

JANUARY 1999

PROVINCIAL EXAMINATION

MINISTRY OF EDUCATION

PRINCIPLES OF MATHEMATICS 12

GENERAL INSTRUCTIONS

1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above and on the **back** cover of this booklet. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this booklet.**
2. Ensure that in addition to this examination booklet, you have an **Examination Response Form**. Follow the directions on the front of the Response Form.
3. **Disqualification** from the examination will result if you bring books, paper, notes or unauthorized electronic devices into the examination room.
4. All multiple-choice answers must be entered on the Response Form using an **HB pencil**. Multiple-choice answers entered in this examination booklet will **not** be marked.
5. For each of the written-response questions, write your answer in the space provided in this booklet.
6. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

END OF EXAMINATION.

7. At the end of the examination, place your Response Form inside the front cover of this booklet and return the booklet and your Response Form to the supervisor.

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PRINCIPLES OF MATHEMATICS 12 PROVINCIAL EXAMINATION

	Value	Suggested Time
1. This examination consists of two parts:		
PART A: 45 multiple-choice questions	45	75
PART B: 8 written-response questions	25	45
	Total: 70 marks	120 minutes

- Aside from an approved calculator, electronic devices including dictionaries and pagers are **not** permitted in the examination room.
- The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae, Rough Work for Graphing, and Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.
- You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.
- A graphing calculator is essential for the Principles of Mathematics 12 Provincial Examination.** The calculator must be a hand-held device designed primarily for mathematical computations involving logarithmic and trigonometric functions as well as for graphing functions. Computers, calculators with a QWERTY keyboard, and electronic writing pads will not be allowed. Students must not bring any external devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or external keyboards. Students may have more than one calculator available during the examination of which one may be a scientific calculator. Calculators may not be shared and must not have the ability to either transmit or receive electronic signals. In addition to an approved calculator, students will be allowed to use rulers, compasses, and protractors during the examination.
- If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.
- When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.
- This examination is designed to be completed in **two hours**. Students may, however, take up to **20 minutes** of additional time to finish.

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PART A: MULTIPLE CHOICE

Value: 45 marks

Suggested Time: 75 minutes

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the Response Form provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

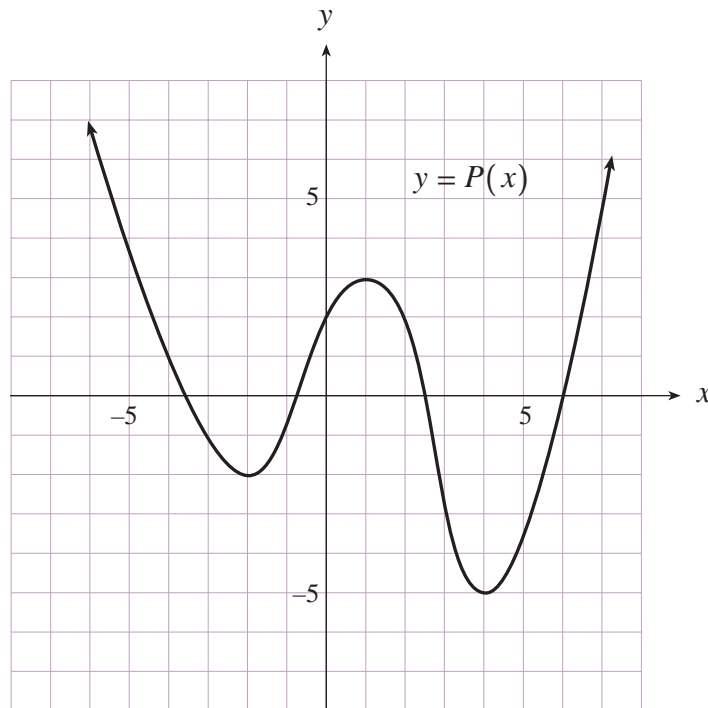
1. If 5 is a zero of the polynomial $P(x)$, then which of the following must be true?
 - A. $P(x) = 5$
 - B. $P(5) = 0$
 - C. $P(0) = 5$
 - D. $P(-5) = 0$

2. According to the Rational Root Theorem, determine all possible rational roots of $4x^5 - 3x^3 + 6x - 2 = 0$.
 - A. $\pm 1, \pm 2$
 - B. $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}$
 - C. $\pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{4}$
 - D. $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}, \pm \frac{1}{4}$

