

**AUGUST 1999**

## **PROVINCIAL EXAMINATION**

**MINISTRY OF EDUCATION**

# **PRINCIPLES OF MATHEMATICS 12**

### **GENERAL INSTRUCTIONS**

1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above and on the **back** cover of this booklet. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this booklet.**
2. Ensure that in addition to this examination booklet, you have an **Examination Response Form**. Follow the directions on the front of the Response Form.
3. **Disqualification** from the examination will result if you bring books, paper, notes or unauthorized electronic devices into the examination room.
4. All multiple-choice answers must be entered on the Response Form using an **HB pencil**. Multiple-choice answers entered in this examination booklet will **not** be marked.
5. For each of the written-response questions, write your answer in the space provided in this booklet.
6. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

**END OF EXAMINATION**.

7. At the end of the examination, place your Response Form inside the front cover of this booklet and return the booklet and your Response Form to the supervisor.

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## PRINCIPLES OF MATHEMATICS 12 PROVINCIAL EXAMINATION

- |   | Value                  | Suggested Time     |
|---|------------------------|--------------------|
| 1. This examination consists of <b>two</b> parts: |                        |                    |
| PART A: 45 multiple-choice questions              | 45                     | 75                 |
| PART B: 8 written-response questions              | 25                     | 45                 |
|   | <b>Total: 70 marks</b> | <b>120 minutes</b> |
- Aside from an approved calculator, electronic devices including dictionaries and pagers are **not** permitted in the examination room.
  - The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae, Rough Work for Graphing, and Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.
  - You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.
  - A graphing calculator is essential for the Principles of Mathematics 12 Provincial Examination.** The calculator must be a hand-held device designed primarily for mathematical computations involving logarithmic and trigonometric functions as well as for graphing functions. Computers, calculators with a QWERTY keyboard, and electronic writing pads will not be allowed. Students must not bring any external devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or external keyboards. Students may have more than one calculator available during the examination of which one may be a scientific calculator. Calculators may not be shared and must not have the ability to either transmit or receive electronic signals. In addition to an approved calculator, students will be allowed to use rulers, compasses, and protractors during the examination.
  - If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.
  - When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.
  - This examination is designed to be completed in **two hours**. Students may, however, take up to **20 minutes** of additional time to finish.

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**PART A: MULTIPLE CHOICE**

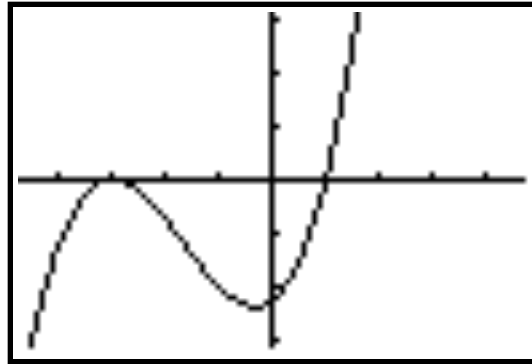
**Value: 45 marks**

**Suggested Time: 75 minutes**

**INSTRUCTIONS:** For each question, select the **best** answer and record your choice on the Response Form provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

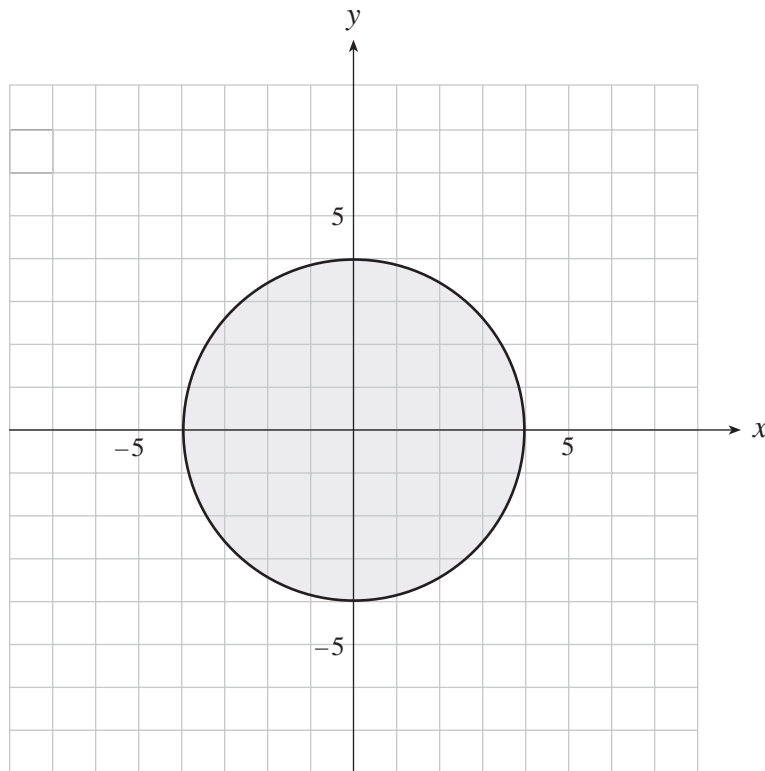
1. When  $P(x)$  is divided by  $x - 1$ , the remainder is 4. Which of the following must be true?
  - A.  $P(1) = 4$
  - B.  $P(4) = 1$
  - C.  $P(-1) = 4$
  - D.  $P(-4) = 1$
  
2. According to the Rational Root Theorem, give all possible rational roots of  $4x^3 - 2x^2 + x - 2 = 0$ .
  - A.  $\pm 1, \pm 2$
  - B.  $\pm 1, \pm 2, \pm 4$
  - C.  $\pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{4}$
  - D.  $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}$
  
3. If  $x^3 - 2x + 3$  is divided by  $x + 2$ , determine the quotient.
  - A.  $x^2 - 4x + 11$
  - B.  $x^2 + 3$
  - C.  $x^2 + 2x + 2$
  - D.  $x^2 - 2x + 2$
  
4. Determine the number of real zeros for the function  $f(x) = x^4 - 20x^3 + 7x + 3$ .
  - A. 1
  - B. 2
  - C. 3
  - D. 4

5. Which of the following conditions is true for the 3<sup>rd</sup> degree polynomial function  $y = f(x)$  graphed below? (Each mark on the axes represents 1 unit.)



