Principles of Mathematics 12

August 2001 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers	**Sub-Organizers**
1. Problem Solving	A Problem Set
2. Patterns and Relations	B Sequences and Series
	C Polynomials
	D Logarithms and Exponents
	E Quadratic Relations
	F Quadratic Systems
3. Shape and Space	G Trigonometry
	H Geometry

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	D	K	1.5	2	C5	23.	C	U	1.5	2	B2
2.	D	U	1.5	2	C1	24.	B	U	1.5	2	B4
3.	D	U	1.5	2	C4	25.	D	U	1.5	2	B4
4.	C	U	1.5	2	C2	26.	B	U	1.5	2	B7
5.	A	U	1.5	2, 1	C9, A7	27.	A	U	1.5	2	B5
6.	A	H	1.5	2	C4	28.	C	H	1.5	2	B4
7.	D	K	1.5	2	E6	29.	C	U	1.5	3	G1
8.	B	K	1.5	2	E6	30.	B	K	1.5	3	G5
9.	D	U	1.5	2	E4	31.	B	K	1.5	3	G5
10.	C	U	1.5	2	E6	32.	C	U	1.5	3	G9
11.	B	U	1.5	2	E5	33.	C	U	1.5	3	G7
12.	C	U	1.5	2	F2	34.	A	U	1.5	3	G8
13.	D	H	1.5	2	F5, C9	35.	A	U	1.5	3	G8
14.	B	H	1.5	2	F1	36.	D	H	1.5	3	G6
15.	B	U	1.5	2	D5	37.	C	H	1.5	3, 2	G3, B6
16.	A	K	1.5	2	D5	38.	C	U	1.5	3	H2
17.	A	U	1.5	2	D1	39.	A	U	1.5	3	H2
18.	C	U	1.5	2	D2	40.	A	U	1.5	3	H4
19.	A	U	1.5	2	D6	41.	D	H	1.5	3	H3
20.	D	H	1.5	2	D5	42.	A	U	1.5	1	A3
21.	B	H	1.5	2, 1	D5	43.	B	U	1.5	1	A3
22.	C	K	1.5	2	B6	44.	A	U	1.5	1	A1

Multiple Choice = 66 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1.	1	U	4	2	C6
2.	2	U	4	2	F1
3.	3	U	4	2, 1	D5, A7
4.	4	U	5	2	E5
5.	5	U	4	3, 1	G9, A7
6.	6	U	4	3	H4
7.	7	U	4	1	A3
8.	8	H	5	3	H3

Written Response = 34 marks

Multiple Choice = 66 (44 questions)
Written Response = 34 (8 questions)
EXAMINATION TOTAL = 100 marks

LEGEND:

Q = Question Number	**K** = Keyed Response	**C** = Cognitive Level
B = Score Box Number	**S** = Score	**CO** = Curriculum Organizer
PLO = Prescribed Learning Outcome		

1. If 2 is a root of the polynomial equation $6x^3 + kx^2 + x + 2 = 0$, determine the other roots.

(4 marks)

Solution

$$6x^3 + kx^2 + x + 2 = 0$$

$$\tfrac{1}{2}\textbf{ mark} \rightarrow \quad \begin{array}{r|rrrr} \underline{2} & 6 & k & 1 & 2 \\ & \downarrow & 12 & -2 & -2 \\ \hline & 6 & -1 & -1 & 0 \end{array} \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$6x^2 - x - 1 = 0 \quad \leftarrow \textbf{1 mark}$$

$$\underset{\textbf{1 mark}}{\underset{\uparrow}{x = -\frac{1}{3}}}, \quad \underset{\textbf{1 mark}}{\underset{\uparrow}{x = \frac{1}{2}}}$$

Alternate Solution

$$\overset{\overset{\tfrac{1}{2}\textbf{ mark}}{\downarrow}}{6(2)^3} + k(2)^2 + 2 + 2 = 0 \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$48 + 4k + 4 = 0$$

$$4k = -52$$

$$k = -13 \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$\underline{6x^3 - 13x^2 + x + 2 = 0} \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