3. Determine the remainder when $x^{12} - 2x^7 + 6x^2 - 4$ is divided by $x + 1$.
 - A. 0
 - B. 1
 - C. 4
 - D. 5

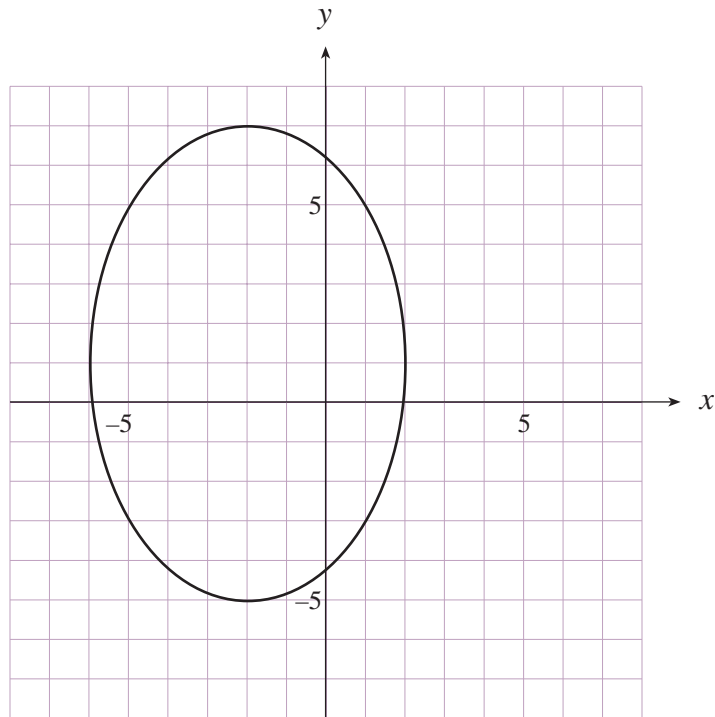
4. Which of the following is a real zero of the polynomial function $f(x) = x^3 - 3x + 3$?
 - A. -2.10
 - B. -2.00
 - C. 0.82
 - D. 3.00

5. The graph of a polynomial function $y = P(x)$ is shown below. If $f(x) = P(x) + k$, determine all values of k such that $f(x)$ will have two unequal real zeros and no other real zeros.



- A. $k < -3$ or $k > 2$
 B. $2 < k < 5$
 C. $-2 < k < 5$
 D. $2 < k < 5$ or $k < -3$
6. Which conic is represented by the equation $3x^2 + 6y^2 - 12y - 4 = 0$?
- A. circle
 B. ellipse
 C. parabola
 D. hyperbola
7. Determine the midpoint of the line segment that has endpoints $(-28, 15)$ and $(16, -7)$.
- A. $(-6, 4)$
 B. $(-12, 8)$
 C. $(-22, 11)$
 D. $(-11, 5.5)$

8. Determine an equation of the ellipse graphed below.



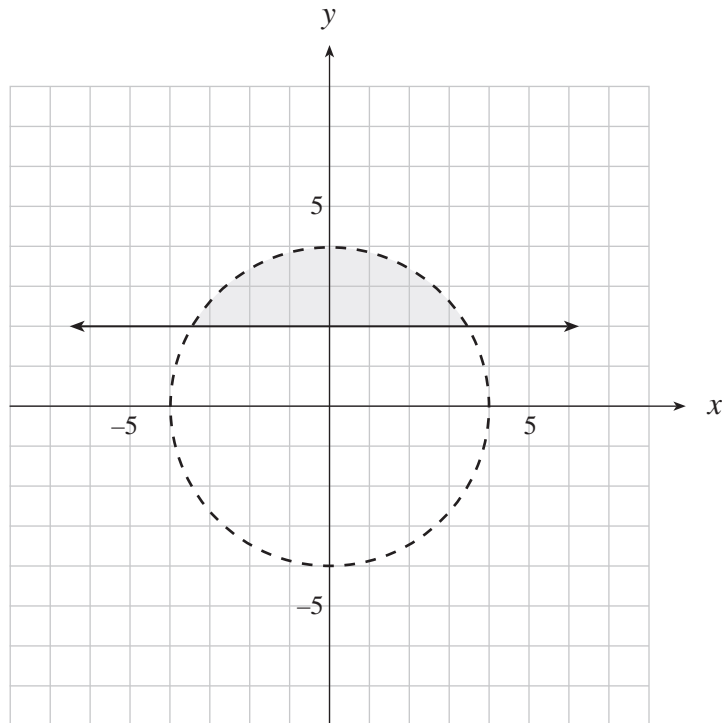
A. $\frac{(x-2)^2}{36} + \frac{(y+1)^2}{16} = 1$

B. $\frac{(x+2)^2}{36} + \frac{(y-1)^2}{16} = 1$

C. $\frac{(x-2)^2}{16} + \frac{(y+1)^2}{36} = 1$

D. $\frac{(x+2)^2}{16} + \frac{(y-1)^2}{36} = 1$

9. Which system describes the shaded region below?



A. $x^2 + y^2 < 16$
 $y \leq 2$

B. $x^2 + y^2 < 16$
 $y \geq 2$

C. $x^2 + y^2 > 16$
 $y \leq 2$

D. $x^2 + y^2 > 16$
 $y \geq 2$

10. Solve: $|5x + 20| > 30$

A. $x < -10$ or $x > 2$

B. $-10 < x < 2$

C. $x < -2$ or $x > 10$

D. $-2 < x < 10$

11. Which system will solve the following problem:

Determine the length ℓ and the width w of a rectangle with a diagonal of 8 and a perimeter of 20.

A. $w\ell = 16$
 $2w + 2\ell = 20$

B. $w + \ell = 20$
 $w^2 + \ell^2 = 64$

C. $w + \ell = 8$
 $2w + 2\ell = 20$

D. $2w + 2\ell = 20$
 $w^2 + \ell^2 = 64$

12. A point $P(x, y)$ moves such that it is equidistant from the point $(3, 8)$ and the line $y = 0$. Determine an equation of this locus.

