

JUNE 1999

PROVINCIAL EXAMINATION

MINISTRY OF EDUCATION

PRINCIPLES OF MATHEMATICS 12

GENERAL INSTRUCTIONS

1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above and on the **back** cover of this booklet. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this booklet.**
2. Ensure that in addition to this examination booklet, you have an **Examination Response Form**. Follow the directions on the front of the Response Form.
3. **Disqualification** from the examination will result if you bring books, paper, notes or unauthorized electronic devices into the examination room.
4. All multiple-choice answers must be entered on the Response Form using an **HB pencil**. Multiple-choice answers entered in this examination booklet will **not** be marked.
5. For each of the written-response questions, write your answer in the space provided in this booklet.
6. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

END OF EXAMINATION.

7. At the end of the examination, place your Response Form inside the front cover of this booklet and return the booklet and your Response Form to the supervisor.

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PRINCIPLES OF MATHEMATICS 12 PROVINCIAL EXAMINATION

- | | Value | Suggested Time |
|---|------------------------|--------------------|
| 1. This examination consists of two parts: | | |
| PART A: 45 multiple-choice questions | 45 | 75 |
| PART B: 8 written-response questions | 25 | 45 |
| | Total: 70 marks | 120 minutes |
- Aside from an approved calculator, electronic devices including dictionaries and pagers are **not** permitted in the examination room.
 - The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae, Rough Work for Graphing, and Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.
 - You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.
 - A graphing calculator is essential for the Principles of Mathematics 12 Provincial Examination.** The calculator must be a hand-held device designed primarily for mathematical computations involving logarithmic and trigonometric functions as well as for graphing functions. Computers, calculators with a QWERTY keyboard, and electronic writing pads will not be allowed. Students must not bring any external devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or external keyboards. Students may have more than one calculator available during the examination of which one may be a scientific calculator. Calculators may not be shared and must not have the ability to either transmit or receive electronic signals. In addition to an approved calculator, students will be allowed to use rulers, compasses, and protractors during the examination.
 - If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.
 - When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.
 - This examination is designed to be completed in **two hours**. Students may, however, take up to **20 minutes** of additional time to finish.

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PART A: MULTIPLE CHOICE

Value: 45 marks

Suggested Time: 75 minutes

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the Response Form provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. When the polynomial $p(x)$ is divided by $x - 4$, the remainder is 6. Which of the following must be true?

- A. $p(4) = 6$
- B. $p(-4) = 6$
- C. $p(6) = 4$
- D. $p(-6) = 4$

2. Solve: $x^3 - 7x - 6 = 0$

- A. $-1, -2, 3$
- B. $-1, 2, -3$
- C. $1, -2, 3$
- D. $1, 2, -3$

3. Determine the largest root of $x^3 - 30x^2 + 235x - 430 = 0$.

- A. 2.64
- B. 8.74
- C. 18.62
- D. 18.75

4. According to the Rational Root Theorem, which of the following equations has possible rational roots of $\pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$?

- A. $3x^3 - 4x^2 + 5x + 1 = 0$
- B. $6x^3 - 4x^2 + 5x + 1 = 0$
- C. $2x^3 - 4x^2 + 5x + 3 = 0$
- D. $3x^3 - 4x^2 + 5x + 2 = 0$

5. Which of the following is a polynomial function with zeros of $-\sqrt{2}$, $\sqrt{2}$ and -1 ?

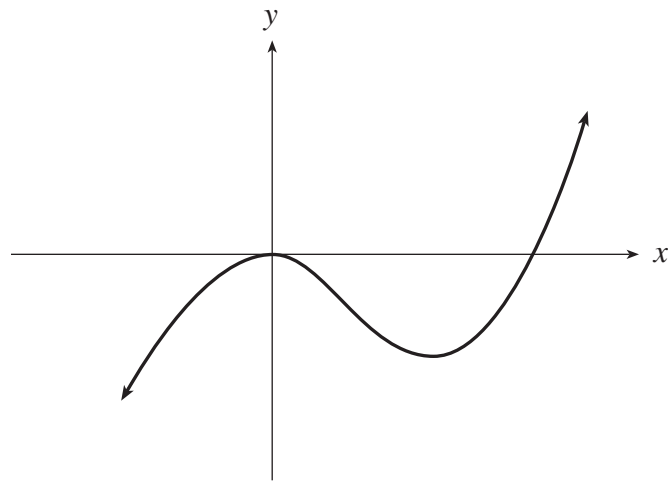
A. $p(x) = x^3 - x^2 - 2x + 2$

B. $p(x) = x^3 + x^2 - 2x - 2$

C. $p(x) = x^3 - x^2 - 4x + 4$

D. $p(x) = x^3 + x^2 - 4x - 4$

6. The graph of the polynomial function $p(x) = ax^3 + bx^2 + cx + d$, where a , b , c and d are constants, is shown below. What are the conditions on c and d ?



