Principles of Mathematics 12

January 2000 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		Sub-Organizers	
1.	Problem Solving	A	Problem Set
2.	Patterns and Relations	B	Sequences and Series
		C	Polynomials
		D	Logarithms and Exponents
		E	Quadratic Relations
		F	Quadratic Systems
3.	Shape and Space	G	Trigonometry
		H	Geometry

Part A: Multiple Choice

Q	K	C	CO	PLO	Q	K	C	CO	PLO
1.	D	K	2	C5	24.	D	U	2	B2
2.	B	U	2	C6	25.	B	U	2	B4
3.	D	U	2	C3	26.	A	U	2	B5
4.	B	U	2	C7	27.	A	H	2	B6
5.	A	U	2	C9	28.	B	H	2	B4
6.	A	H	2	C4	29.	B	U	3	G1
7.	C	K	2	E5	30.	C	U	3	G2
8.	B	U	2	E3	31.	A	K	3	G2
9.	C	U	2	E6	32.	D	K	3	G7
10.	A	H	2	E4	33.	B	U	3	G5
11.	A	U	2	F1	34.	C	U	3	G3
12.	D	U	2	F4	35.	C	U	3	G2
13.	C	U	2, 1	F2, A7	36.	A	U	3	G8
14.	B	U	2	E7	37.	B	U	3, 1	G3, A7
15.	D	K	2	D4	38.	D	H	3	G5
16.	A	U	2	D5	39.	C	U	3	H1
17.	D	U	2	D3	40.	C	U	3	H1
18.	D	U	2	D2	41.	C	U	3	H1
19.	D	U	2	D1	42.	D	U	3	H3
20.	C	H	2	D5	43.	B	U	1	A3
21.	B	H	2	D5	44.	C	U	1	A3
22.	C	U	2	B2	45.	C	H	1	A3
23.	B	K	2	B3					

Multiple Choice = 45 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1.	1	U	3	2, 1	C6, A7
2.	2	U	3	2	D6
3.	3	U	3	2	E4
4.	4	U	3	3	G8
5.	5	U	3	2, 1	F2, A7
6.	6	U	3	3	H4
7.	7	H	3	1	A3
8.	8	H	4	3	H2

Written Response = 25 marks

Multiple Choice = 45 (45 questions)
Written Response = 25 (8 questions)
EXAMINATION TOTAL = 70 marks

LEGEND:

Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
CO = Curriculum Organizer
PLO = Prescribed Learning Outcome

PART B: WRITTEN RESPONSE

Value: 25 marks **Suggested Time: 45 minutes**

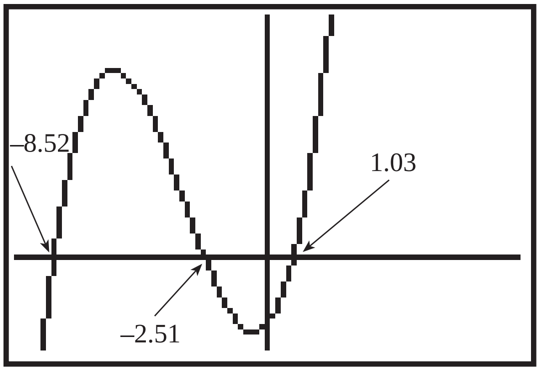
INSTRUCTIONS:	Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided. If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions. When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution. **Full marks will NOT be given for the final answer only.**

1. Solve the following equation using a graphing calculator. **(3 marks)**

$$x^3 + 10x^2 = 22 - 10x$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window.