- A.  $f(x) < 0$  when  $x > -3$   
B.  $f(x) < 0$  when  $x > 1$   
C.  $f(x) > 0$  when  $x > -3$   
D.  $f(x) > 0$  when  $x > 1$
6. Which conic is described by the equation  $5x^2 + y^2 + 5x + y = 0$  ?
- A. circle  
B. ellipse  
C. parabola  
D. hyperbola
7. Find the midpoint of the line segment joining  $A(-3, 5)$  and  $B(7, 1)$ .
- A.  $(2, 3)$   
B.  $(5, 2)$   
C.  $(4, 6)$   
D.  $(10, 4)$

8. Which inequality represents the shaded region below?



- A.  $x^2 + y^2 \geq 4$
  - B.  $x^2 + y^2 \leq 4$
  - C.  $x^2 + y^2 \geq 16$
  - D.  $x^2 + y^2 \leq 16$
9. Determine an equation for the set of all points  $P(x, y)$  such that the sum of the distances from  $P$  to  $A(2, 5)$  and from  $P$  to  $B(2, -7)$  is 20.

- A.  $(x+2)^2 + (y+5)^2 + (x+2)^2 + (y-7)^2 = 400$
- B.  $(x-2)^2 + (y-5)^2 + (x-2)^2 + (y+7)^2 = 400$
- C.  $\sqrt{(x+2)^2 + (y+5)^2} + \sqrt{(x+2)^2 + (y-7)^2} = 20$
- D.  $\sqrt{(x-2)^2 + (y-5)^2} + \sqrt{(x-2)^2 + (y+7)^2} = 20$

10. A parabola has a vertex of  $(1, -3)$  and passes through the points  $(7, 1)$  and  $(7, -7)$ . Determine an equation of this parabola.

- A.  $y = \frac{1}{9}(x-1)^2 - 3$   
B.  $y = -\frac{1}{9}(x-1)^2 - 3$   
C.  $x = \frac{3}{8}(y+3)^2 + 1$   
D.  $x = \frac{3}{8}(y-1)^2 - 3$

11. The following table gives values for  $Y_1 = 1.5X - 2$  and  $Y_2 = -(X-1)^2 + 2$ . From the table, determine an intersection point of the two graphs,  $Y_1$  and  $Y_2$ .

X	Y <sub>1</sub>	Y <sub>2</sub>
-1	-3.5	-2
0	-2	1
1	-0.5	2
2	1	1
3	2.5	-2
4	4	-7
5	5.5	-14

- A.  $(0, -2)$   
B.  $(-2, 1)$   
C.  $(2, 1)$   
D.  $(4, -7)$

12. Solve the system:

$$xy = 12.6$$

$$y = 0.251x^2 - 0.64x + 1$$

- A.  $(2.73, 4.62)$   
B.  $(4.28, 8.32)$   
C.  $(4.32, 2.92)$   
D.  $(6.12, 6.48)$



13. If  $(1, 3)$  is a solution for the following system, determine the value of  $k$ .

$$x^2 + y^2 = 10$$

$$3x^2 - 5y^2 = k$$

- A. -42
- B. -12
- C. 4
- D. 22

14. An ellipse, with centre in quadrant I, is tangent to both axes. The centre of the ellipse is on the line  $y = \frac{4}{3}x$ . Which of the following could be an equation of this ellipse?

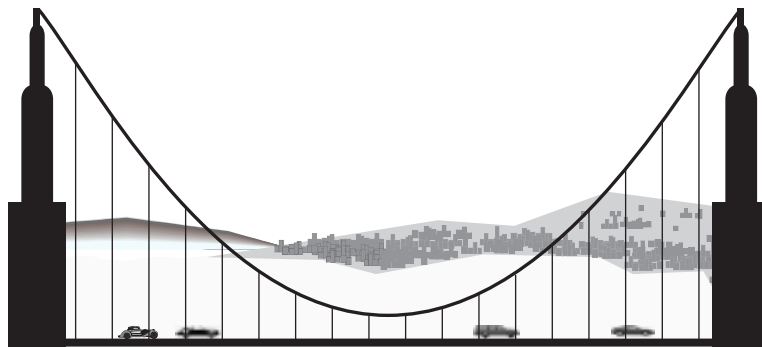
A.  $\frac{(x-3)^2}{16} + \frac{(y-4)^2}{9} = 1$

B.  $\frac{(x-3)^2}{9} + \frac{(y-4)^2}{16} = 1$

C.  $\frac{(x-4)^2}{16} + \frac{(y-3)^2}{9} = 1$

D.  $\frac{(x-4)^2}{9} + \frac{(y-3)^2}{16} = 1$

15. A suspension bridge has a cable in the shape of a parabola, as shown below. The tops of the towers at each end of the cable are 50 m above the road and 400 m apart. If the cable is 4 m above the road at the halfway point, which function could represent the shape of the cable?