A. $c = 0, d = 0$

B. $c = 0, d > 0$

C. $c > 0, d = 0$

D. $c \neq 0, d = 0$

7. Determine the midpoint of the line segment joining the points $A(-8, -7)$ and $B(4, -3)$.

A. $(-6, -2)$

B. $(-2, -5)$

C. $(-4, -10)$

D. $(6, 2)$

8. Which absolute value inequality describes the solution shown?



- A. $|x - 1| < 4$
- B. $|x + 1| < 4$
- C. $|x - 1| > 4$
- D. $|x + 1| > 4$

9. For which of the following values of the constant A will the equation $Ax^2 + 6y^2 = 36$ represent an ellipse?

- A. -12
- B. -6
- C. 0
- D. 12

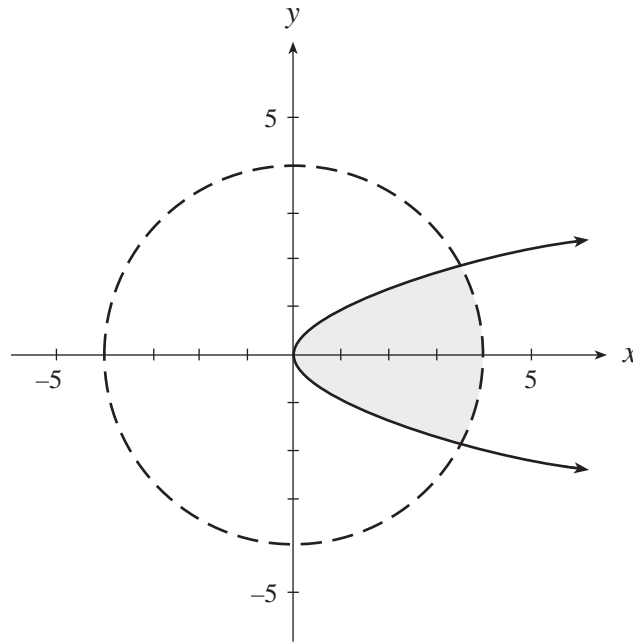
10. What is the length of the conjugate axis of the hyperbola $\frac{x^2}{16} - \frac{y^2}{25} = 1$?

- A. 4
- B. 5
- C. 8
- D. 10

11. Change $2y^2 - 4y - x - 2 = 0$ to standard form.

- A. $x = (y - 1)^2 - 1$
- B. $x = (y - 1)^2 - 2$
- C. $x = 2(y - 1)^2 - 4$
- D. $x = 2(y - 1)^2 - 3$

12. Which system has the shaded region shown below as its solution?



A. $x \geq y^2$
 $x^2 + y^2 < 16$

B. $x \leq y^2$
 $x^2 + y^2 > 16$

C. $x \leq y^2$
 $x^2 + y^2 < 16$

D. $x \geq y^2$
 $x^2 + y^2 > 16$

13. Two separate square pig pens are constructed. One has sides of length x and the other has sides of length y . A total of 76 metres of fencing is used to construct the two pens. The total area of these two pens is 205 square metres. Which of the following systems could be used to find x and y ?

A. $2x + 2y = 76$
 $2xy = 205$

B. $x + y = 76$
 $x^2 + y^2 = 205$

C. $x + y = 205$
 $x^2 + y^2 = 76$

D. $4x + 4y = 76$
 $x^2 + y^2 = 205$

14. A hyperbola with vertices $(-2, 5)$ and $(-2, -1)$, has an asymptote that passes through the point $(2, 5)$. Determine an equation of the hyperbola.

A. $\frac{(y-2)^2}{25} - \frac{(x+2)^2}{4} = 1$

B. $\frac{(y+2)^2}{9} - \frac{(x-2)^2}{16} = 1$

C. $\frac{(y+2)^2}{25} - \frac{(x-2)^2}{4} = 1$

D. $\frac{(y-2)^2}{9} - \frac{(x+2)^2}{16} = 1$

15. Which of the following is an arithmetic sequence?

- A. 1, 4, 7
- B. 1, 4, 8
- C. 1, 4, 9
- D. 1, 4, 16

16. Determine the number of terms in the series:

$$\sum_{k=4}^{67} (3k - 5)$$

- A. 63
- B. 64
- C. 67
- D. 68

17. Find the sum of the infinite geometric series: $175 - 70 + 28 - \dots$

- A. 125
- B. 133
- C. 291.67
- D. no finite sum

18. Determine the 3rd term of the sequence given by the following recursive definition:

$$t_1 = 12$$

$$t_2 = 6$$

$$t_n = t_{n-1} + 2t_{n-2}, \quad n > 2$$

- A. 1
- B. 24
- C. 30
- D. 96

19. Given a geometric series with a first term of 14 and a common ratio of 1.8, determine the sum of the first 10 terms.

- A. -4 986.45
- B. 2 777.03
- C. 3 453.79
- D. 6 230.82

20. Determine the number of terms in the following geometric sequence:

$$\frac{a^2}{b}, a, b, \dots, \frac{b^{15}}{a^{14}}$$

- A. 14
- B. 15
- C. 16
- D. 17

21. The 1st term of an arithmetic sequence is 9. If $t_{10} + t_{11} = 50.3$, find the common difference.

- A. 1.7
- B. 1.8
- C. 1.9
- D. 2.0

22. Change $y = \log_5 x$ to exponential form.

- A. $x = 5y$
- B. $x = 5^y$
- C. $x = y^5$
- D. $x = \left(\frac{1}{5}\right)^y$

23. Express $\log a - \log b + 2 \log c$ as a single logarithm.

- A. $\log \frac{ac^2}{b}$
- B. $\log \frac{a}{bc^2}$
- C. $\log \frac{ab}{2c}$
- D. $\log \frac{a}{2bc}$

24. Solve for x : $3^x = 18$

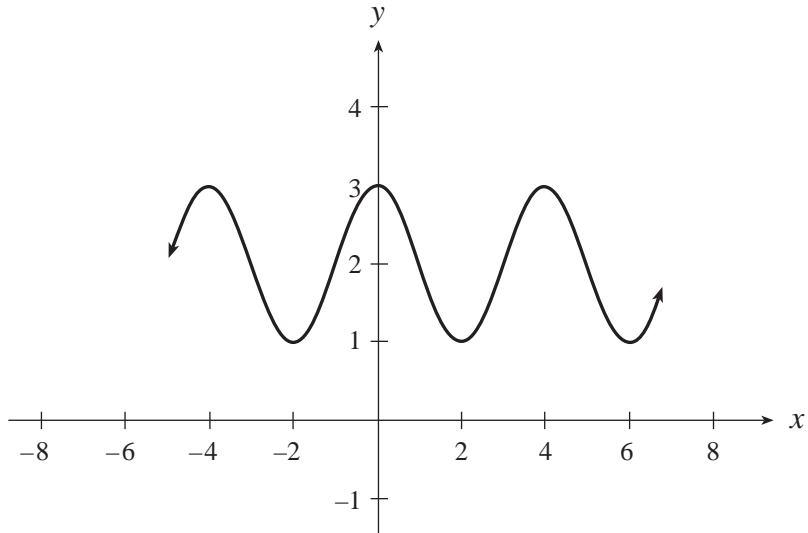
- A. $x = \log 6$
- B. $x = 3 \log 18$
- C. $x = \frac{\log 18}{\log 3}$
- D. $x = \log 18 - \log 3$