Solution



$x\ [-10,\ 10]$ $y\ [-30,\ 80]$

$Y_1 = x^3 + 10x^2 + 10x - 22$ ← $\frac{1}{2}$ **mark** for equation

$\frac{1}{2}$ **mark** for appropriate window domain (i.e. includes roots)

$\frac{1}{2}$ **mark** for appropriate window range (i.e. includes max and min)

$$\underset{\frac{1}{2}\textbf{ mark}}{\underset{\uparrow}{x = -8.52}}\ ,\quad \underset{\frac{1}{2}\textbf{ mark}}{\underset{\uparrow}{-2.51}}\ ,\quad \underset{\frac{1}{2}\textbf{ mark}}{\underset{\uparrow}{1.03}}$$

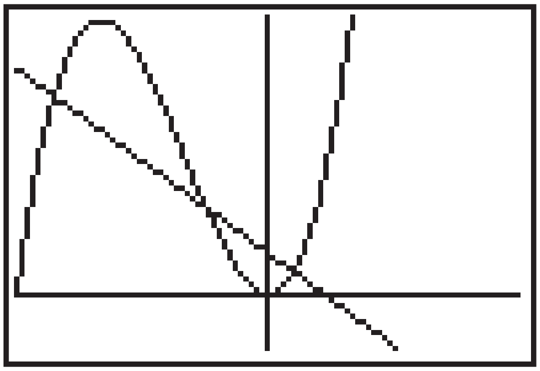
1. Solve the following equation using a graphing calculator. **(3 marks)**

$$x^3 + 10x^2 = 22 - 10x$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window.

Alternate Solution

$$\left.\begin{array}{l} \mathrm{Y}_1 = x^3 + 10x^2 \\ \mathrm{Y}_2 = 22 - 10x \end{array}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}\ \text{for equations}$$



$\leftarrow \tfrac{1}{2}$ **mark** for graph

$x\ [-10,\ 10]$ $y\ [-30,\ 150]$ $\leftarrow \tfrac{1}{2}$ **mark** for window dimensions

$$x = \underset{\tfrac{1}{2}\textbf{mark}}{\underset{\uparrow}{-8.52}}\ ,\quad \underset{\tfrac{1}{2}\textbf{mark}}{\underset{\uparrow}{-2.51}}\ ,\quad \underset{\tfrac{1}{2}\textbf{mark}}{\underset{\uparrow}{1.03}}$$

2. A population of wolves decreases by 2% each year. At the present time, there are 8 000 wolves. How long will it take for the population to become 500 wolves? (Answer to the nearest year.)

(3 marks)

Solution

$\frac{1}{2}$ **mark** → $8\,000\overset{\overset{\frac{1}{2}\textbf{ mark}}{\downarrow}}{(0.98)^t} = 500$

$(0.98)^t = 0.0625$ ← $\frac{1}{2}$ **mark**

$\frac{1}{2}$ **mark** → **either** $\begin{cases} t = \log_{0.98} 0.0625 \\ t = \dfrac{\log 0.0625}{\log 0.98} \end{cases}$

$\frac{1}{2}$ **mark** → $t = 137.24$

$\frac{1}{2}$ **mark** → ∴ it will take 137 years
or 138 years

3. A point $P(x,\ y)$ moves such that it is always the same distance from $A(12,\ 0)$ as it is from $B(3,\ 1)$. Determine an equation, in standard form, of this locus. **(3 marks)**