A. $(x - 3)^2 + (y - 8)^2 = x^2$

B. $(x - 3)^2 + (y - 8)^2 = y^2$

C. $y = 4$

D. $(x + 3)^2 + (y + 8)^2 = y^2$

13. Change $2x^2 + y^2 - 8x - 6y + 1 = 0$ to standard form.

A. $\frac{(x - 2)^2}{6} + \frac{(y - 3)^2}{12} = 1$

B. $\frac{(x - 2)^2}{8} + \frac{(y - 3)^2}{16} = 1$

C. $\frac{(x - 4)^2}{12} + \frac{(y - 3)^2}{24} = 1$

D. $\frac{(x - 4)^2}{20} + \frac{(y - 3)^2}{24} = 1$

14. Determine the value of k such that the following system has exactly three different real solutions.

$$x^2 + y^2 = 16$$

$$x = 2y^2 + k$$

- A. -16
- B. -4
- C. 4
- D. 16

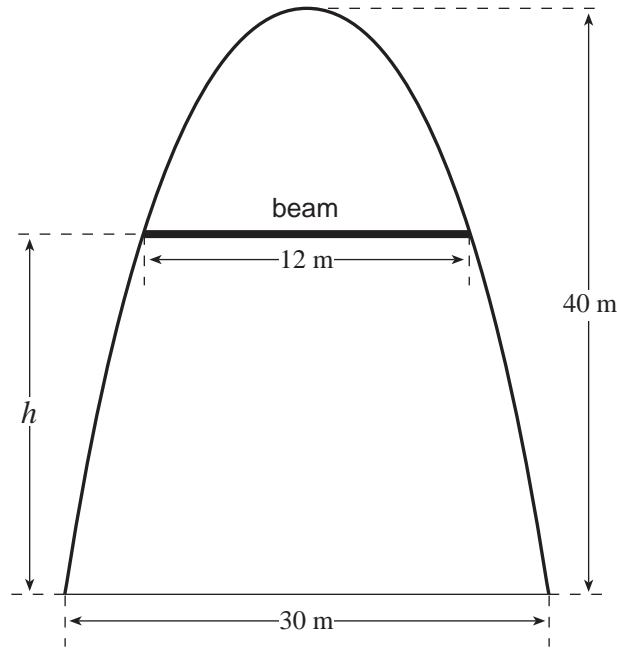
15. Solve the following system:

$$y^2 - x^2 = 4$$

$$y = 6 \log x$$

- A. (0.43, -2.19)
- B. (0.46, -2.05)
- C. (0.47, -1.94) , (1.59, 1.21)
- D. no solution

16. A parabolic arch of maximum height 40 m is 30 m wide at ground level. A horizontal beam of length 12 m is placed across the arch as shown in the diagram below. Calculate the height, h , of the beam above the ground.



- A. 14.4 m
B. 16 m
C. 32 m
D. 33.6 m
17. Evaluate: $\log_{12} 8000$
- A. 0.33
B. 2.82
C. 3.62
D. 3.90
18. Express $3 \log a + \log b - \log c$ as a single logarithm.
- A. $\log\left(\frac{a^3 b}{c}\right)$
B. $\log(a^3 + b - c)$
C. $3 \log\left(\frac{ab}{c}\right)$
D. $\log\left(\frac{3ab}{c}\right)$

19. The point $(-3, 2)$ is on the graph of the function $f(x)$. Which point must be on the graph of $f^{-1}(x)$, the inverse of $f(x)$?
- A. $(-2, 3)$
 - B. $(2, -3)$
 - C. $(3, -2)$
 - D. $\left(-\frac{1}{3}, \frac{1}{2}\right)$
20. Determine the range of the function $y = 7^{x+2} - 4$.
- A. $y > -4$
 - B. $y > -2$
 - C. $y > 2$
 - D. $y > 4$
21. Solve: $\log_3(2x+3) - \log_3(x+1) = 2$
- A. $-\frac{6}{7}$
 - B. $-\frac{3}{4}$
 - C. 0
 - D. no real roots
22. Solve for x : $(\log_x 7)(\log_7 5) = 2$
- A. $\frac{5}{2}$
 - B. $\sqrt{5}$
 - C. $\frac{2\sqrt{3}}{7}$
 - D. $\frac{\sqrt{35}}{7}$

23. Simplify: $8^{\log_2 6x}$

- A. $18x$
- B. $6x^3$
- C. $216x^3$
- D. $1296x^4$

24. Which of the following is an arithmetic sequence?

- A. 6, 10, 14, 18
- B. 6, 10, 15, 21
- C. 6, 12, 18, 30
- D. 6, 12, 24, 48

25. Determine the number of terms in the arithmetic sequence: $-12, -9, -6, \dots, 303$

- A. 97
- B. 98
- C. 105
- D. 106

26. Find the 3rd term of the sequence defined by the following recursive definition:

$$t_1 = 4$$

$$t_n = (n+3)t_{n-1}, \quad n > 1$$

- A. 24
- B. 80
- C. 120
- D. 840

27. Find the sum of the infinite geometric series: $8 - 6 + \frac{9}{2} - \frac{27}{8} + \dots$

- A. -32
- B. $-\frac{32}{7}$
- C. $\frac{32}{7}$
- D. 32

28. Which of the following is equivalent to $\sum_{k=1}^{20} 3^{k+2}$?

- A. $\frac{27(1-3^{20})}{-2}$
- B. $\frac{27(1-3^{19})}{-2}$
- C. $\frac{3(1-3^{20})}{-2}$
- D. $\frac{3(1-3^{22})}{-2}$