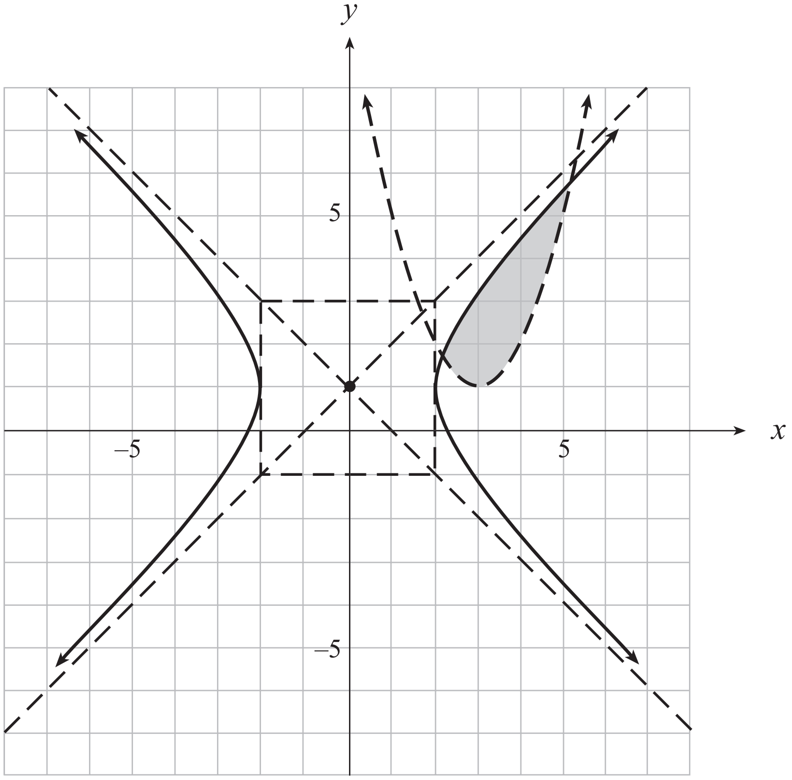
$$x = 2\ , \quad \underset{\textbf{1 mark}}{\underset{\nearrow}{-\frac{1}{3}}}\ , \quad \underset{\textbf{1 mark}}{\underset{\nwarrow}{\frac{1}{2}}}$$

2. Graph the solution of the following system of inequalities on the grid provided. **(4 marks)**

$$x^2 - (y-1)^2 \geq 4$$

$$y > (x-3)^2 + 1$$

Solution



1 mark hyperbola
1 mark parabola
1 mark shading (each $\frac{1}{2}$)
1 mark dotted and solid (each $\frac{1}{2}$)

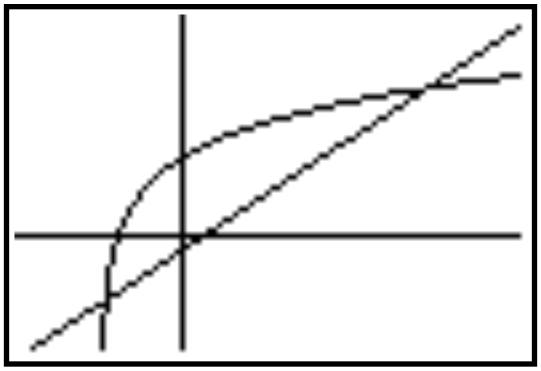
3. Solve the following system using a graphing calculator. **(4 marks)**

$$y = 5\log_3(x+5)$$

$$y = x - 1$$

Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the function(s) and all intersection points are visible.

Solution



$$\left.\begin{aligned} Y_1 &= \frac{5\log(x+5)}{\log 3} \\ Y_2 &= x-1 \end{aligned}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

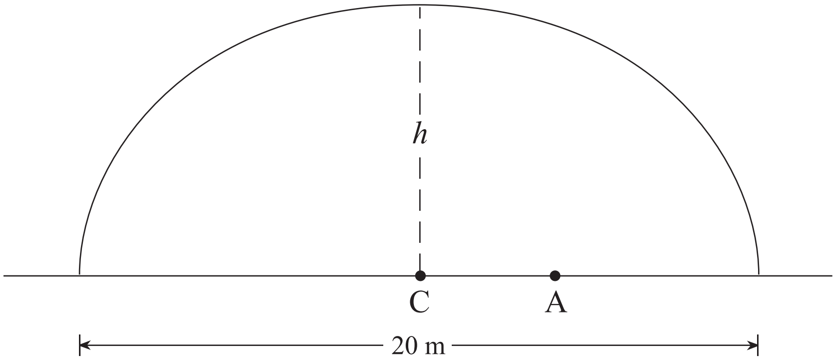
← **1 mark** for graph $\left(\tfrac{1}{2}\right.$ **mark** for each equation$\left.\right)$