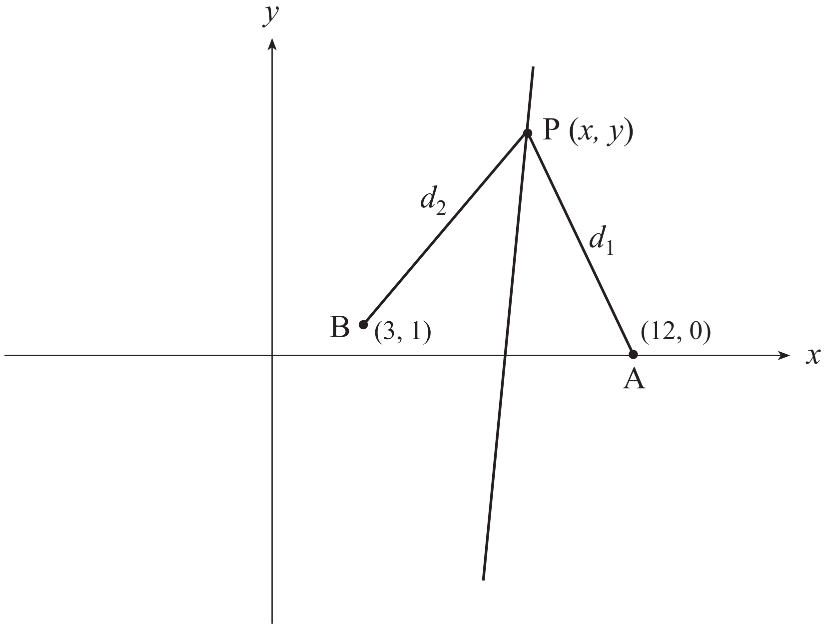
Solution

$$d_1 = d_2$$

$$\sqrt{(x-12)^2+(y-0)^2} = \sqrt{(x-3)^2+(y-1)^2} \quad \leftarrow \textbf{1 mark}$$

$$x^2-24x+144+y^2 = x^2-6x+9+y^2-2y+1 \quad \leftarrow \textbf{1 mark}$$

$$\left.\begin{aligned} 2y-18x &= -134 \\ y &= 9x-67 \\ 9x-y &= 67 \\ 9x-y-67 &= 0 \end{aligned}\right\} \quad \leftarrow \textbf{1 mark}$$



3. A point $\mathrm{P}(x,\ y)$ moves such that it is always the same distance from $\mathrm{A}(12,\ 0)$ as it is from $\mathrm{B}(3,\ 1)$. Determine an equation, in standard form, of this locus. **(3 marks)**

Alternate Solution

midpoint $\left(\frac{12+3}{2},\ \frac{0+1}{2}\right)=\left(\frac{15}{2},\ \frac{1}{2}\right)$ ← **½ mark**

slope of AB $m=\frac{y_2-y_1}{x_2-x_1}=\frac{0-1}{12-3}=-\frac{1}{9}$ ← **½ mark**

$m_{\perp}=9$ ← **½ mark**

$$y-y_0=m(x-x_0)$$

$$\left.\begin{aligned} y-\frac{1}{2}&=9\left(x-\frac{15}{2}\right)\\ y-\frac{1}{2}&=9x-\frac{135}{2}\\ y&=9x-\frac{134}{2}\\ y&=9x-67\\ \text{or}\quad 9x-y&=67\\ \text{or}\quad 9x-y-67&=0 \end{aligned}\right\}\leftarrow \mathbf{1\tfrac{1}{2}\ marks}$$

4. Prove the identity: **(3 marks)**

$$\csc^2 x + \sec^2 x \qquad = \qquad \csc^2 x \sec^2 x$$

Solution

$$\csc^2 x + \sec^2 x \qquad = \qquad \csc^2 x \sec^2 x$$

	LEFT SIDE		RIGHT SIDE	
$\frac{1}{2}$ **mark** →	$\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}$	← $\frac{1}{2}$ **mark**	$\frac{1}{\sin^2 x} \cdot \frac{1}{\cos^2 x}$	← $\frac{1}{2}$ **mark**
1 mark → common denominator	$\frac{\cos^2 x + \sin^2 x}{\sin^2 x \cos^2 x}$		$\frac{1}{\sin^2 x \cos^2 x}$	
$\frac{1}{2}$ **mark** →	$\frac{1}{\sin^2 x \cos^2 x}$			

LS = RS

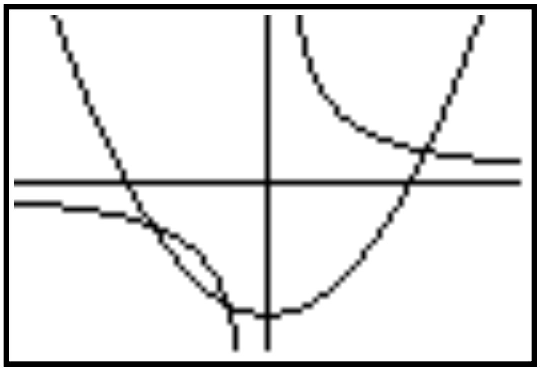
5. Solve the following system using a graphing calculator. **(3 marks)**

$$xy = 12$$

$$y = \frac{1}{4}x^2 - 8$$

Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the function(s) and all intersection points are visible.

