the sum of the two dice is equal to 6. **(1 mark)**

solution

$\frac{3}{16}$ ← **1 mark**

b) the product of the two dice is a multiple of 3. **(1 mark)**

solution

$\frac{7}{16}$ ← **1 mark**

c) the number showing up on the first die is greater than the number showing up on the second die. **(1 mark)**

solution

$\frac{6}{16}$ ← **1 mark**

d) the sum of the two dice is equal to 6 or the product of the two dice is a multiple of 3. **(1 mark)**

solution

$\frac{9}{16}$ ← **1 mark**

Deduct $\frac{1}{2}$ mark for showing a) and b)

Full marks if a) and b) incorrect but the process in d) correct

e) the first die is a 4 given that the sum of the two dice is equal to 6. **(1 mark)**

solution

$\frac{1}{3}$ ← **1 mark**

3. Strontium-90 is a radioactive substance with a half-life of 28 days. How many days will it take for a 200 gram sample of strontium-90 to be reduced to 8 grams? (Solve algebraically using logarithms.)
(5 marks)

solution

$$\underbrace{\underset{\uparrow}{200}\overset{\tfrac{1}{2}\textbf{ mark}\downarrow}{(0.5)}^{\overset{\tfrac{1}{2}\textbf{ mark}\downarrow}{\frac{t}{28}}} = \underset{\uparrow}{8}}_{\tfrac{1}{2}\textbf{ mark}}$$

$$(0.5)^{\frac{t}{28}} = 0.04 \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$\log(0.5)^{\frac{t}{28}} = \log(0.04) \qquad \leftarrow \textbf{1 mark}$$

$$\frac{t}{28}\log(0.5) = \log(0.04) \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$t = \frac{28\log(0.04)}{\log(0.5)} \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$t = 130.03 \text{ days} \qquad \leftarrow \textbf{1 mark}$$

alternate solution

$$0.5 = e^{28k} \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$\ln 0.5 = 28k$$

$$\frac{\ln 0.5}{28} = k$$

$$8 = 200e^{\frac{\ln 0.5}{28}t} \qquad \leftarrow \textbf{1 mark}$$

$$0.04 = e^{\frac{\ln 0.5}{28}t} \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$\ln 0.04 = \ln e^{\frac{\ln 0.5}{28}t} \qquad \leftarrow \textbf{1 mark}$$

$$\ln 0.04 = \frac{\ln 0.5}{28}t \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$\frac{28\ln 0.04}{\ln 0.5} = t \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$130.03 \text{ days} = t \qquad \leftarrow \textbf{1 mark}v$$

Note: 130 or 131 days will also be accepted for full marks in this question.

4. Solve $2\cos^2 x + \cos x - 1 = 0$ algebraically over the set of real numbers. (Give the general solution using exact values.) **(5 marks)**

solution

$$2\cos^2 x + \cos x - 1 = 0$$

$(2\cos x - 1)(\cos x + 1) = 0$ or use quadratic formula

$\cos x = \frac{1}{2}$ $\leftarrow$ **1 mark** $\qquad$ $\cos x = -1$ $\leftarrow$ **1 mark**

$x = \frac{\pi}{3}$ $\leftarrow$ **$\frac{1}{2}$ mark** $\qquad$ $x = \pi$ $\leftarrow$ **$\frac{1}{2}$ mark**

$x = \frac{5\pi}{3}$ $\leftarrow$ **$\frac{1}{2}$ mark**

$\Big\}$ At this stage accept exact or decimal values

$x = 1.05,\ 3.14,\ 5.24$

$x = \frac{\pi}{3} + 2n\pi$ $\leftarrow$ **$\frac{1}{2}$ mark**

$x = \frac{5\pi}{3} + 2n\pi$ $\leftarrow$ **$\frac{1}{2}$ mark**

$\Big\}$ **or** $2n\pi \pm \frac{\pi}{3}$ $\leftarrow$ **1 mark**

$x = \pi + 2n\pi$ $\leftarrow$ **$\frac{1}{2}$ mark**

$= (2n+1)\pi$

$(n$ is any integer$)$

If generalizations wrong award **$\frac{1}{2}$ mark** for some attempt / knowledge of general solution.