$x\ [-10,\ 20]$ $y\ [-10,\ 20]$ ← $\tfrac{1}{2}$ **mark** for window dimensions

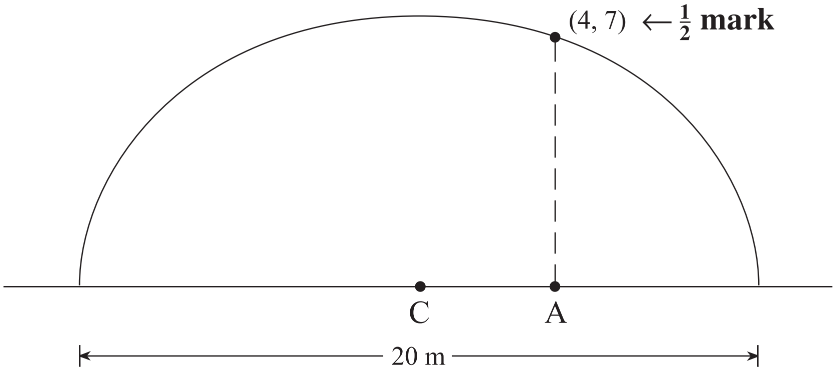
$$(\underset{\uparrow}{14.52},\ \underset{\uparrow}{13.52})\quad(\underset{\uparrow}{-4.72},\ \underset{\uparrow}{-5.72})$$

$\tfrac{1}{2}$ **mark each** $\tfrac{1}{2}$ **mark each**

4. An arch is semi-elliptical in shape with a maximum width of 20 m, as shown in the diagram. Point A is 4 m from the centre, C. If the height of the arch at point A is 7 m, determine the maximum height, h, of the arch. **(5 marks)**



Solution



(4, 7) ← **$\frac{1}{2}$ mark**

$$\frac{x^2}{100} + \frac{y^2}{h^2} = 1 \qquad \leftarrow \mathbf{1\tfrac{1}{2}\ marks}$$

$$\frac{16}{100} + \frac{49}{h^2} = 1 \qquad \leftarrow \mathbf{1\ mark}$$

$$\left.\begin{aligned} \frac{49}{h^2} &= 1 - \frac{16}{100} \\ \frac{49}{h^2} &= 0.84 \\ \frac{49}{0.84} &= h^2 \\ h^2 &= 58.\dot{3} \end{aligned}\right\} \leftarrow \mathbf{1\ mark}$$

$$h = 7.64 \qquad \leftarrow \mathbf{1\ mark}$$

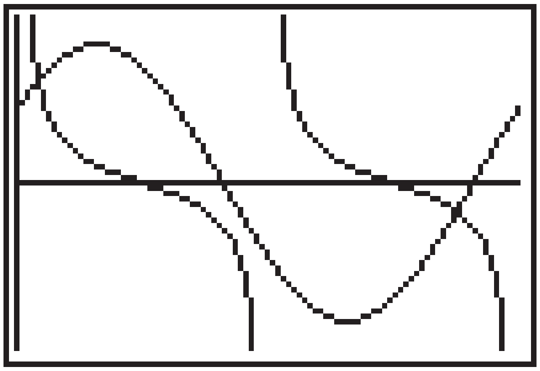
$\therefore$ the maximum height of the arch is 7.64 m.

5. Solve the following equation using a graphing calculator: **(4 marks)**

$$5\cos(x-1) = \cot x\,, \quad 0 \le x < 2\pi$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that all intersection points or zeros are visible within the viewing window.