Solution



$$\left.\begin{aligned} Y_1 &= \frac{12}{x} \\ Y_2 &= \frac{1}{4}x^2 - 8 \end{aligned}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}\ \text{for equations}$$

$\leftarrow \frac{1}{2}$ **mark** for graph

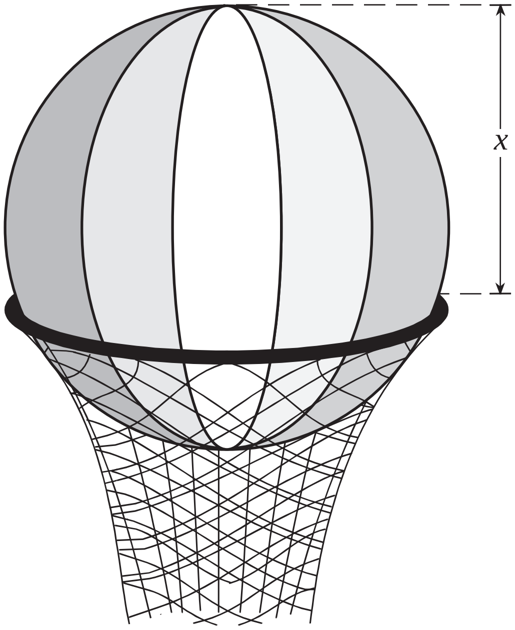
$x\ [-10,\ 10]$ $y\ [-10,\ 10]$ $\leftarrow \frac{1}{2}$ **mark** for window dimensions only if graph is shown

$(-1.64,\ -7.33)$ $(-4.66,\ -2.58)$ $(6.29,\ 1.91)$ $\leftarrow$ **$1\frac{1}{2}$ marks**

TRACE Method: within 0.1 $\leftarrow$ **$2\frac{1}{2}$ marks**

otherwise $\leftarrow$ **2 marks**

6. A child throws her beach ball with radius 28 cm into a basketball hoop with an inside diameter of 46 cm. The ball is too big and gets stuck, as shown in the diagram. What is the vertical distance x from the top of the ball to the level of the hoop? **(3 marks)**



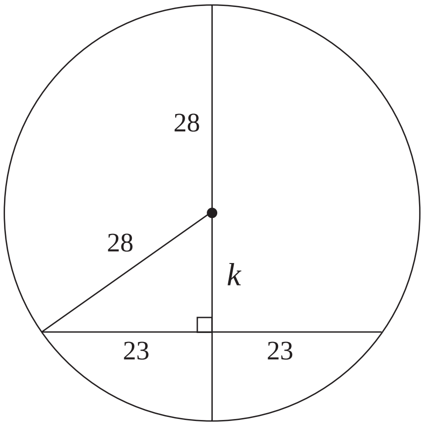
Solution

$28^2 = k^2 + 23^2$ ← **1 mark**

$15.97 = k$ ← **$\frac{1}{2}$ mark**

$x = k + 28$

$x = 15.97 + 28 = 43.97$ cm ← **$\frac{1}{2}$ mark**



← **1 mark**

7. Given $\frac{1}{\log_y 4} = \log_{\frac{1}{4}} \frac{1}{8x}$, express y as a polynomial function of x. State the restrictions on x and y. **(3 marks)**

Solution

$$\frac{1}{\log_y 4} = \log_{\frac{1}{4}} \frac{1}{8x}$$

$\log_4 y = \log_4 8x$ ← **1½ marks**

$y = 8x$ ← **½ mark**

$y > 0$, $y \neq 1$

$x > 0$

Restrictions ← **1 mark**

Number of restrictions:

$\frac{1}{3}$ ← **no marks**

$\frac{2}{3}$ ← **½ mark**

$\frac{3}{3}$ ← **1 mark**

Note: The restriction $x \neq \frac{1}{8}$ need not be stated.
Equivalent to $y \neq 1$.
Either one acceptable to count as one of the three.