25. Determine an equation of the asymptote of the graph of $y = 4 \log_5(x - 4) + 5$.
- A. $x = 4$
 - B. $x = 5$
 - C. $y = 4$
 - D. $y = 5$
26. Solve: $\log_2(4 - x) - \log_2 x = 1$
- A. 1
 - B. $\frac{4}{3}$
 - C. $\frac{3}{2}$
 - D. 2
27. Determine an expression equivalent to $\frac{1}{\log_a x} - \frac{1}{\log_b x}$.
- A. $\log_a x - \log_b x$
 - B. $\log_x a - \log_x b$
 - C. $\log_b x - \log_a x$
 - D. $\log_x b - \log_x a$
28. The point (m, n) is on the graph of $f(x) = a^x$. Which of the following must be a point on the graph of $g(x) = \log_a x$ ($a > 0$)?
- A. $(-m, -n)$
 - B. $(-n, -m)$
 - C. (m, n)
 - D. (n, m)

29. Convert 200° to radians.

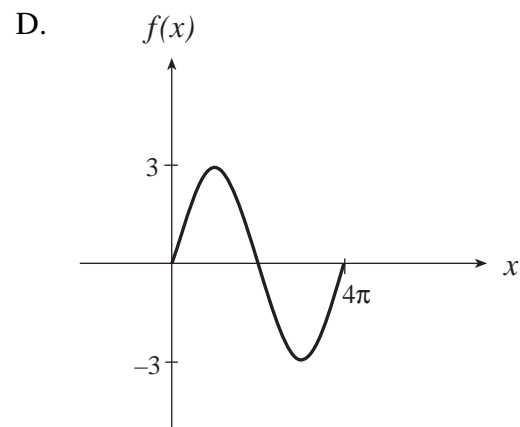
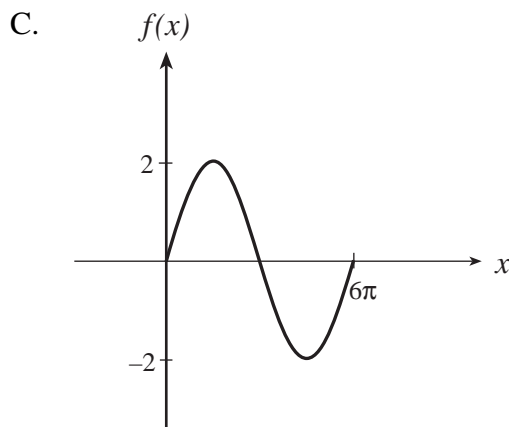
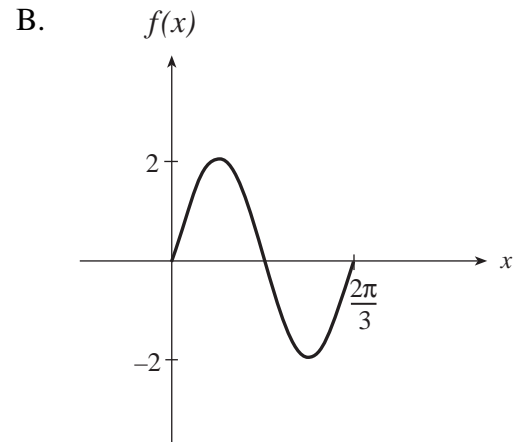
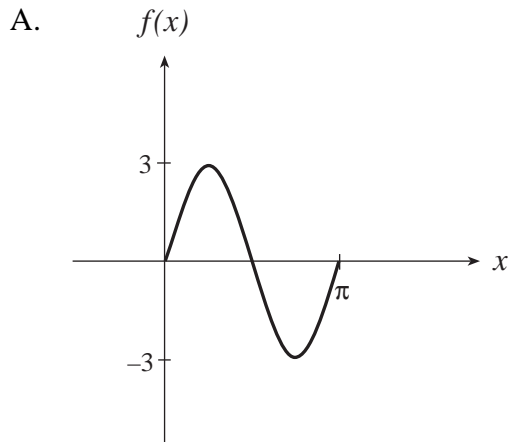
- A. 2.83
- B. 3.49
- C. 3.83
- D. 4.49

30. Determine the period of the trigonometric function graphed below.



- A. 1
- B. 2
- C. 3
- D. 4

31. Which of the following shows one period of the graph of $f(x) = 2 \sin 3x$?



32. Evaluate: $\sec \frac{2\pi}{5}$

- A. 0.70
- B. 1.05
- C. 1.43
- D. 3.24

33. If $\cos \theta = \frac{5}{13}$, where θ is in quadrant IV, determine the value of $\cot \theta$.
- A. $-\frac{12}{5}$
- B. $-\frac{5}{12}$
- C. $\frac{5}{12}$
- D. $\frac{12}{5}$
34. How many solutions does $\tan^2 x + 5 \cos x - 8 = 0$ have over the interval $0 \leq x < 2\pi$?
- A. 1
- B. 2
- C. 3
- D. 4
35. Solve: $\tan^2 x = \tan x$, $0 \leq x < 2\pi$
- A. 0, 0.79
- B. 0.79, 3.93
- C. 0, 0.79, 3.14, 3.93
- D. 0, 2.36, 3.14, 5.50
36. Determine all restrictions for the expression:
- $$\frac{\sec x}{4 \sin^2 x - 1}$$
- A. $\sin x \neq \pm \frac{1}{4}$
- B. $\sin x \neq \pm \frac{1}{2}$
- C. $\cos x \neq 0$, $\sin x \neq \pm \frac{1}{4}$
- D. $\cos x \neq 0$, $\sin x \neq \pm \frac{1}{2}$