Solution



$Y_1 = 5\cos(x-1)$ ← $\frac{1}{2}$ **mark** for equation

$Y_2 = \dfrac{1}{\tan(x)}$ ← $\frac{1}{2}$ **mark** for equation

← $\frac{1}{2}$ **mark** for graph

$x\ [0,\ 2\pi]$ $\quad y\ [-6,\ 6]$ ← $\frac{1}{2}$ **mark** for window dimensions

$$x = \underset{\substack{\uparrow \\ \textbf{1 mark}}}{0.26}\ ,\quad \underset{\substack{\uparrow \\ \textbf{1 mark}}}{5.51}$$

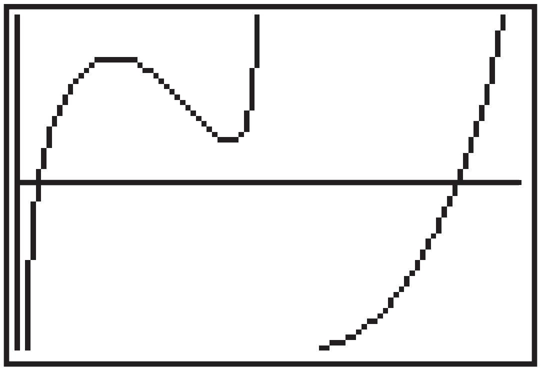
5. Solve the following equation using a graphing calculator: **(4 marks)**

$$5\cos(x-1) = \cot x, \quad 0 \le x < 2\pi$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that all intersection points or zeros are visible within the viewing window.

Alternate Solution

$Y_1 = 5\cos(x-1) - \dfrac{1}{\tan(x)}$ ← **1 mark** for equation



← $\frac{1}{2}$ **mark** for graph

$x\ [0,\ 2\pi]$ $y\ [-6,\ 6]$

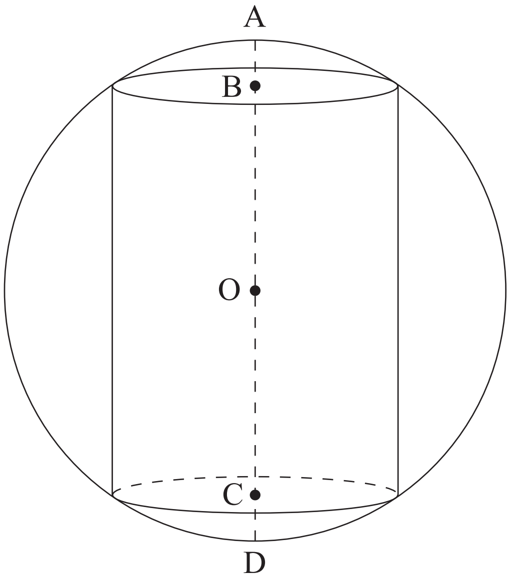
← $\frac{1}{2}$ **mark** for window dimensions

$x = 0.26\ ,\quad 5.51$

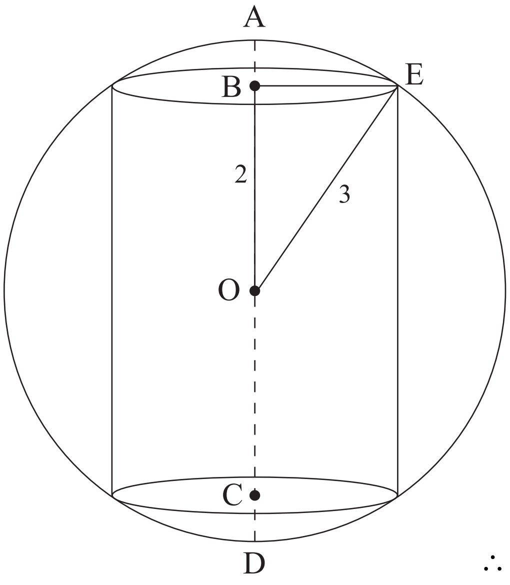
$\uparrow$ **1 mark** $\uparrow$ **1 mark**

6. A sphere with centre O has a volume of 36π cm^3. A cylinder is inscribed in the sphere so that AB = CD = 1 cm, as shown in the diagram. If the diameter AD passes through the centre of the cylinder, determine the volume of the cylinder. **(4 marks)**