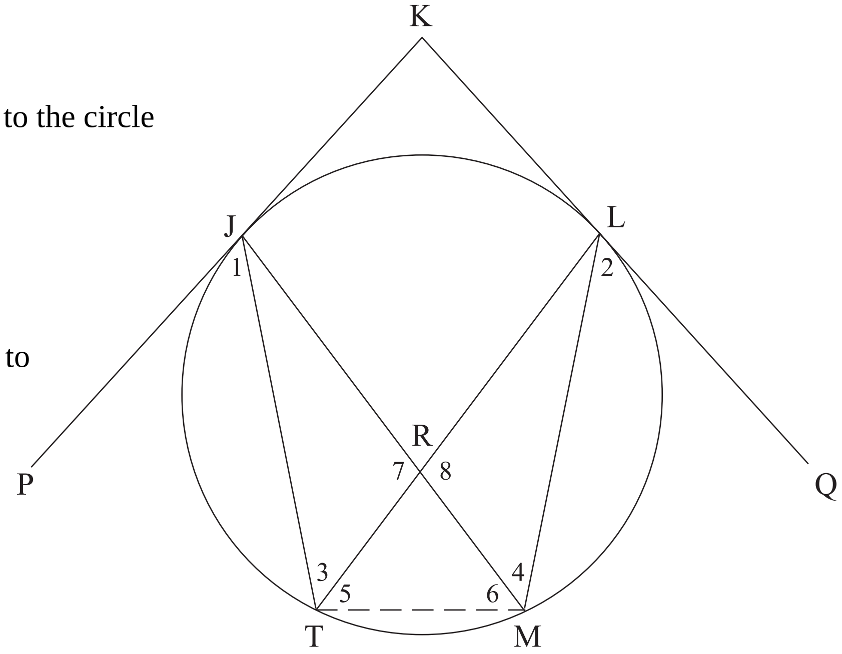
Students should choose one or the other method of proof.

8. Complete the proof. **(4 marks)**

Given: PK and QK are tangents to the circle
$\angle 1 = \angle 2$

Prove: JR = LR

Note: Students are encouraged to number angles.



Solution

Paragraph proof method:

Since PK and QK are tangents, $\angle 1 = \angle 6$ ($\frac{1}{2}$ **mark**) and $\angle 2 = \angle 5$ ($\frac{1}{2}$ **mark**) by ∠ between tangent and chord (with reason).

Given that $\angle 1 = \angle 2$, $\angle 6 = \angle 5$ because they = ∠s that are = ($\frac{1}{2}$ **mark**).

This produces an isosceles Δ where RT = RM because sides opposite = ∠s are = ($\frac{1}{2}$ **mark**).

$\angle 3 = \angle 4$ since they are inscribed on the same arc ($\frac{1}{2}$ **mark**) and $\angle 7 = \angle 8$ because vertically opposite ∠s are = .

This gives ΔJTR ≅ ΔLMR ($\frac{1}{2}$ **mark**) by ASA ($\frac{1}{2}$ **mark**) and therefore JR = LR ($\frac{1}{2}$ **mark**) since they are corresponding sides of these ≅ Δs.