29. Convert 322° to radians.

- A. 0.66
- B. 2.81
- C. 5.62
- D. 11.24

30. Which expression is equivalent to $2 \csc \frac{\pi}{7}$?

A. $2 \sin \frac{7}{\pi}$

B. $\frac{1}{2 \sin \frac{\pi}{7}}$

C. $\frac{2}{\sin \frac{7}{\pi}}$

D. $\frac{2}{\sin \frac{\pi}{7}}$

31. Solve: $3 \tan x + \sqrt{15} = 0$ where $0 \leq x < 2\pi$

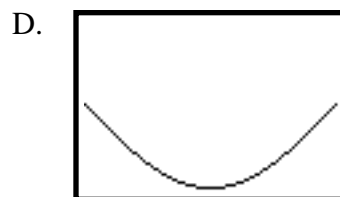
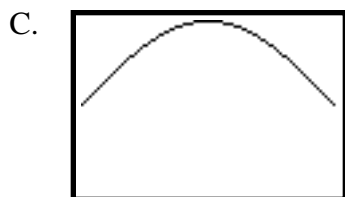
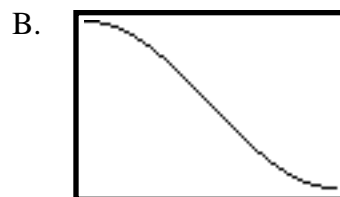
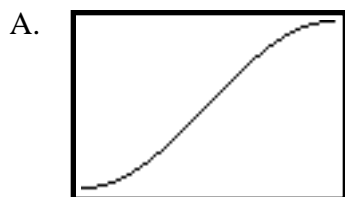
A. 0.91 , 4.05

B. 2.23 , 4.05

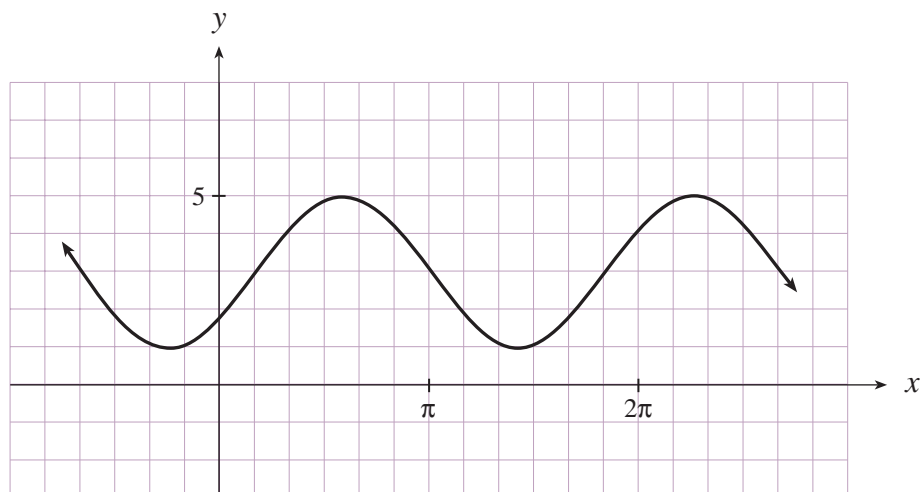
C. 2.23 , 5.37

D. 4.05 , 5.37

32. Which graph illustrates $y = \cos(\pi x) + 15$ for $13 \leq x \leq 14$ and $14 \leq y \leq 16$?



Use the following graph to answer questions 33 and 34.



33. Determine the amplitude of the sine function shown in the graph above.

- A. 2
- B. 3
- C. 4
- D. 5

34. Determine the period of the sine function shown in the graph above.

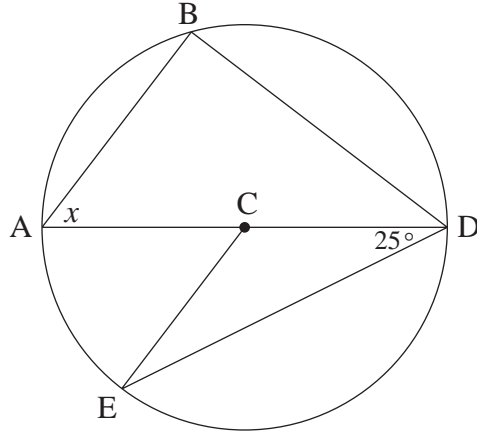
- A. $\frac{5\pi}{6}$
- B. π
- C. $\frac{5\pi}{3}$
- D. $\frac{11\pi}{6}$