$V_{\text{sphere}} = \frac{4}{3}\pi r^3 \qquad V_{\text{cylinder}} = \pi r^2 h$



Solution



$V = \frac{4}{3}\pi r^3$

$36\pi = \frac{4}{3}\pi r^3$ ← $\frac{1}{2}$ **mark**

$27 = r^3$

$3 = r$ ← $\frac{1}{2}$ **mark**

since AB = 1 ⇒ BO = 2 ← $\frac{1}{2}$ **mark**

$BE^2 + 2^2 = 3^2$

$BE^2 + 4 = 9$

$BE^2 = 5$

$BE = \sqrt{5}$ = radius of cylinder ← **1 mark**

$h = 4$ ← $\frac{1}{2}$ **mark**

∴ volume of cylinder $= \pi r^2 h$

$= \pi\left(\sqrt{5}\right)^2 4$

$= 20\pi$ cm^3 ← **1 mark**

7. State the restrictions on x and y in the following equation, then express y as a polynomial function of x. **(4 marks)**

$$\frac{1}{\log_y 7} = \log_7 4 - \log_7\left(\frac{1}{x^2}\right)$$

Solution

$$\frac{1}{\log_y 7} = \log_7 4 - \log_7\left(\frac{1}{x^2}\right)$$

$$\underset{\substack{\uparrow \\ \textbf{1 mark}}}{\log_7 y} = \log_7 4 + \underset{\substack{\uparrow \\ \textbf{\tfrac{1}{2} mark}}}{\log_7 x^2}$$

$$y = 4x^2 \quad \leftarrow \textbf{1 mark}$$

restriction $y > 0 \quad y \neq 1$ ← **1 mark**

$x \neq 0$ ← **$\frac{1}{2}$ mark**

Note: It is not necessary to say $x \neq \pm\frac{1}{2}$

Students must choose one or the other method of proof.

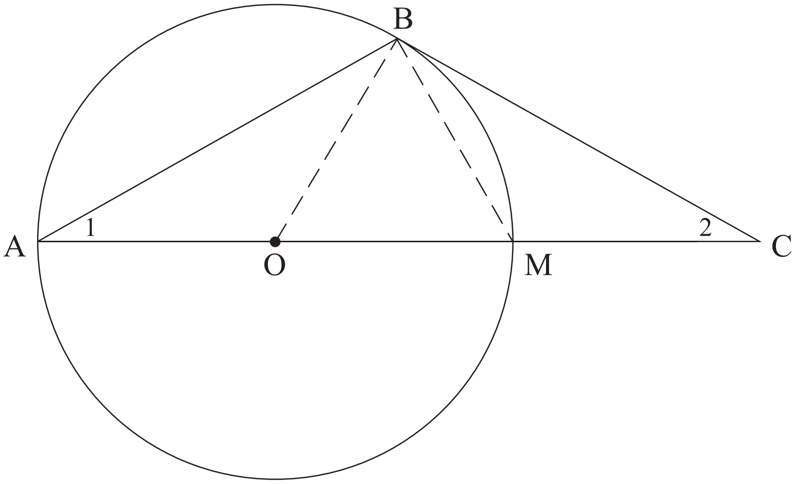
8. Complete the proof. **(5 marks)**

Diagram clarification: O is the centre of the circle
A, O, M, C are collinear

Given: AB = BC
BC is tangent to the circle

Prove: AM = OC

Note: Students are encouraged to number angles.



Solution

Paragraph proof method:

Since AB = BC, then $\angle 1 = \angle 2$ because they are angles opposite equal sides $\left(\frac{1}{2}\ \textbf{mark}\right)$. Since BC is a tangent, then $\angle OBC = 90°$ **(1 mark)** and $\angle ABM = 90°$ **(1 mark)** because it is an inscribed angle on a diameter, therefore $\angle OBC = \angle ABM$ by substitution $\left(\frac{1}{2}\ \textbf{mark}\right)$. So $\Delta ABM \cong \Delta CBO$ by ASA $\left(\frac{1}{2}\ \textbf{mark}\right)$. Therefore AM = OC $\left(1\frac{1}{2}\ \textbf{marks}\right)$.