- A.  $y = 0.00115x^2$   
B.  $y = 0.00125x^2$   
C.  $y = 0.0002875x^2$   
D.  $y = 0.0003125x^2$
16. Which absolute value inequality has the same solution as the quadratic inequality  $x^2 - 8x - 20 > 0$  ?
- A.  $|x - 6| > 4$   
B.  $|x - 4| > 6$   
C.  $|x + 4| > 6$   
D.  $|x + 6| > 4$
17. Change to exponential form:  $\log_2 p = t$
- A.  $p = 2^t$   
B.  $p = t^2$   
C.  $t = 2^p$   
D.  $t = p^2$

18. Which expression is equivalent to  $\log \sqrt{x}$  ?
- A.  $\log \frac{x}{2}$
  - B.  $\frac{1}{2} \log x$
  - C.  $2 \log x$
  - D.  $\frac{\log x}{\log \frac{1}{2}}$
19. When a quantity of ice is added to a bowl of water, the water's temperature,  $T$  (in degrees Celsius) is given by  $T = 23(0.84)^n$ , where  $n$  is the number of minutes after the ice is added. How many minutes will it take for the temperature of the water to reach  $5^\circ$  Celsius?
- A. 0.54
  - B. 1.84
  - C. 8.75
  - D. 9.62
20. Determine the intersection point of the graphs of  $y = 5 \log(x - 3)$  and  $y = -\log x + 3$ .
- A. (3.68, -0.83)
  - B. (5.80, 2.24)
  - C. (9.21, 3.96)
  - D. (10, 2)
21. Given  $f(x) = 10^{x+1}$ , determine  $f^{-1}(x)$ , the inverse of  $f(x)$ .
- A.  $f^{-1}(x) = \log x - 1$
  - B.  $f^{-1}(x) = \log(x - 1)$
  - C.  $f^{-1}(x) = \frac{1}{10^{x+1}}$
  - D.  $f^{-1}(x) = 10^{\frac{1}{x+1}}$

22. If  $2 < \log_a 3$ , which of the following must be true?

- A.  $2 < a < 3$
- B.  $a < 2$  or  $a > 3$
- C.  $100 < a < 1\,000$
- D.  $a < 100$  or  $a > 1\,000$

23. Given  $\log_a 2 = x$  and  $\log_a 3 = y$ , determine an expression for  $\log_a \left(\frac{6}{a}\right)$  in terms of  $x$  and  $y$ .

- A.  $x + y$
- B.  $xy$
- C.  $x + y - 1$
- D.  $xy - 1$

24. Which of the following is a geometric sequence?

- A.  $-1, 2, 4$
- B.  $-1, 2, -4$
- C.  $-1, -2, 4$
- D.  $1, -2, -4$

25. Determine the arithmetic mean between 3 and 12.

- A. 4
- B. 4.5
- C. 6
- D. 7.5

26. Determine the 3<sup>rd</sup> term of the sequence given by the following recursive definition:

$$t_1 = 5$$

$$t_n = 2t_{n-1} + n, \quad n > 1$$

- A. 12
- B. 25
- C. 26
- D. 27

27. If  $\sum_{k=1}^n k = 10$ , solve for  $n$ .

- A. 1
- B. 4
- C. 5
- D. 10

28. Given the geometric sequence  $b^2, b^{2.25}, b^{2.5}, b^{2.75}, b^3, \dots$ , determine an expression for the  $n^{\text{th}}$  term.

- A.  $b^{0.25n+1.75}$
- B.  $b^{2.25-0.25n}$
- C.  $b^{0.25n+2}$
- D.  $b^{n-0.25}$

29. Convert  $315^\circ$  to radians.

- A.  $\frac{7\pi}{4}$
- B.  $\frac{4\pi}{7}$
- C.  $\frac{\pi}{28}$
- D.  $\frac{28}{\pi}$

30. Determine the phase shift of the function  $y = 2 \cos(3x - 12)$ .

- A. 2 to the right
- B. 3 to the right
- C. 4 to the right
- D. 12 to the right

31. Evaluate:  $\csc \frac{\pi}{7}$

- A. -0.61
- B. 0.79
- C. 1.11
- D. 2.30

32. In which quadrant is  $\cot \theta > 0$  and  $\sec \theta < 0$  ?

- A. I
- B. II
- C. III
- D. IV

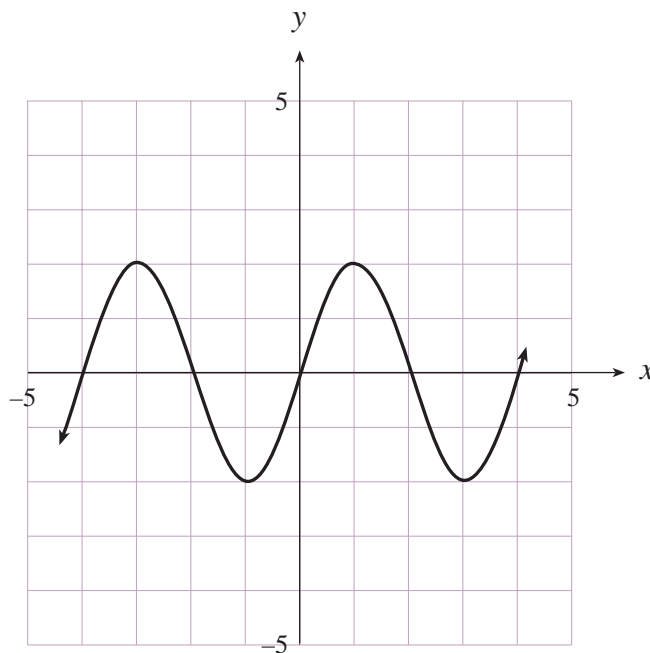
33. Solve:  $\sec x = 5$ , where  $0 \leq x < 2\pi$

- A. 0.20 , 2.94
- B. 0.98 , 5.30
- C. 1.37 , 1.77
- D. 1.37 , 4.91

34. Determine the range of the function  $y = 5 \cos 2x + 1$ .

- A.  $-5 \leq y \leq 5$
- B.  $-4 \leq y \leq 6$
- C.  $-2 \leq y \leq 2$
- D.  $-1 \leq y \leq 3$

35. Determine an equation of the sine function graphed below.



- A.  $y = \sin \pi x$
- B.  $y = 2 \sin \pi x$
- C.  $y = \sin \frac{\pi}{2} x$
- D.  $y = 2 \sin \frac{\pi}{2} x$