35. Which expression is equivalent to $\frac{\sin \theta + \cos \theta \cot \theta}{\cot \theta}$?

- A. $\csc \theta$
- B. $\cos \theta$
- C. $\sin \theta$
- D. $\sec \theta$

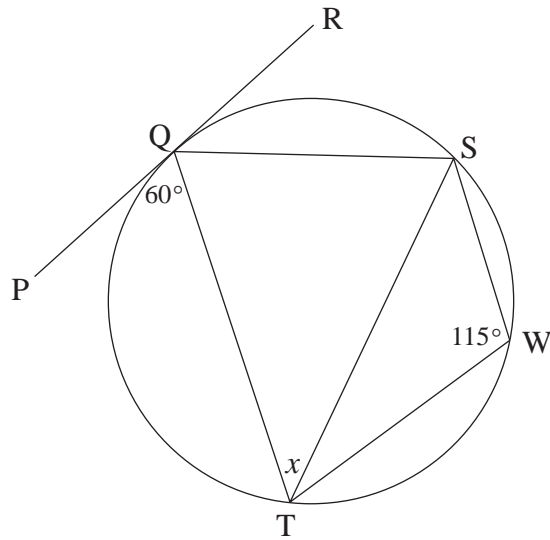
36. Solve: $\sin 2\theta + 2 \cos \theta = 0$, where $0 \leq \theta < 2\pi$
- A. 4.71
 - B. 1.57, 4.71
 - C. 1.57, 3.14
 - D. 1.57, 3.14, 4.71
37. If $\sec \theta = -\frac{5}{3}$ and angle θ terminates in quadrant III, which point must be on the terminal side of θ ?
- A. $(-6, -10)$
 - B. $(-10, -6)$
 - C. $(-8, -6)$
 - D. $(-6, -8)$
38. Let A be an angle in standard position such that $0 < A < \frac{\pi}{2}$. If $\sin A = n$ and $\cos A = m$, determine an expression for $\sin(\pi + A) + \cos(\pi + A)$.
- A. $-m - n$
 - B. $-m + n$
 - C. $m - n$
 - D. $m + n$

39. In a circle with centre C, AD is a diameter and $AB \parallel EC$. Find the measure of $\angle x$.



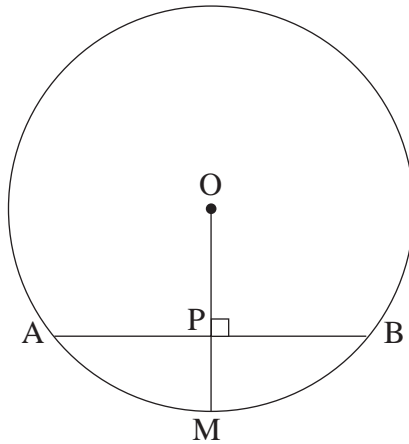
- A. 40°
- B. 50°
- C. 60°
- D. 70°

40. In the diagram below, PR is tangent to the circle at Q. Find the measure of $\angle x$.



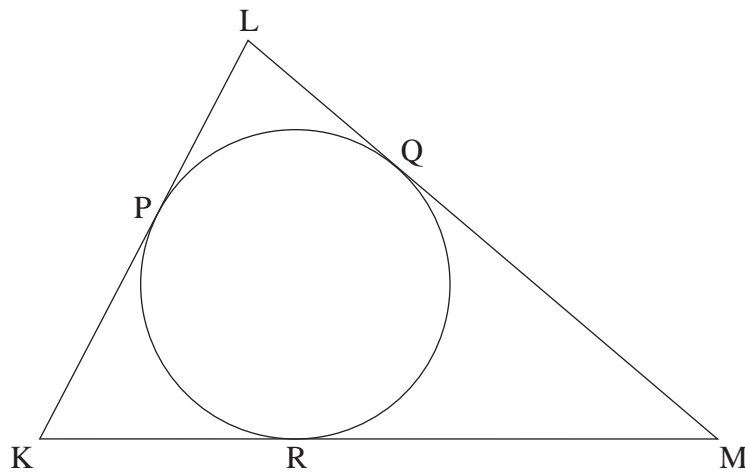
- A. 50°
- B. 55°
- C. 60°
- D. 65°

41. In the diagram below, a circle with centre O has a radius of 12. If $PM = 5$ and $OM \perp AB$, determine the length of chord AB .