8. Complete the proof. **(5 marks)**

Diagram clarification: O is the centre of the circle
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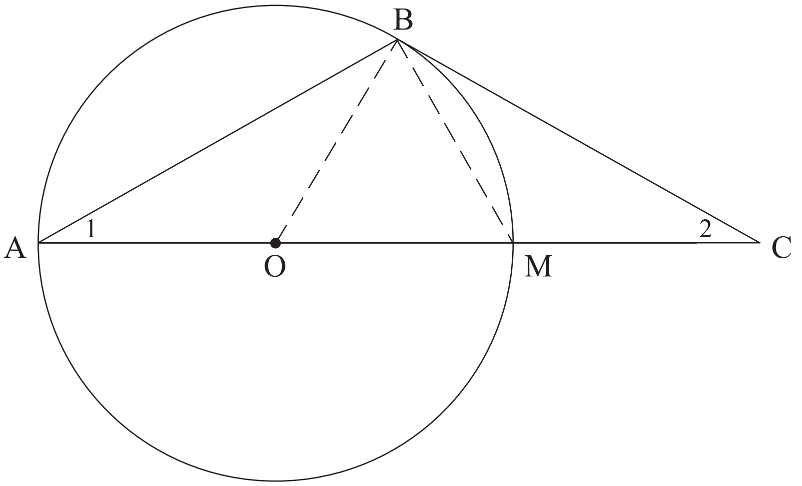


Solution

Two-column proof method:

STATEMENT	REASON
join OB and BM	construction (deduct $\frac{1}{2}$ **mark** if missing)
AB = BC	given (deduct $\frac{1}{2}$ **mark** if either given is missing)
$\angle 1 = \angle 2$	$\angle$s opposite = sides are = ← $\frac{1}{2}$ **mark**
BC is tangent	given
$\angle OBC = 90°$	tangent $\perp$ radius ← **1 mark**
$\angle ABM = 90°$	inscribed $\angle$s on a diameter = 90° ← **1 mark**
$\angle OBC = \angle ABM$	substitution (both = 90°) ← $\frac{1}{2}$ **mark**
$\Delta ABM \cong \Delta CBO$	ASA ← $\frac{1}{2}$ **mark**
AM = OC	CPCTC ← **1** $\frac{1}{2}$ **marks**

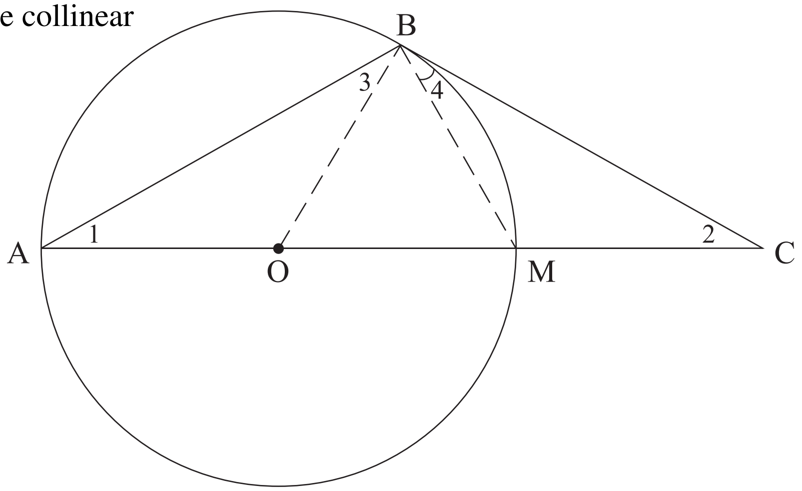
8. Complete the proof. **(5 marks)**

Diagram clarification: O is the centre of the circle
A, O, M, C are collinear

Given: AB = BC
BC is tangent to the circle

Prove: AM = OC

Note: Students are encouraged to number angles.



Alternate Solution

Two-column proof method:

STATEMENT	REASON
join OB and BM	construction (deduct $\frac{1}{2}$ **mark** if missing)
AB = BC	given (deduct $\frac{1}{2}$ **mark** if either given is missing)
∠1 = ∠2	∠s opposite = sides are = ← $\frac{1}{2}$ **mark**
OA = OB	radii ← $\frac{1}{2}$ **mark**
∠1 = ∠3	∠s opposite = sides are = ← $\frac{1}{2}$ **mark**
BC is a tangent	given
∠4 = ∠1	insc. ∠ between chord and tangent ← **1 mark**
∠3 = ∠4	substitution ← $\frac{1}{2}$ **mark**
ΔOAB ≅ ΔMCB	ASA ← $\frac{1}{2}$ **mark**
OA = MC	CPCTC ← $\frac{1}{2}$ **mark**
OA + OM = MC + OM	addition prop. of equ. ← $\frac{1}{2}$ **mark**
AM = OC	substitution ← $\frac{1}{2}$ **mark**

END OF KEY