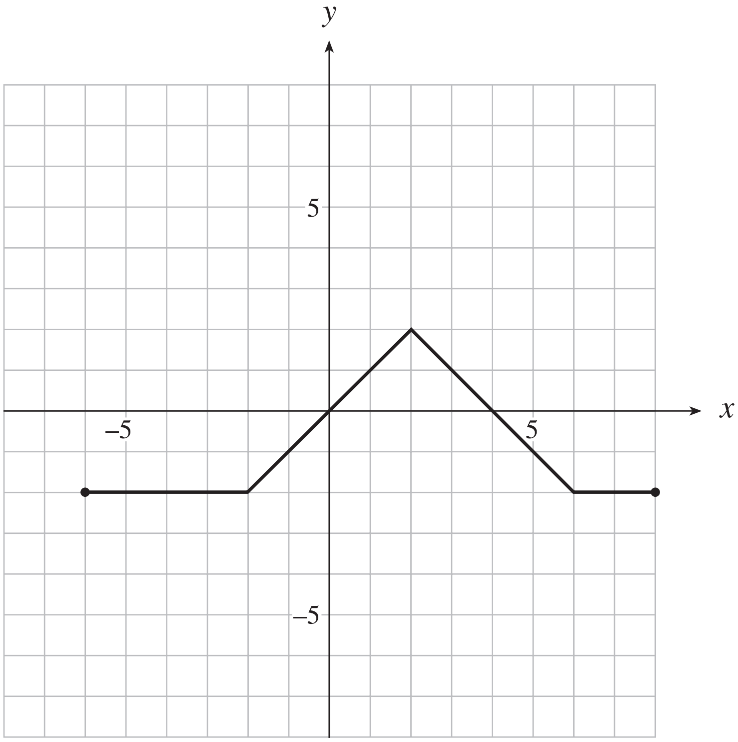
or $x = \frac{\pi}{3} + \frac{2\pi}{3}n$ $\leftarrow$ **$1\frac{1}{2}$ marks** $\qquad$ $x = (2n+1)\frac{\pi}{3}$ $\qquad$ Deduct **$\frac{1}{2}$ mark** for correct answers in **degrees**

Note: No deduction of marks if n is not specified as being an integer.

Note: If students provide a graphical solution, they may receive part marks if they provide a general solution in terms of n.

Cap at $4\frac{1}{2}$ marks	**Cap at 4 marks**	**Cap at 3 marks**	**Cap at $3\frac{1}{2}$ marks**
• Degrees instead of radians	• Not exact values but generalized correctly • Correct exact values incorrectly generalized • Wrong original factor signs $\rightarrow \frac{2\pi}{3}n$	• Correct solutions with no algebraic support	• Correct exact values but no generalization

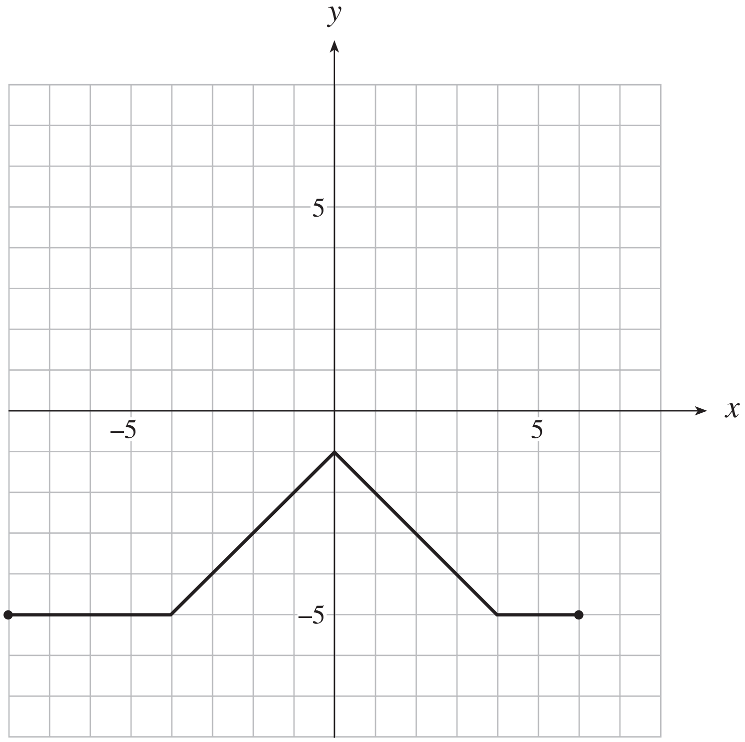
5. The graph of $y = f(x)$ is shown below.