37. The range of the trigonometric function $y = a \cos x + b$ is $-2 \leq y \leq 8$. Determine the value of b .

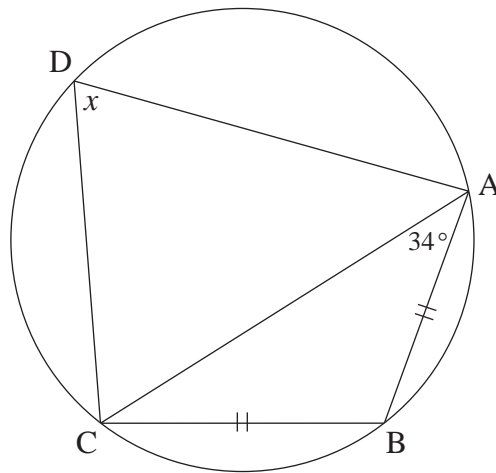
- A. 3
- B. 5
- C. 6
- D. 10

38. In an arithmetic sequence, $t_1 = \sin(A - B)$ and $t_2 = \sin(A + B)$. Determine an expression for t_5 .

- A. $\sin A \cos B + 4 \cos A \sin B$
- B. $\sin A \cos B + 5 \cos A \sin B$
- C. $\sin A \cos B + 6 \cos A \sin B$
- D. $\sin A \cos B + 7 \cos A \sin B$

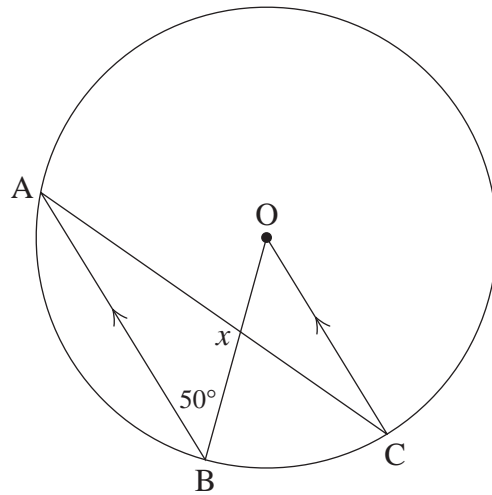
For questions 39 to 42, diagrams are not drawn to scale.

39. In the diagram below, $AB = BC$ and $\angle BAC = 34^\circ$. Determine the measure of $\angle x$.



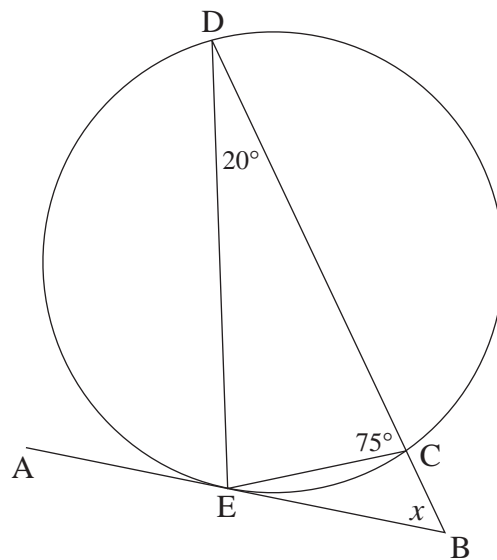
- A. 34°
- B. 60°
- C. 68°
- D. 112°

40. In the diagram below, O is the centre of the circle, and $AB \parallel OC$. Determine the measure of $\angle x$.



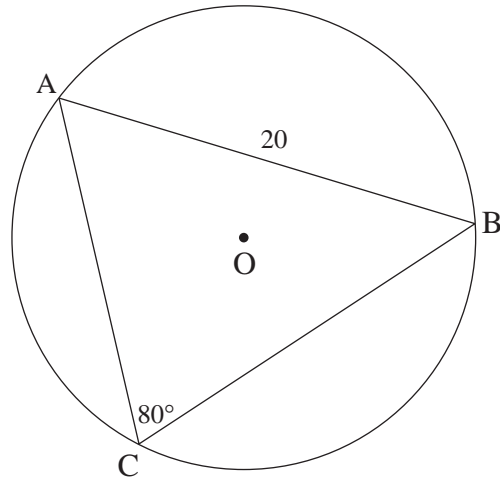
- A. 75°
- B. 80°
- C. 90°
- D. 105°

41. In the diagram below, AB is tangent to the circle at E. Determine the measure of $\angle x$.



- A. 45°
- B. 50°
- C. 55°
- D. 60°

42. In the diagram below, O is the centre of the circle, and $AB = 20$. Find the radius of the circle.



- A. 10.15
- B. 11.62
- C. 15.56
- D. 17.43

43. How many points of intersection are there for the following system?

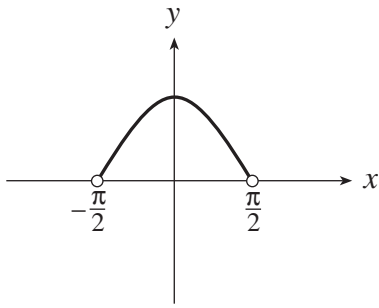
$$y = \sin \pi x$$

$$y = 0.2x$$

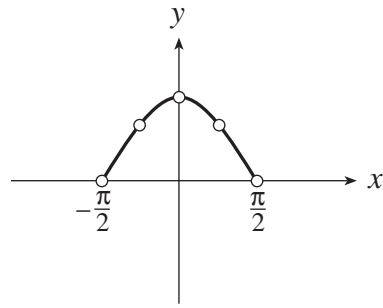
- A. 9
- B. 10
- C. 11
- D. 12

44. Which of the following represents the graph of the function $\log_{|x|} y = \log_{|x|} \cos x$ over the interval $-\pi \leq x \leq \pi$?