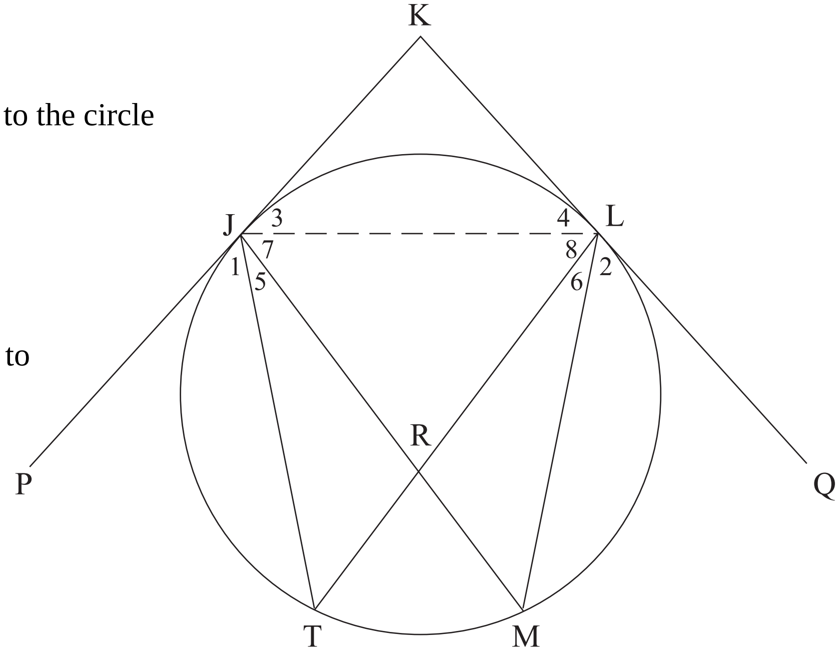
Students should choose one or the other method of proof.

8. Complete the proof. **(4 marks)**

Given: PK and QK are tangents to the circle
$\angle 1 = \angle 2$

Prove: JR = LR

Note: Students are encouraged to number angles.



Alternate Solution

Paragraph proof method:

Given that PK and QK are tangents, JK = KL since tangents from an external point are = ($\frac{1}{2}$ **mark**).

This means $\angle 3 = \angle 4$ since ∠s opposite = sides are = **(1 mark)**.

Given that $\angle 1 = \angle 2$, $\angle 5 = \angle 6$ since they are inscribed on the same arc ($\frac{1}{2}$ **mark**).

Since $[\angle 1 + \angle 5 + \angle 7 + \angle 3 = 180°$ because ∠s on a line = 180°, $\angle 2 + \angle 6 + \angle 8 + \angle 4$ is also = 180° ($\frac{1}{2}$ **mark**)$]$ $\therefore$ the remaining ∠s must be equal: $\angle 7 = \angle 8$ ($\frac{1}{2}$ **mark**).

This gives JR = LR because they are sides opposite these = ∠s (**1 mark**).

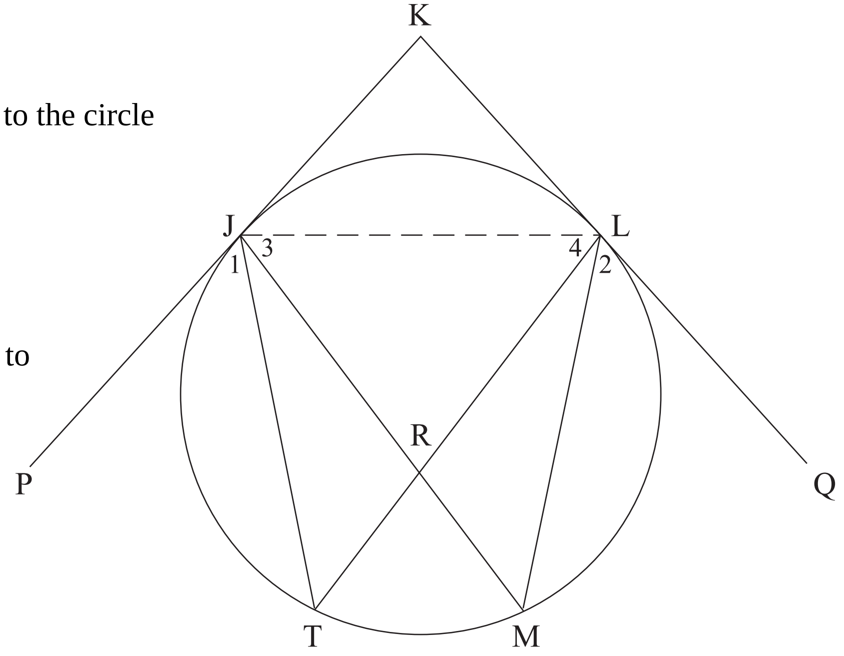
Students should choose one or the other method of proof.