On the grids provided, sketch the graphs of:

a) $y = f(x+2) - 3$ **(2 marks)**

solution



← **1 mark** for 2 left
← **1 mark** for 3 down

One-time deduction	**Each time deduction**
deduct $\frac{1}{2}$ mark if arrowheads	**deduct $\frac{1}{2}$ mark** for each point(s) (different) incorrect
deduct $\frac{1}{2}$ mark for an end missing	
deduct $\frac{1}{2}$ mark(s) if end incorrect	

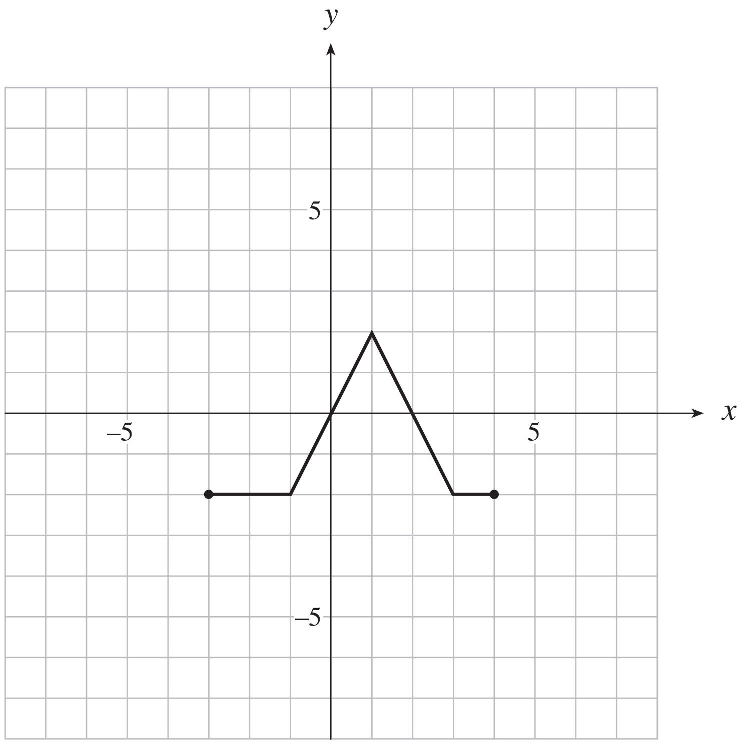
Marks:

1 mark: horizontal shift wrong
1 mark: vertical shift wrong
$\frac{1}{2}$ mark: both shifts wrong (shape preserved)
$\frac{1}{2}$ mark: some other translation has occurred (shape preserved)
$1\frac{1}{2}$ marks: a miscount of 1 square
0 marks: shape not preserved
0 marks: reflection

If more than 1 graph is on the same grid ***without*** labelling but the correct graph is there, deduct **$\frac{1}{2}$ mark**.

b) $y = f(2x)$ **(2 marks)**

solution

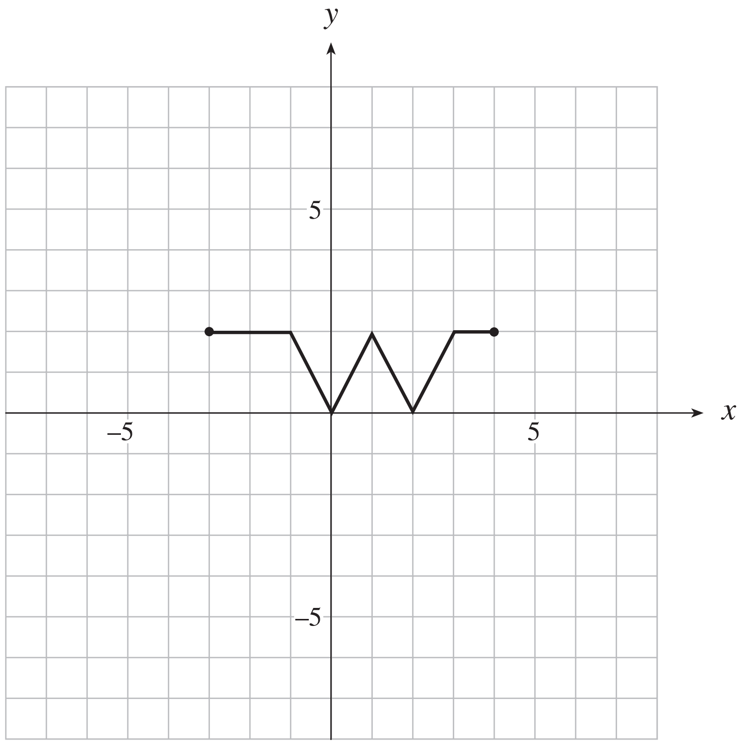


Marks:

1 mark:	horizontal expansion by a factor of 2, i.e. $\left[f\left(\frac{1}{2}x\right)\right]$
$\frac{1}{2}$ **mark**:	vertical expansion by a factor of 2, i.e. $\left[2f(x)\right]$
0 marks:	vertical compression by a factor of $\frac{1}{2}$, i.e. $\left[\frac{1}{2}f(x)\right]$
1 mark:	horizontal compression around vertex
1 mark:	horizontal compression around feet
2 marks:	horizontal compression

c) $y = \left|f(2x)\right|$ **(1 mark)**

solution



Marks:

1 mark:	correct absolute value of **their** b)
$\frac{1}{2}$ **mark**:	$f\left|2x\right|$
$\frac{1}{2}$ **mark**:	absolute value of original function
0 marks:	incorrect absolute value of correct b)
1 mark:	for taking the absolute value of the function obtained in b)

$$\frac{1}{2} - \underset{\uparrow}{\frac{1}{2}} = \frac{1}{2}$$

when deducting
$\frac{1}{2}$ for minor error

6. In a sequence, if $t_n = \sum_{k=1}^{n}\left(\frac{1}{x}\right)^{k-1} + \sum_{k=1}^{n}\left(-\frac{1}{x}\right)^{k-1}$, determine the value of t_4 if $x = 3$. **(5 marks)**

solution

$$t_4 = \sum_{k=1}^{4}\left(\frac{1}{3}\right)^{k-1} + \sum_{k=1}^{4}\left(-\frac{1}{3}\right)^{k-1} \quad \Bigg\} \leftarrow \begin{array}{l} \tfrac{1}{2}\ \textbf{mark} \text{ for recognizing } n = 4 \\ \tfrac{1}{2}\ \textbf{mark} \text{ for } n = 4 \text{ substituted in correctly} \\ \tfrac{1}{2}\ \textbf{mark} \text{ for } x = 3 \end{array}$$

$$= \overbrace{1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27}}^{\substack{\textbf{1 mark} \\ \downarrow}} + \overbrace{1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27}}^{\substack{\textbf{1}\frac{1}{2}\textbf{ marks} \\ \downarrow}}$$

$$= 2 + \frac{2}{9}$$

$$= 2\frac{2}{9} \text{ or } \frac{20}{9} \text{ or } 2.22 \quad \leftarrow \textbf{1 mark}$$

Marks:

Cap at $1\frac{1}{2}$ marks: answer only

Cap at $2\frac{1}{2}$ marks: if all 8 fractions, no adding

Cap at $4\frac{1}{2}$ marks: substitute in $x = 4$ instead of $x = 3$

Cap at $4\frac{1}{2}$ marks: if stopped at $2 + \frac{2}{9}$

Cap at 4 marks: if stopped at $2 + \frac{2}{3^2}$

Full marks: if started with $2 + \frac{2}{x^2}$ and finished

Cap at 3 marks: if pairs of values added, i.e. 2, 0, 0.222, 0, but no sum

Cap at $4\frac{1}{2}$ marks: if showed $2 + 0 + 0.222 + 0$

Cap at 4 marks: wrong fraction, so wrong answer

alternate solution

$$t_4 = \sum_{k=1}^{4}\left(\frac{1}{x}\right)^{k-1} + \sum_{k=1}^{4}\left(-\frac{1}{x}\right)^{k-1} \quad \left.\right\} \leftarrow \begin{array}{l} \tfrac{1}{2}\textbf{ mark}\text{ for recognizing } n = 4 \\ \tfrac{1}{2}\textbf{ mark for } n = 4 \text{ substituted in correctly} \end{array}$$

$$= \overbrace{1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}}^{\overset{\textbf{1 mark}}{\downarrow}} + \overbrace{1 - \frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^3}}^{\overset{\textbf{1}\frac{1}{2}\textbf{ marks}}{\downarrow}}$$

$$= 2 + \frac{2}{x^2}$$

$$= 2 + \frac{2}{3^2} \quad \leftarrow \tfrac{1}{2}\textbf{ mark} \text{ for } x = 3$$

$$= 2\frac{2}{9} \text{ or } \frac{20}{9} \text{ or } 2.22 \quad \leftarrow \textbf{1 mark}$$