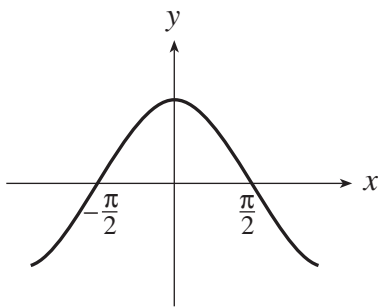
A.



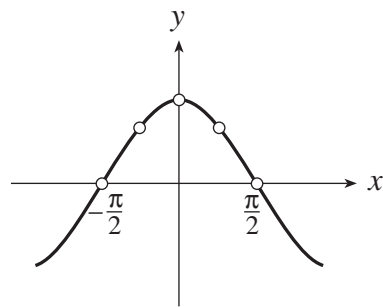
B.



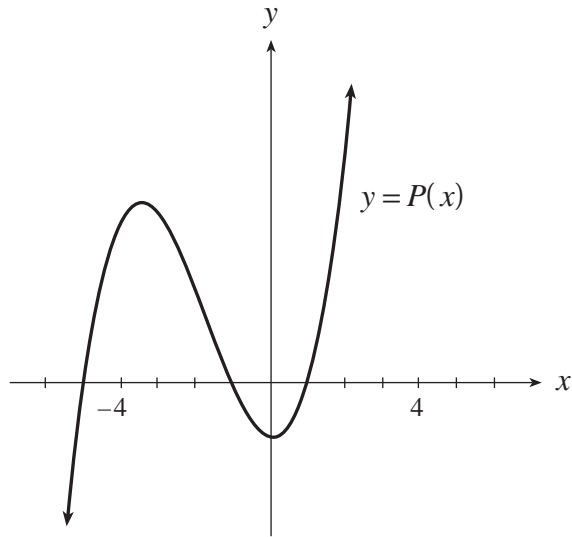
C.



D.



45. The graph of the cubic polynomial function $y = P(x)$ is shown below. Determine the zeros of $y = xP(-x)$.



- A. $-5, -1, 0, 1$
- B. $-5, -1, 1$
- C. $-1, 0, 1, 5$
- D. $-1, 1, 5$

This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.

PART B: WRITTEN RESPONSE

Value: 25 marks

Suggested Time: 45 minutes

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

Full marks will NOT be given for the final answer only.

1. Solve the following system algebraically. Express all solutions as ordered pairs. **(3 marks)**

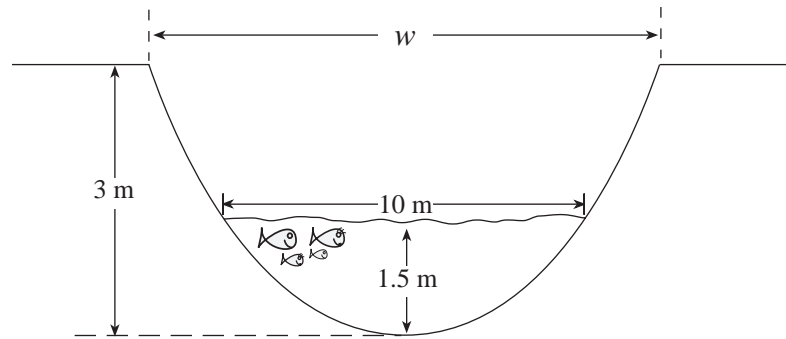
$$3x^2 + 4y^2 = 21$$

$$x^2 - 2y = 1$$

ANSWER:



2. The cross section of a drainage ditch is parabolic in shape, as shown in the diagram below. When the width of the water surface is 10 metres, the maximum depth of the water is 1.5 metres. Determine the width of the water, w , when the maximum depth is 3 metres. **(3 marks)**



ANSWER:



3. Prove the identity:

(3 marks)

$$\frac{\sin \theta + \tan \theta}{1 + \cos \theta} = \frac{\sin 2\theta}{2 \cos^2 \theta}$$

LEFT SIDE

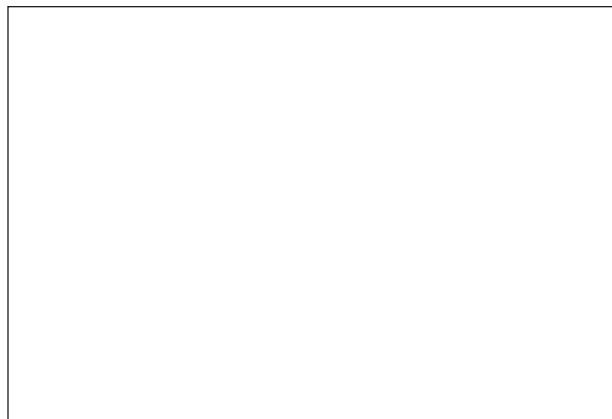
RIGHT SIDE

4. Solve the following inequality using a graphing calculator.

(3 marks)

$$x^3 - 8x^2 > 18x - 20$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window. The solution may be given in algebraic form or shown on a number line.



$Y_1 =$

$Y_2 =$

$Y_3 =$

$Y_4 =$

[,] [,]

x
min x
max

y
min y
max

ANSWER:



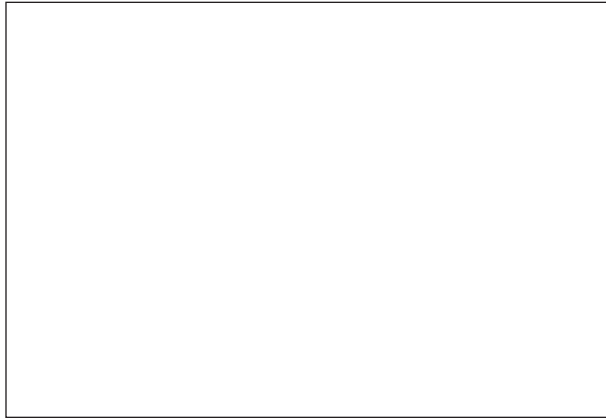
5. Solve the following system using a graphing calculator.

(3 marks)

$$x^2 + y^2 = 4$$

$$y = 10 \log x$$

Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the function(s) and all intersection points are visible.