8. Complete the proof. **(4 marks)**

Given: PK and QK are tangents to the circle
$\angle 1 = \angle 2$

Prove: JR = LR

Note: Students are encouraged to number angles.



Solution

Two-column proof method:

STATEMENT	REASON
join JL	construction
PK and QK are tangents	given
$\frac{1}{2}$ **mark** → $\angle 2 = \angle 3$, $\angle 1 = \angle 4$ ← $\frac{1}{2}$ **mark**	∠ between tangent and chord ← $\frac{1}{2}$ **mark**
$\angle 1 = \angle 2$	given
$\frac{1}{2}$ **mark** → $\angle 3 = \angle 4$	substitution ← **1 mark**
JR = LR	sides opposite = ∠s are = ← **1 mark**

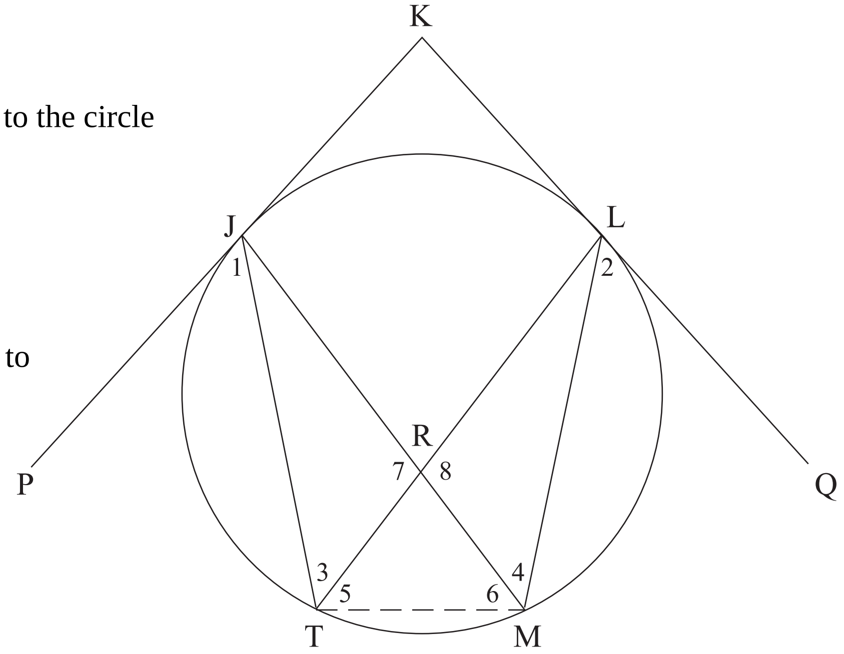
Students should choose one or the other method of proof.

8. Complete the proof. **(4 marks)**

Given: PK and QK are tangents to the circle
$\angle 1 = \angle 2$

Prove: JR = LR

Note: Students are encouraged to number angles.