36. Simplify:  $\cos A \cos B - \sin A \sin B$  where  $A = \frac{\pi}{2} - B$

- A. 0
- B. 1
- C.  $\cos\left(\frac{\pi}{2} - 2B\right)$
- D.  $\cos\left(\frac{\pi}{2} + 2B\right)$

37. Determine the number of solutions of  $2 \cos^3 x - \cos x = 0$ , where  $0 \leq x < 2\pi$ .

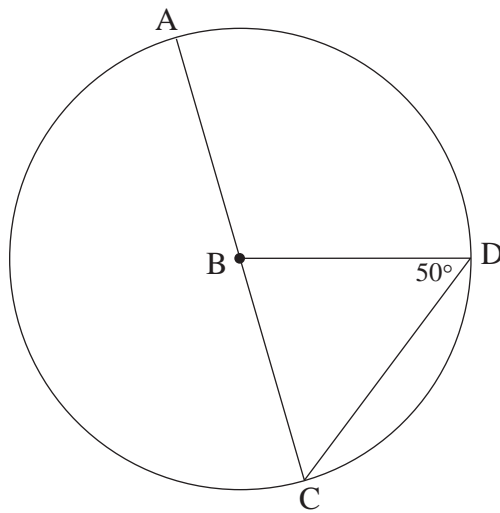
- A. 2
- B. 3
- C. 4
- D. 6

38. Determine the value of  $\sin 2\theta$ , if  $\cos \theta = -\frac{12}{13}$  and  $\pi < \theta < \frac{3\pi}{2}$ .

- A.  $-\frac{10}{13}$
- B.  $\frac{25}{169}$
- C.  $-\frac{120}{169}$
- D.  $\frac{120}{169}$

**For questions 39 and 40, diagrams are not drawn to scale.**

39. AC is the diameter of the circle with centre B. Determine the measure of  $\angle ABD$ .

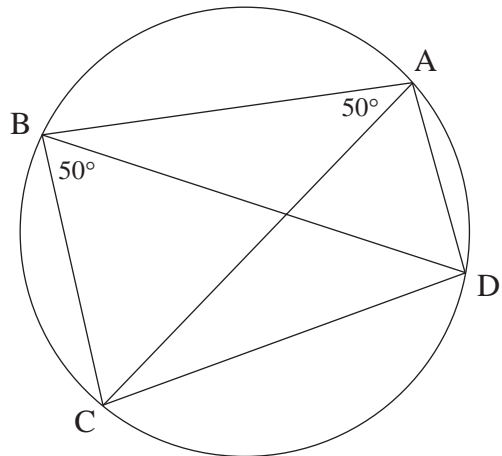


- A.  $100^\circ$
- B.  $110^\circ$
- C.  $120^\circ$
- D.  $130^\circ$



Use the following diagram to answer question 40.

Given:  $\angle BAC = 50^\circ$   
 $\angle CBD = 50^\circ$



40. Determine the measure of  $\angle BCD$ .

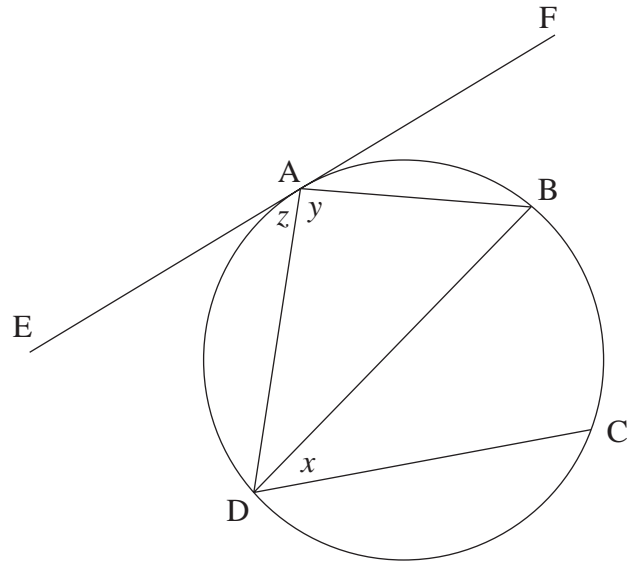
- A.  $50^\circ$
- B.  $80^\circ$
- C.  $90^\circ$
- D.  $100^\circ$

41. In a circle with radius 15, how far from the centre is a chord of length 24 ?

- A. 6
- B. 8
- C. 9
- D. 12

Use the following diagram to answer question 42.

Given:  $\widehat{AB} = \widehat{BC}$   
EF is tangent to the circle at A



42. Express  $y$  in terms of  $x$  and  $z$ .

- A.  $y = x + z$
- B.  $y = 2x + z$
- C.  $y = 180^\circ - x - z$
- D.  $y = 180^\circ - 2x - z$

43. A “Mickey Mouse®” fractal is constructed as follows:

Circle 1 is drawn with radius 8 cm.

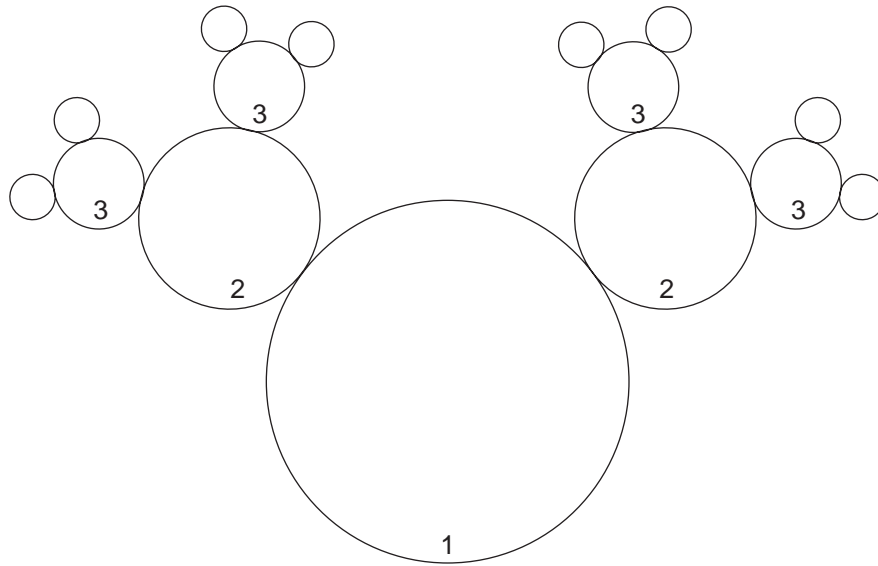
Circle 2 is drawn with a radius that is one half the radius of circle 1.