6. In a sequence, if $t_n = \sum_{k=1}^{n}\left(\frac{1}{x}\right)^{k-1} + \sum_{k=1}^{n}\left(-\frac{1}{x}\right)^{k-1}$, determine the value of t_4 if $x = 3$. **(5 marks)**

solution

$$Y_1 = \mathrm{SUM}\Big(\mathrm{SEQ}\big(\left(X^{-1}\right)\wedge(K-1),\ K,\ 1,\ 4\big)\Big)$$

$$Y_2 = \mathrm{SUM}\Big(\mathrm{SEQ}\big(\left(-X^{-1}\right)\wedge(K-1),\ K,\ 1,\ 4\big)\Big)$$

$$Y_3 = Y_1 + Y_2$$

Table

X	Y_1	Y_2	Y_3	
1	4	0	4	
2	1.875	.625	2.5	
3	1.4815	.74074	2.2222	← **answer**
4	1.3281	.79688	2.125	

alternate solution

$$Y_1 = (1/3)\wedge(X-1) + (-1/3)\wedge(X-1)$$

2nd Trace Value or Table

$X = 1$ $Y = 2$

$X = 2$ $Y = 0$

$X = 3$ $Y = 0.\dot{2}$ **or** $\frac{2}{9}$

$X = 4$ $Y = 0$

Total $= 2\frac{2}{9}$ ← **answer**

7. In a large city the probability that a grade 12 student has a part time job is 0.40. Use the normal approximation to the binomial to determine the probability that in a random sample of 60 grade 12 students at least 20 of them have a part time job. (Show all solution steps. If using a calculator, clearly show the function used and the substitution of the numbers into this function.) **(4 marks)**

solution

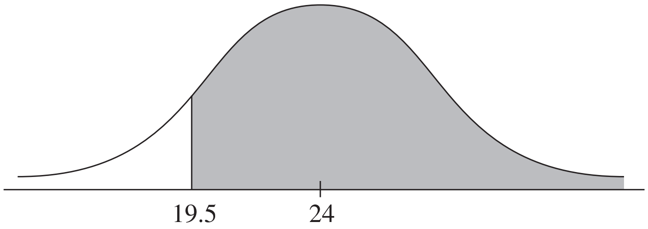
$\mu = 60(0.40) = 24$ ← $\frac{1}{2}$ **mark**

$\sigma = \sqrt{(60)(0.4)(0.6)} = 3.795$ ← **1 mark**

for $x = 20$, use a continuity correction to obtain 19.5 ← $\frac{1}{2}$ **mark**

Using TI-83 calculator:

$$\overset{\frac{1}{2}\textbf{ mark}}{\overset{\downarrow}{\text{normal cdf}}}\left(\underbrace{19.5,\ \overset{\frac{1}{2}\textbf{ mark}}{\overset{\downarrow}{1E99}},\ 24,\ \underbrace{\sqrt{(60)(0.4)(0.6)}}_{\text{or } 3.795}}_{\frac{1}{2}\textbf{ mark} \text{ for 4 parameters of which } \mu \text{ and } \sigma \text{ must be in the correct position}}\right)$$



= 0.88 ← $\frac{1}{2}$ **mark**
or
88%

Note: 1E99 could be any number ≥ 60
Using the Sharp calculator: cdfnorm $\left(19.5,\ 1E99,\ 24,\ \sqrt{(60)(0.4)(0.6)}\right)$

Note: If students use $1 - \text{binomcdf}\,(60,\ 0.40,\ 19)$, they will also get 88% and should receive **1 mark**. This is because the intent of the question is that students demonstrate the use of the normal approximation to the binomial.

7. In a large city the probability that a grade 12 student has a part time job is 0.40. Use the normal approximation to the binomial to determine the probability that in a random sample of 60 grade 12 students at least 20 of them have a part time job. (Show all solution steps. If using a calculator, clearly show the function used and the substitution of the numbers into this function.) **(4 marks)**

alternate solution

Using a z-score table:

$\mu = 60(0.4) = 24$ ← **½ mark**

$\sigma = \sqrt{60(0.4)(0.6)}$ ← **1 mark**

for $x = 20$ use a continuity correction to obtain 19.5 ← **½ mark**

$z = \frac{19.5 - 24}{3.795} = -1.1858 \approx -1.19$ ← **1 mark** $\left(\begin{array}{l}\tfrac{1}{2}\ \textbf{mark}\text{ for using } z = \dfrac{x-\mu}{\sigma}\\ \tfrac{1}{2}\ \textbf{mark}\text{ for answer } (z = -1.19)\end{array}\right)$

area to the left of $z_{-1.19} = 0.1170$ ← **½ mark**

area to the right of $z_{-1.19} = 1 - 0.1170$

$= 0.88$ ← **½ mark**

or

88%

END OF KEY