$Y_1 =$

$Y_2 =$

$Y_3 =$

$Y_4 =$

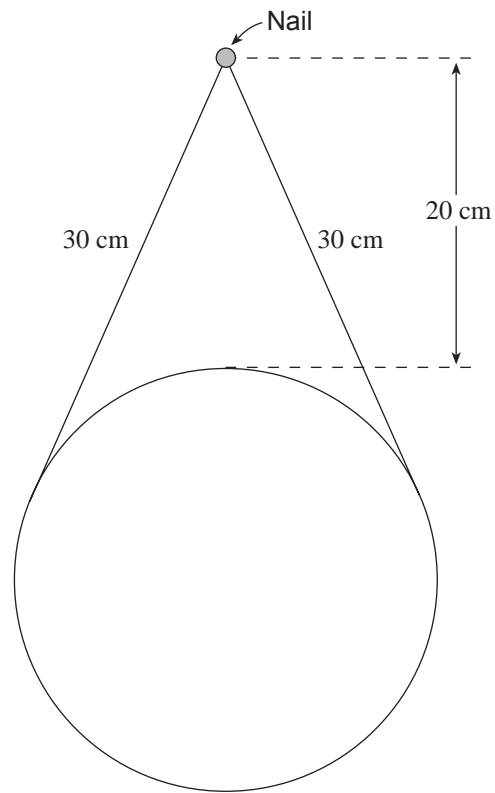
[,] [,]

x x
min max

y y
min max

ANSWER:

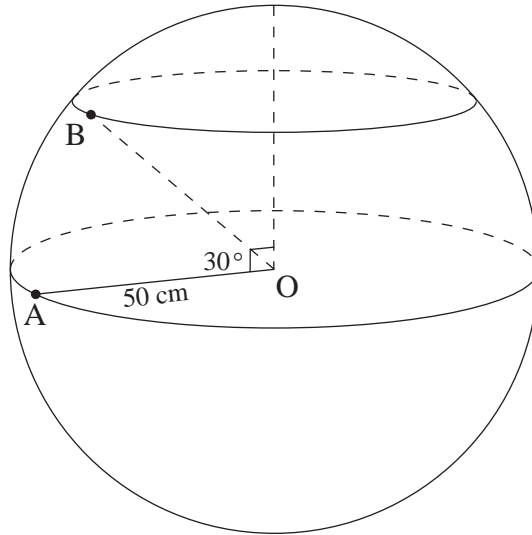
6. A circular mirror is suspended from a nail by two wires, each of length 30 cm, which are tangent to the mirror. If the nail is 20 cm from the top of the mirror, determine the diameter of the mirror. **(3 marks)**



ANSWER:



7. Two circles are drawn around a sphere with centre O and radius 50 cm. The centre circle passing through point A is at 0° latitude, and a smaller circle passing through point B is at 30° latitude, as shown in the diagram below. Determine the circumference of the circle at point B . **(3 marks)**



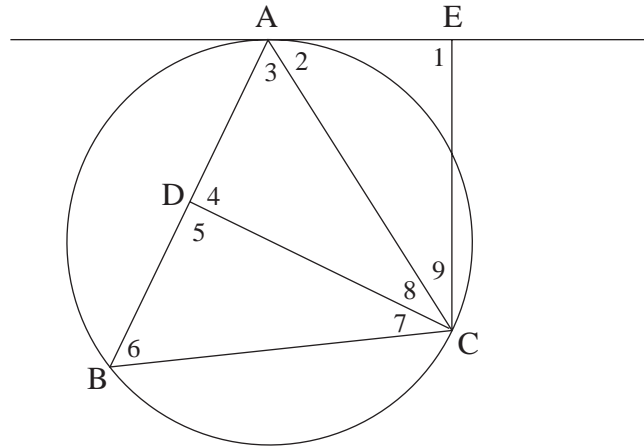
ANSWER:

8. Complete the proof.

(4 marks)

Given: AE is tangent to the circle at A
 $CE \perp AE$
 $CD \perp AB$
 $AC = BC$

Prove: $CE = CD$



PROOF

END OF EXAMINATION

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A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_n = \frac{a - \ell r}{1-r}$$