Two circle 2's are placed as the “ears” of circle 1.

Circle 3 is drawn with a radius that is one half the radius of circle 2.

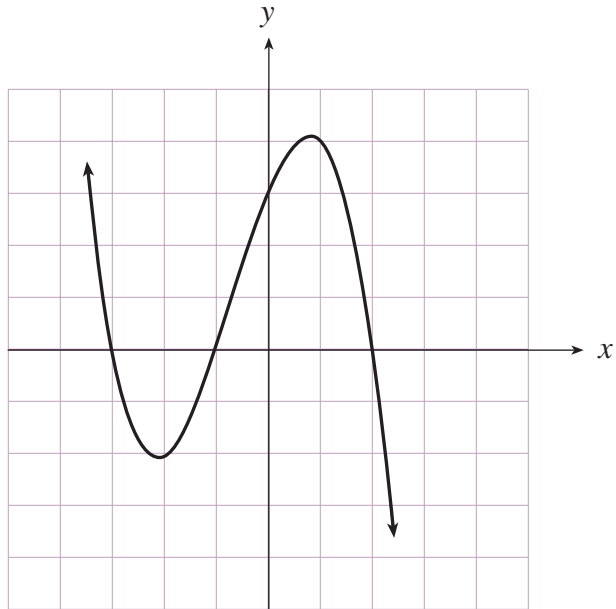
Two circle 3's are placed as the “ears” on each circle 2.

If this process is continued without end, determine the sum of the areas of all of the circles including the area of circle 1.



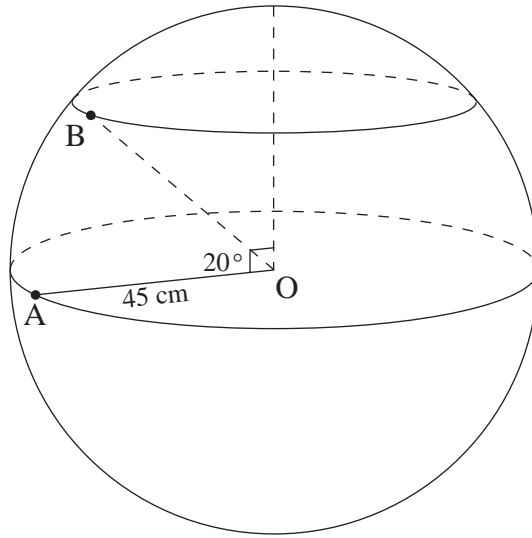
- A.  $64\pi$
- B.  $\frac{256}{3}\pi$
- C.  $120\pi$
- D.  $128\pi$

44. The graph of the cubic polynomial  $y = P(x)$  is shown below. Determine the  $y$ -intercept of  $y = -P(x) + 1$ .



- A. -4
- B. -2
- C. 2
- D. 4

45. Two circles are drawn around a sphere with centre  $O$  and radius  $45$  cm. The centre circle passing through point  $A$  is at  $0^\circ$  latitude, and a smaller circle passing through point  $B$  is at  $20^\circ$  latitude, as shown in the diagram below. Determine the radius of the circle at point  $B$ .



- A.  $15.39$  cm
- B.  $16.38$  cm
- C.  $35.00$  cm
- D.  $42.29$  cm

**This is the end of the multiple-choice section.  
Answer the remaining questions directly in this examination booklet.**

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## PART B: WRITTEN RESPONSE

Value: 25 marks

Suggested Time: 45 minutes

**INSTRUCTIONS:** Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

**Full marks will NOT be given for the final answer only.**

1. In an arithmetic sequence, the 3<sup>rd</sup> term is 7 and the 6<sup>th</sup> term is 11. Determine the 1<sup>st</sup> term of this sequence. **(3 marks)**



ANSWER:



2. Change the following equation to standard form.

**(3 marks)**

$$4x^2 - 9y^2 - 16x - 18y - 29 = 0$$

ANSWER:



3. If  $x + 2$  is a factor of the polynomial  $P(x) = 2x^3 + kx^2 - 32x - 4k^2$ , determine all possible values of  $k$ . **(3 marks)**

ANSWER:



4. Prove the identity:

**(3 marks)**

$$\frac{\sin 2\theta}{\cos \theta} + \frac{\cos 2\theta}{\sin \theta} = \csc \theta$$

LEFT SIDE	RIGHT SIDE

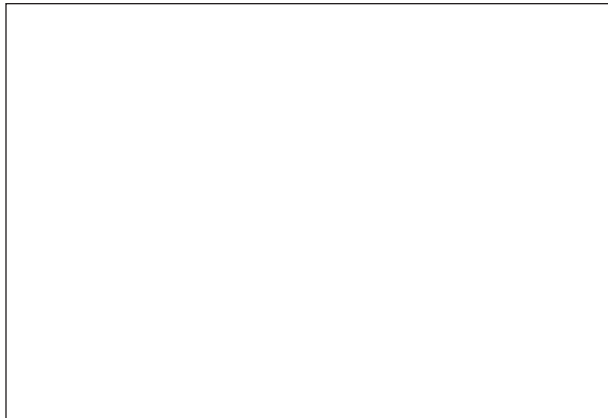


5. Solve the following equation using a graphing calculator.

**(3 marks)**

$$\log_3(x+2) = \frac{1}{2}x$$

Sketch the graph in the viewing window below and state the function(s) used in your graph. Indicate appropriate window dimensions that will provide enough of the graph so that all intersection points or all zeros are visible.