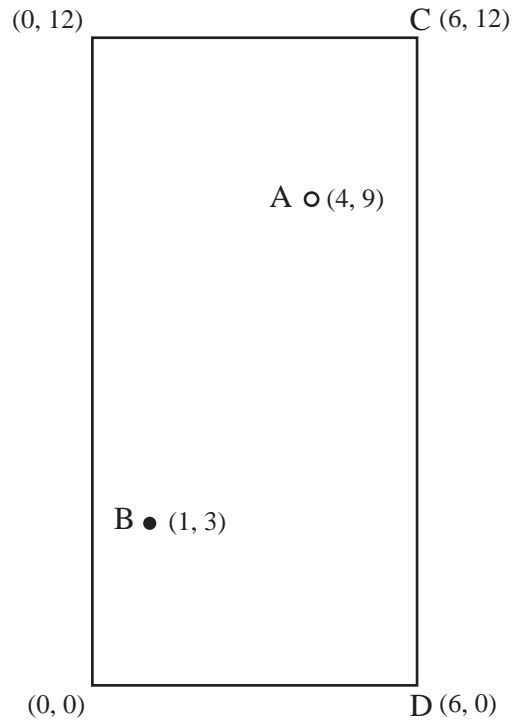
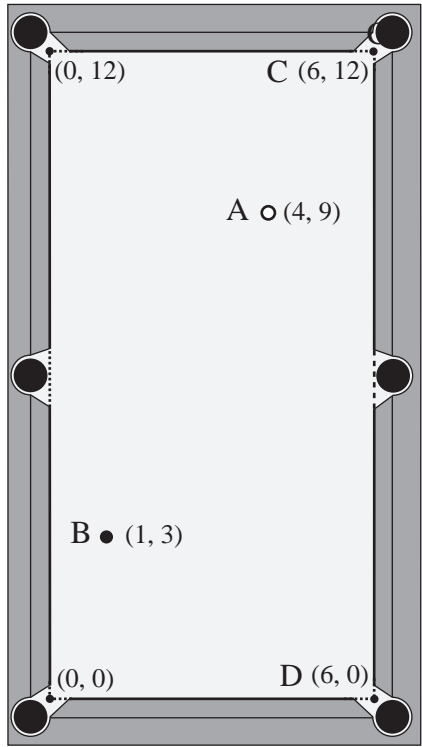
- A. 9.75
- B. 10.91
- C. 19.49
- D. 21.82

42. A circle is inscribed in $\triangle KLM$ as shown in the diagram below. If $LK = 10$, $LM = 13$, and $KM = 17$, find the length of PL .



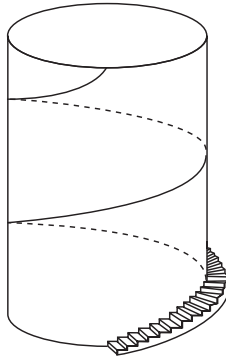
- A. 2
- B. 2.5
- C. 3
- D. 3.5

43. Jack and Jill are playing snooker on a 6×12 snooker table. To establish the position of the balls, consider a coordinate system with $(0, 0)$ at the bottom left corner and the other corners at $(6, 0)$, $(6, 12)$, and $(0, 12)$. The cue ball is at $A(4, 9)$ and the ball that must be hit is at $B(1, 3)$. If the cue ball at A must bounce off side CD and then hit the ball at B , determine the coordinates of the point on side CD that the cue ball must hit.



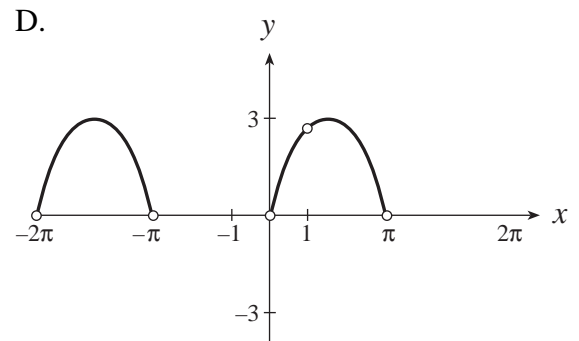
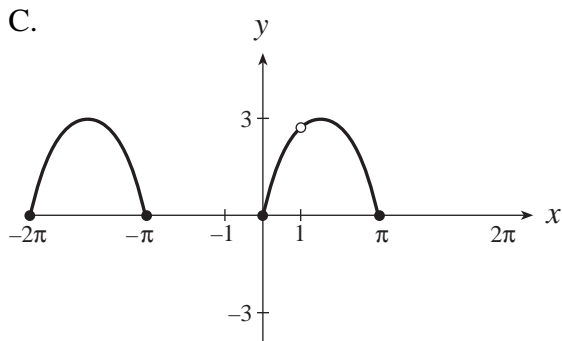
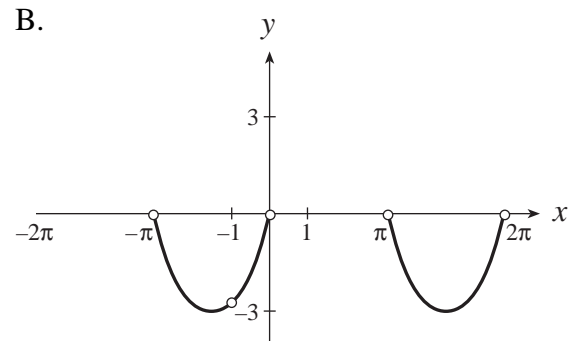
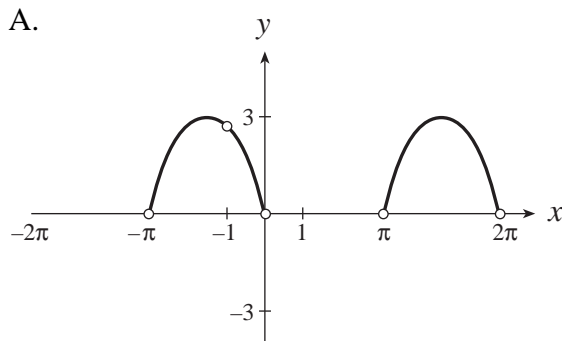
- A. $\left(6, \frac{21}{5}\right)$
 B. $\left(6, \frac{30}{7}\right)$
 C. $\left(6, \frac{33}{5}\right)$
 D. $\left(6, \frac{51}{7}\right)$

44. A cylindrical storage tank has a radius of 4 m and a height of 24 m. A staircase allowing access from the ground to the top of the tank, circles the tank twice. Determine the length of the inner edge of this staircase.