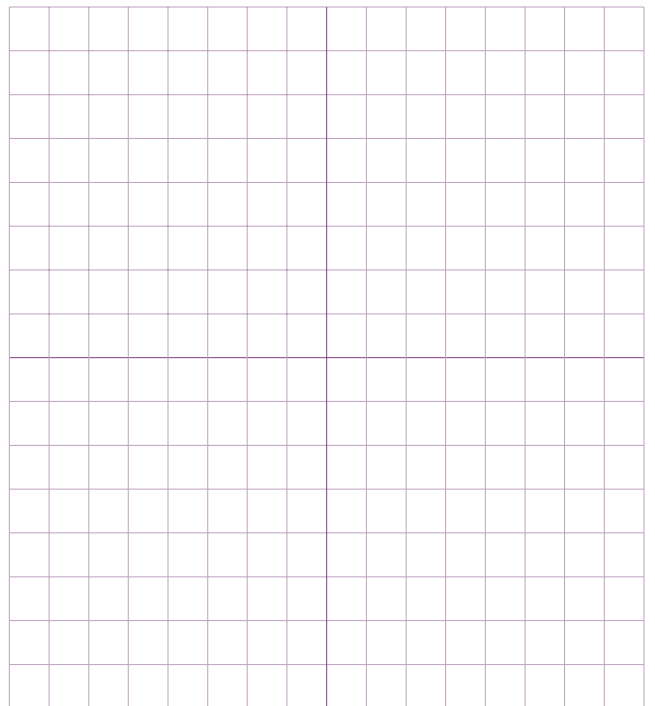
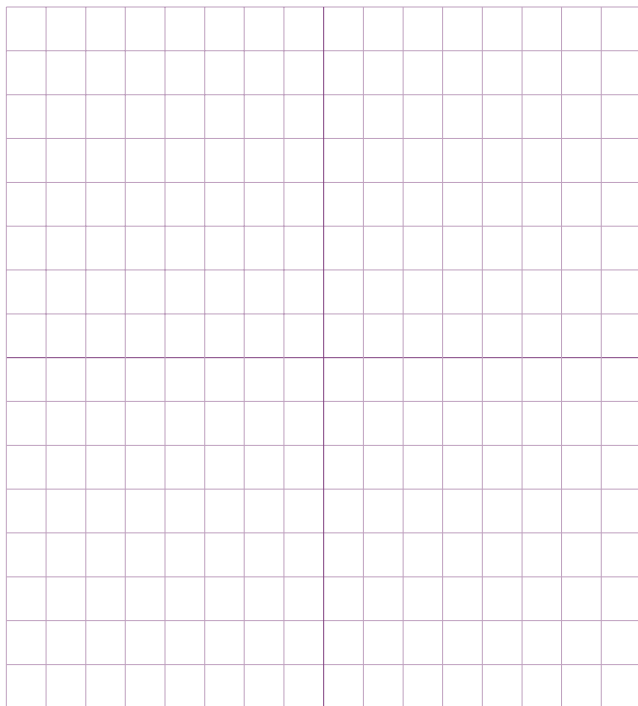
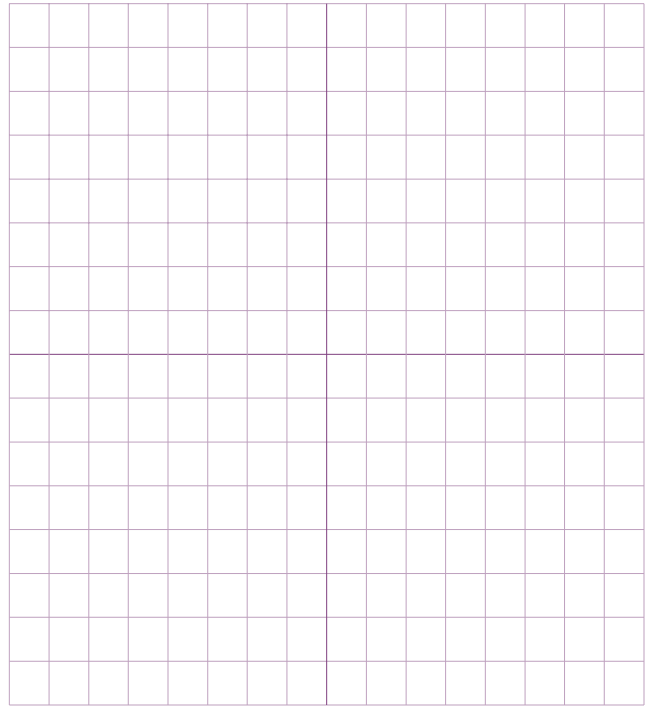
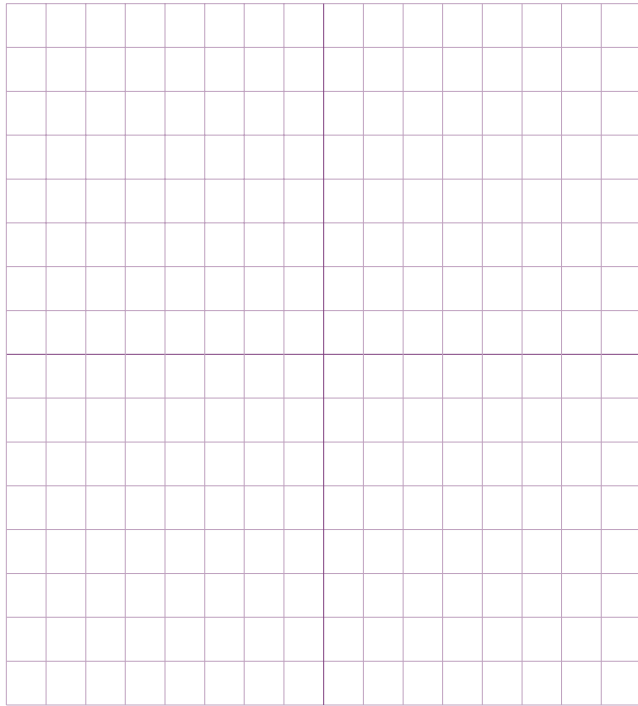
$$S = \frac{a}{1-r}$$