Y<sub>1</sub> =

Y<sub>2</sub> =

Y<sub>3</sub> =

Y<sub>4</sub> =

[            ,            ]            [            ,            ]

$x$   
min       $x$   
max

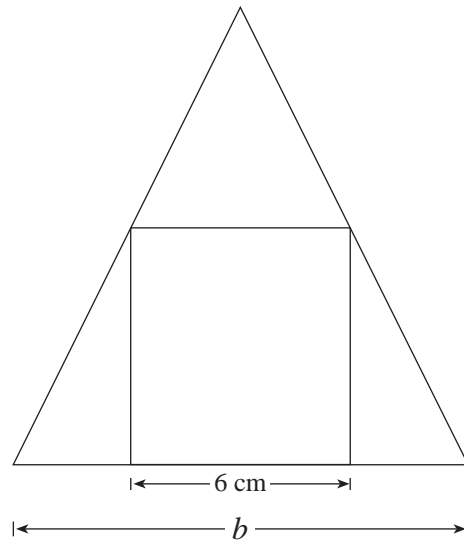
$y$   
min       $y$   
max



ANSWER:



6. A rectangle with width 6 cm and height 7 cm is inscribed in an isosceles triangle of height 12 cm, as shown in the diagram below. Determine the length of base  $b$  of the isosceles triangle. **(3 marks)**



ANSWER:

7. Solve the following system using a graphing calculator. Express all solutions as ordered pairs. **(3 marks)**

$$y = \sin \frac{\pi}{5} x$$

$$y = \frac{1}{4} x + 0.4$$

Sketch the graph in the viewing window below. State the functions that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the functions and all intersection points are visible. (*Note:* Graph at least one period of the sine curve.)



Y<sub>1</sub> =

Y<sub>2</sub> =

Y<sub>3</sub> =

Y<sub>4</sub> =

[            ,            ]  
 $x_{\min}$              $x_{\max}$

[            ,            ]  
 $y_{\min}$              $y_{\max}$

ANSWER:



8. Complete the proof.

(4 marks)

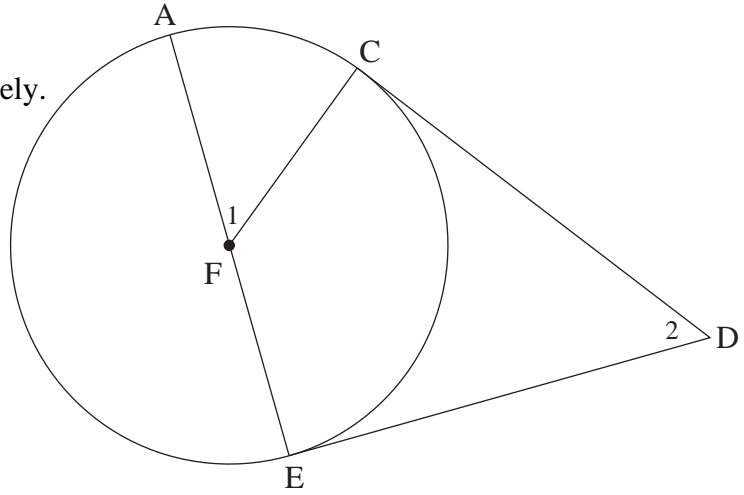
Given: DC and DE are tangents to the circle at points C and E respectively.

F is the centre

AE is a diameter

Prove:  $\angle 1 = \angle 2$

Students are encouraged to number angles.



PROOF

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**END OF EXAMINATION**

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## A SUMMARY OF BASIC IDENTITIES AND FORMULAE

### Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

### Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

### Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

### Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

### Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_n = \frac{a - \ell r}{1-r}$$