- A. 34.75 m
 B. 50.27 m
 C. 55.70 m
 D. 103.36 m

45. Which of the following best represents the graph of the function $\log_{|x|} y = \log_{|x|} 3 + \log_{|x|}(\sin x)$ over the interval $-2\pi \leq x \leq 2\pi$?



**This is the end of the multiple-choice section.
 Answer the remaining questions directly in this examination booklet.**

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PART B: WRITTEN RESPONSE

Value: 25 marks

Suggested Time: 45 minutes

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

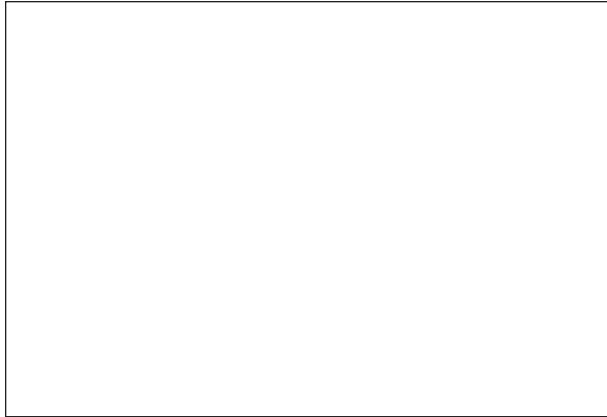
Full marks will NOT be given for the final answer only.

1. Solve the following inequality using a graphing calculator.

(3 marks)

$$x^3 - 8x^2 \geq -4x + 20$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window. The solution may be given in algebraic form or shown on a number line.



$Y_1 =$

$Y_2 =$

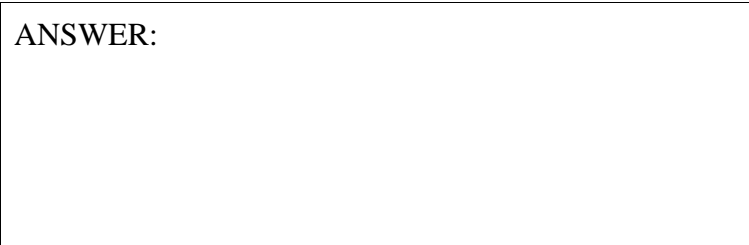
$Y_3 =$

$Y_4 =$

[,]
 x x
min max

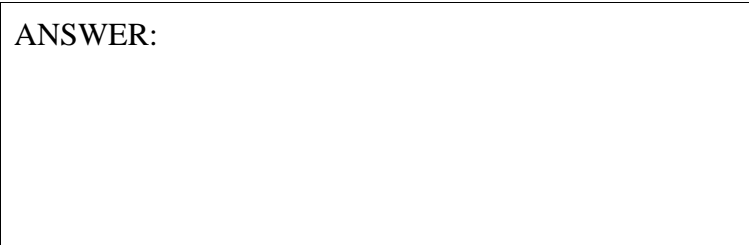
[,]
 y y
min max

ANSWER:



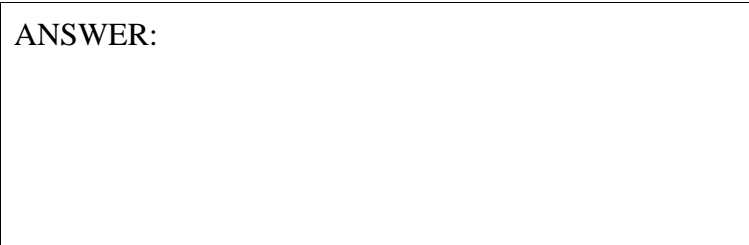
2. A population of frogs doubles every 20 weeks. If the present population is 400 frogs, how long will it take for the population to reach 10 000 ? **(3 marks)**

ANSWER:



3. A hyperbola has centre $(1, 4)$, and one vertex at $(1, 10)$. If the asymptotes have slopes $\pm \frac{3}{2}$, determine the equation of the hyperbola in standard form. **(3 marks)**

ANSWER:



4. Pat's new job paid $\$x$ for the first month, and increased by $\$50$ each month after the first month. In a total of 24 months, including the first month, Pat earned $\$60\,000$. Determine the value of x . **(3 marks)**

ANSWER:

5. Prove the following identity:

(3 marks)