**You may detach this page for convenient reference.
Exercise care when tearing along perforations.**

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ROUGH WORK FOR GRAPHING

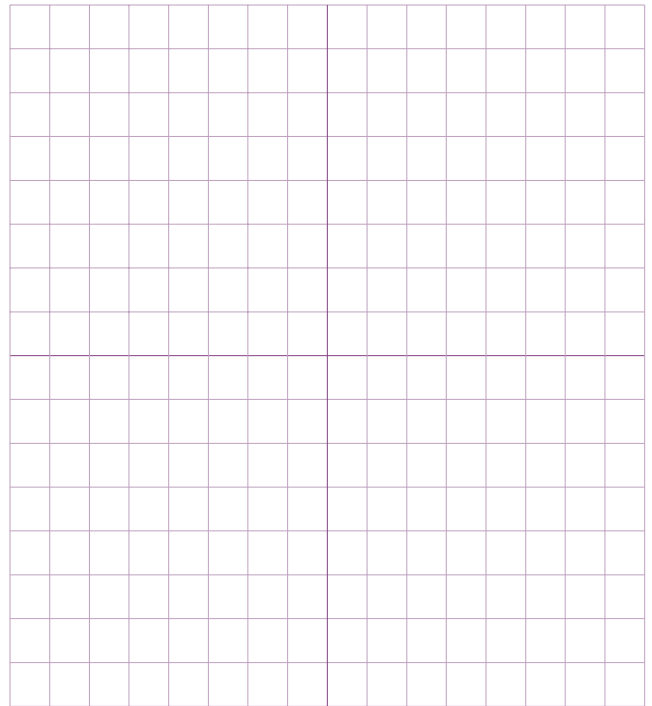
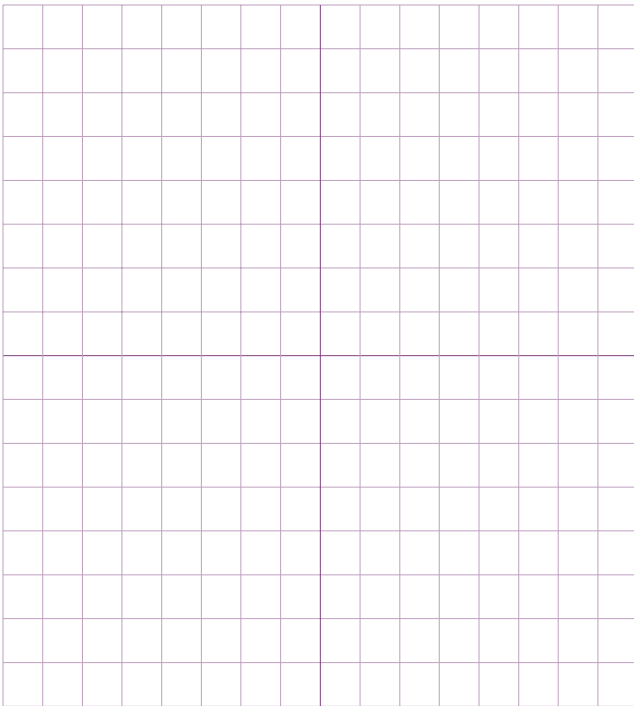
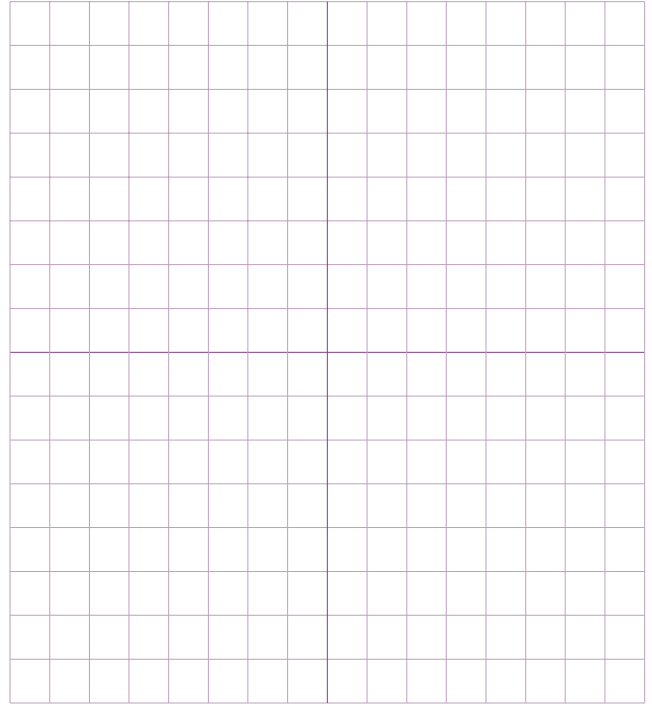
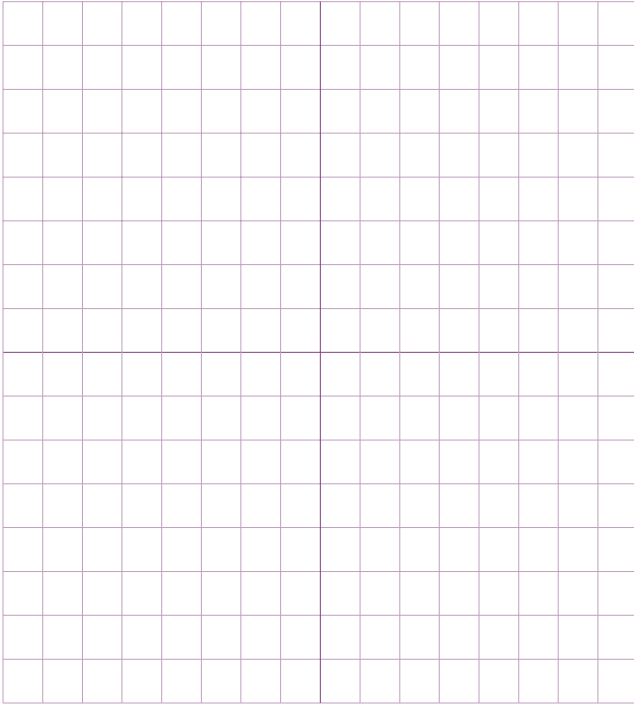
(No marks will be given for work done on this page.)



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ROUGH WORK FOR GRAPHING

(No marks will be given for work done on this page.)

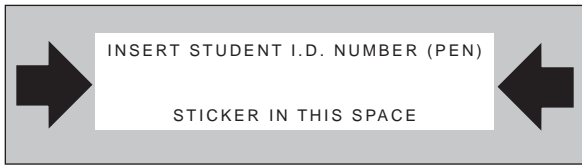


ROUGH WORK FOR MULTIPLE-CHOICE

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ROUGH WORK FOR MULTIPLE-CHOICE





PRINCIPLES OF MATHEMATICS 12

June 1999

Course Code = MA

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**PRINCIPLES OF
MATHEMATICS 12**

June 1999

Course Code = MA

Score for
Question 1:

1. $\frac{\quad}{(3)}$

Score for
Question 8:

8. $\frac{\quad}{(4)}$

Score for
Question 2:

2. $\frac{\quad}{(3)}$

Score for
Question 3:

3. $\frac{\quad}{(3)}$

Score for
Question 4:

4. $\frac{\quad}{(3)}$

Score for
Question 5:

5. $\frac{\quad}{(3)}$

Score for
Question 6:

6. $\frac{\quad}{(3)}$

Score for
Question 7:

7. $\frac{\quad}{(3)}$