Alternate Solution 1

Two-column proof method:

STATEMENT	REASON
$\angle 1 = \angle 2$	given
A $\angle 3 = \angle 4$ ← $\frac{1}{2}$ **mark**	inscribed $\angle$s on same arc are =
draw TM	construction
PK and QK are tangents	given
$\angle 1 = \angle 6$ ← $\frac{1}{2}$ **mark**	$\angle$ between tangent and chord
$\angle 2 = \angle 5$ ← $\frac{1}{2}$ **mark**	$\angle$ between tangent and chord
$\angle 5 = \angle 6$ ← $\frac{1}{2}$ **mark**	= to = $\angle$s
S RT = RM ← $\frac{1}{2}$ **mark**	sides opposite = $\angle$s are =
A $\angle 7 = \angle 8$	vertically opposite $\angle$s are =

Cap at $2\frac{1}{2}$

$\Delta JTR \cong \Delta LMR$ ← $\frac{1}{2}$ **mark**	ASA ← $\frac{1}{2}$ **mark**
JR = LR	CPCTC ← $\frac{1}{2}$ **mark**

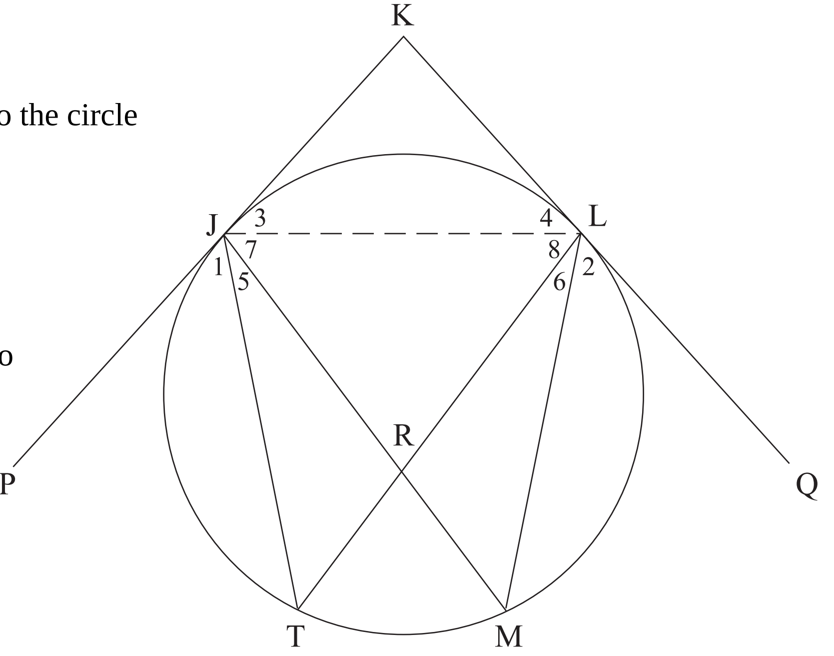
Students should choose one or the other method of proof.

8. Complete the proof. **(4 marks)**

Given: PK and QK are tangents to the circle
$\angle 1 = \angle 2$

Prove: JR = LR

Note: Students are encouraged to number angles.



Alternate Solution 2

Two-column proof method:

	STATEMENT	REASON
	Join JL	
	$\angle 1 = \angle 2$	given
	PK and QK are tangents	given
	JK = KL	tangents from an external point are =
1 mark →	$\angle 3 = \angle 4$	∠s opposite = sides are =
$\frac{1}{2}$ mark →	$\angle 5 = \angle 6$	inscribed ∠s on same arc are =
$\frac{1}{2}$ mark →	$\left\{ \begin{array}{l} \angle 1 + \angle 5 + \angle 7 + \angle 3 = 180^\circ \\ \angle 2 + \angle 6 + \angle 8 + \angle 4 = 180^\circ \end{array} \right.$	∠s on a line are supplementary ∠s on a line are supplementary
$\frac{1}{2}$ mark →	$\angle 7 = \angle 8$	supplements to = ∠s are = ← **$\frac{1}{2}$ mark**
		Cap at 3
	JR = LR	sides opposite = ∠s are = ← **1 mark**

END OF KEY