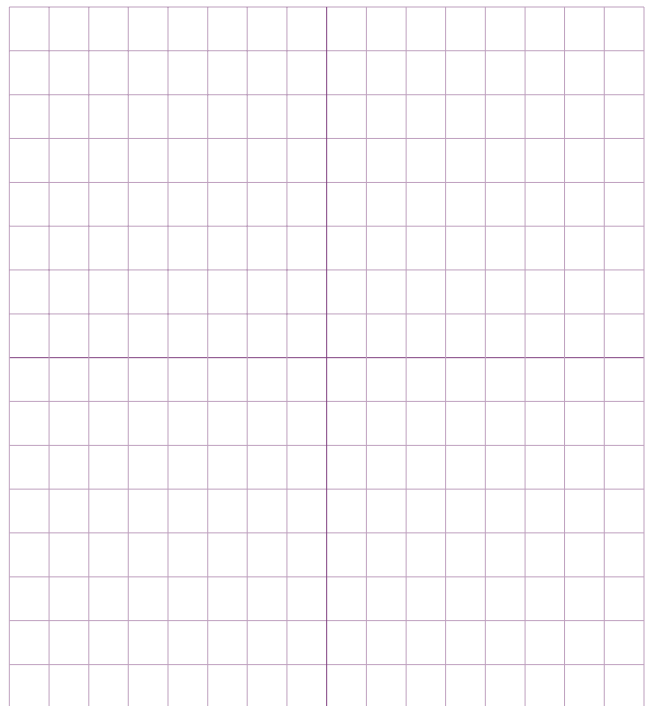
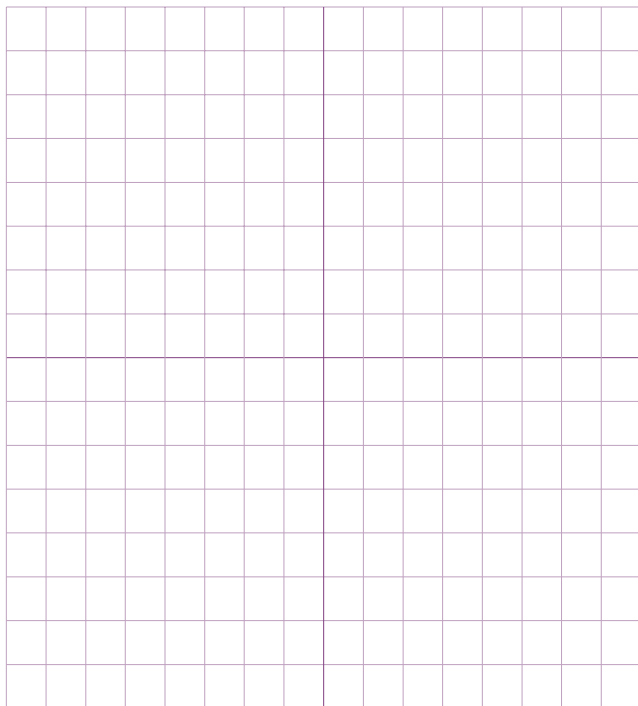
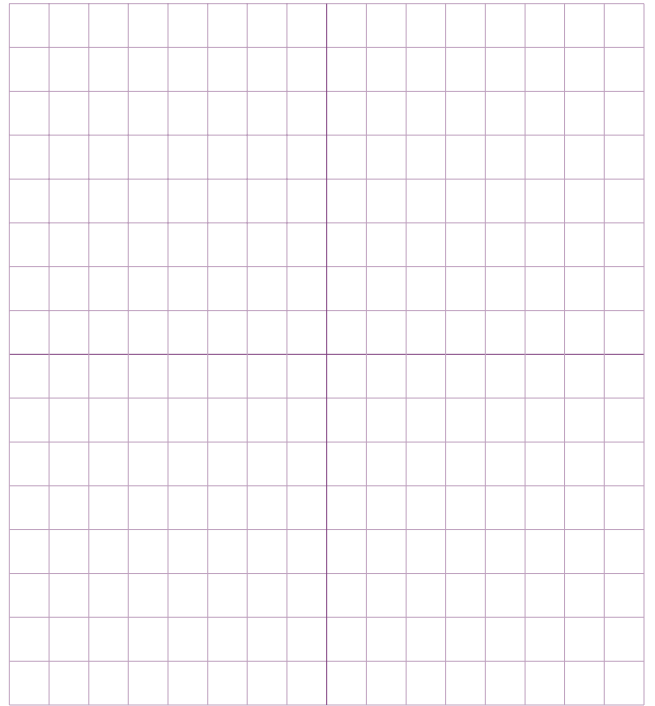
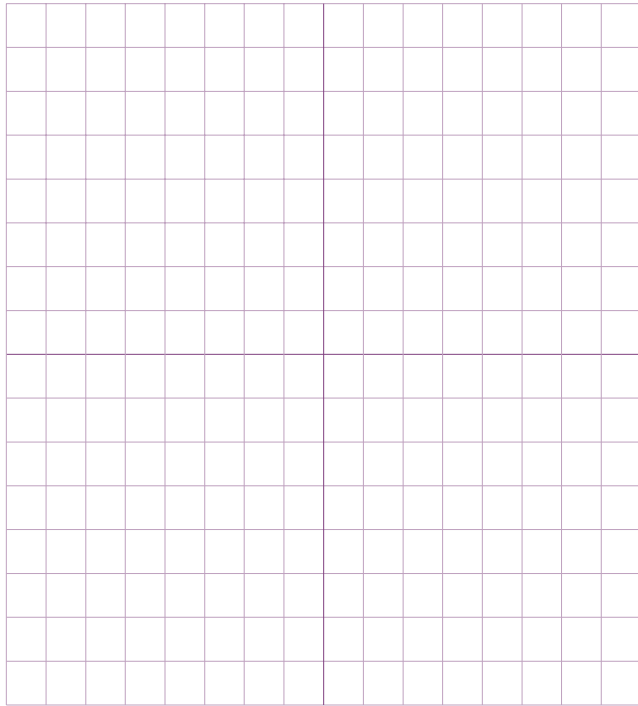
$$S = \frac{a}{1-r}$$

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Exercise care when tearing along perforations.**

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**ROUGH WORK FOR GRAPHING**

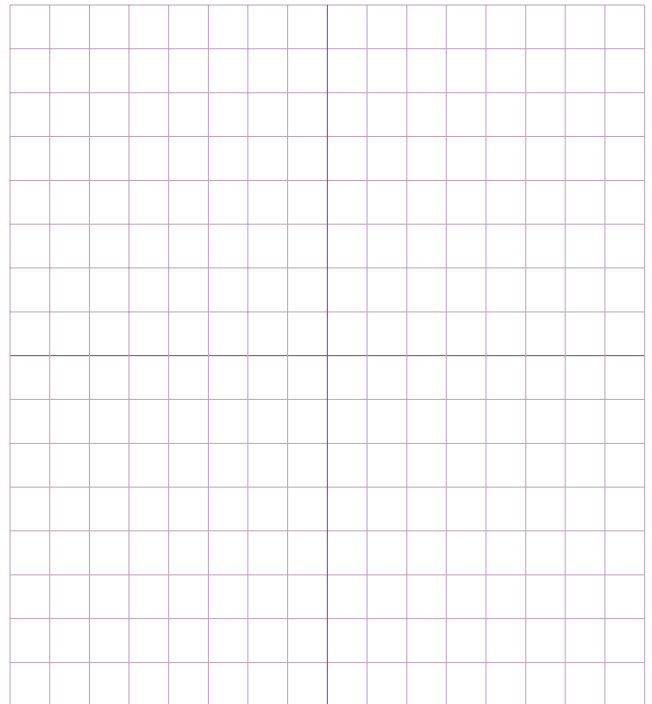
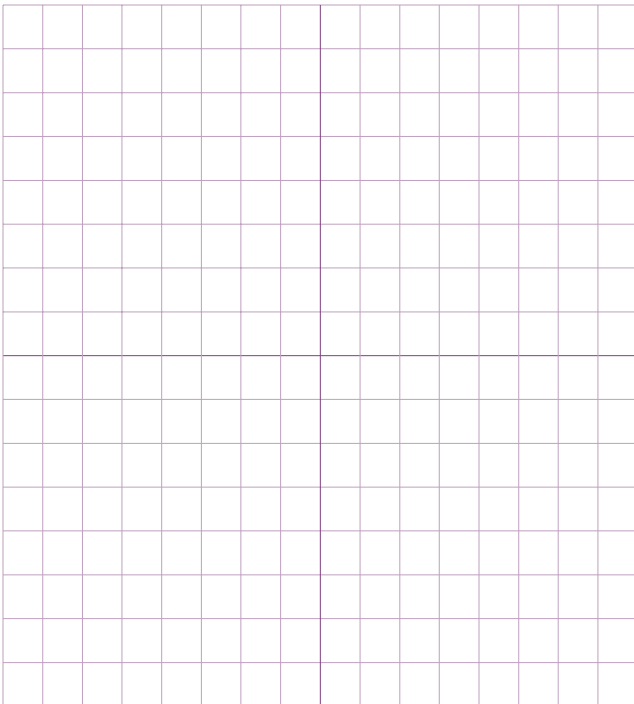
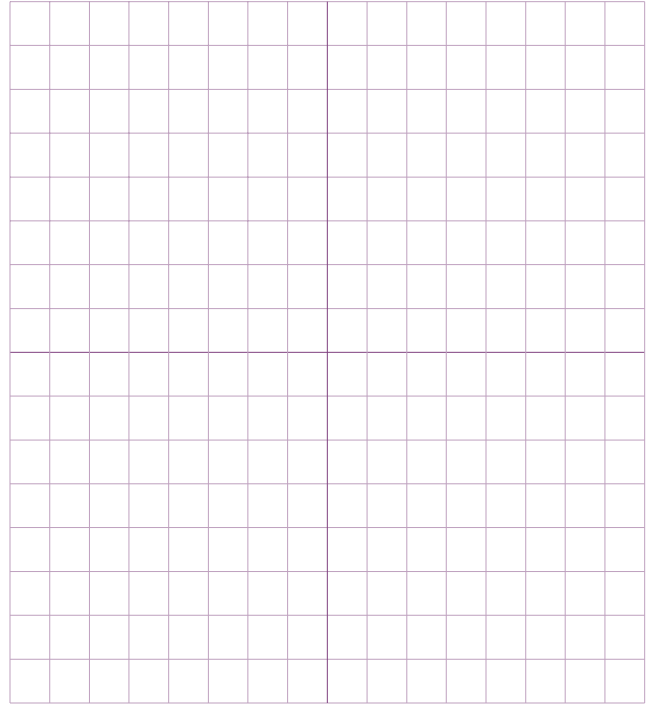
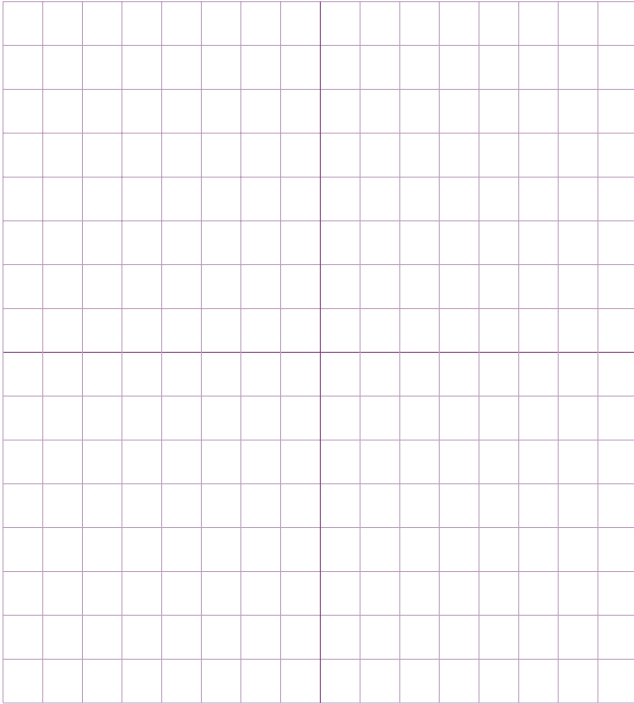
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# ROUGH WORK FOR GRAPHING

(No marks will be given for work done on this page.)

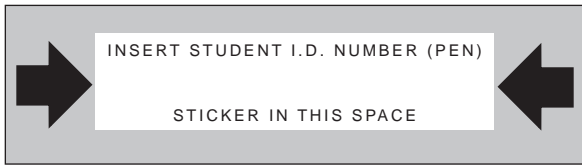


**ROUGH WORK FOR MULTIPLE-CHOICE**

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## ROUGH WORK FOR MULTIPLE-CHOICE





# **PRINCIPLES OF MATHEMATICS 12**

**August 1999**

Course Code = MA



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**PRINCIPLES OF  
MATHEMATICS 12**

**August 1999**

Course Code = MA

Score for  
Question 1:

1.  $\frac{\quad}{(3)}$

Score for  
Question 8:

8.  $\frac{\quad}{(4)}$

Score for  
Question 2:

2.  $\frac{\quad}{(3)}$

Score for  
Question 3:

3.  $\frac{\quad}{(3)}$

Score for  
Question 4:

4.  $\frac{\quad}{(3)}$

Score for  
Question 5:

5.  $\frac{\quad}{(3)}$

Score for  
Question 6:

6.  $\frac{\quad}{(3)}$

Score for  
Question 7:

7.  $\frac{\quad}{(3)}$