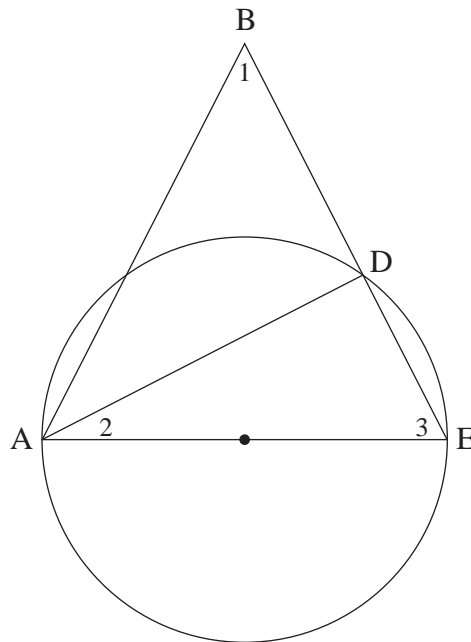
$$\frac{\cot \theta}{\csc \theta - 1} = \frac{\csc \theta + 1}{\cot \theta}$$

LEFT SIDE

RIGHT SIDE

ANSWER:

7. In the diagram below, AE is a diameter, $AB = 10$, $BD = 6$, $\angle 1 = \angle 2$ and B, D, E are collinear. Find the measure of $\angle 3$. (Answer to the nearest degree.) **(3 marks)**



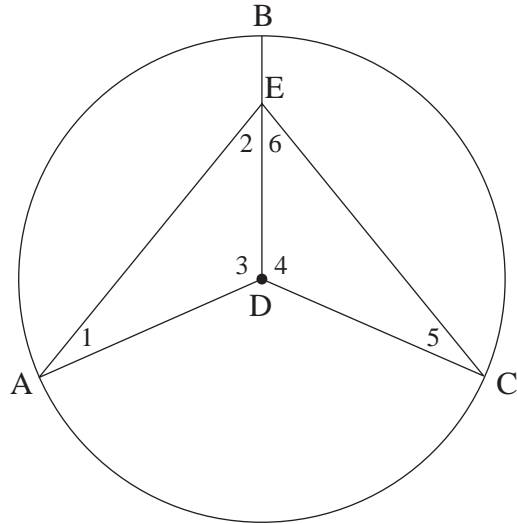
ANSWER:

8. Complete the following proof.

(4 marks)

Given: D is the centre of the circle
 $\widehat{AB} = \widehat{BC}$
B, E, D are collinear

Prove: $\angle 1 = \angle 5$



PROOF

END OF EXAMINATION

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A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_n = \frac{a - \ell r}{1 - r}$$

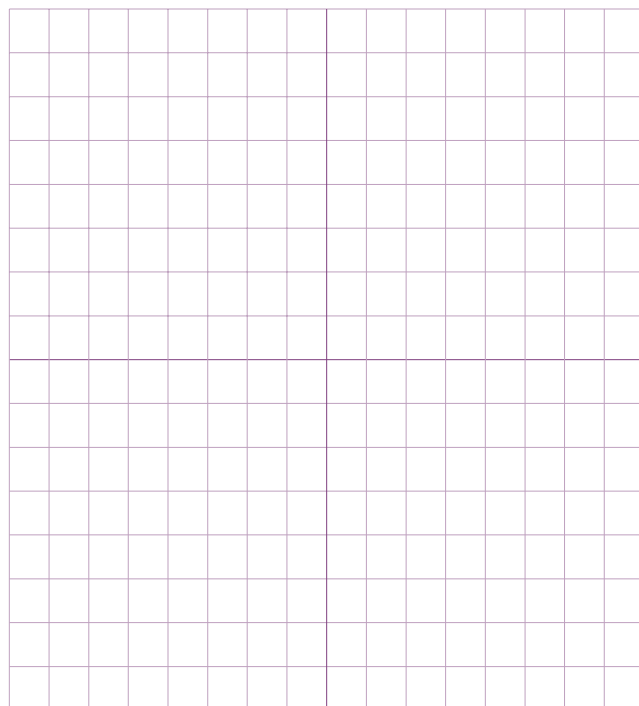
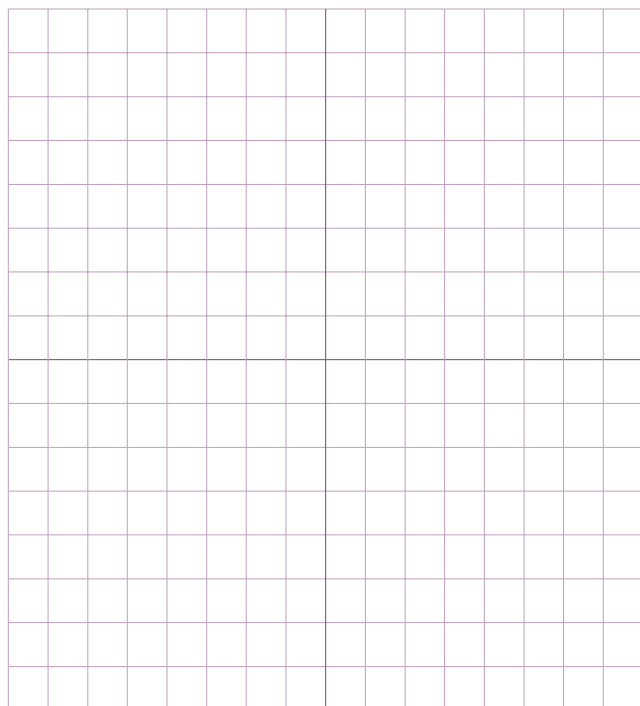
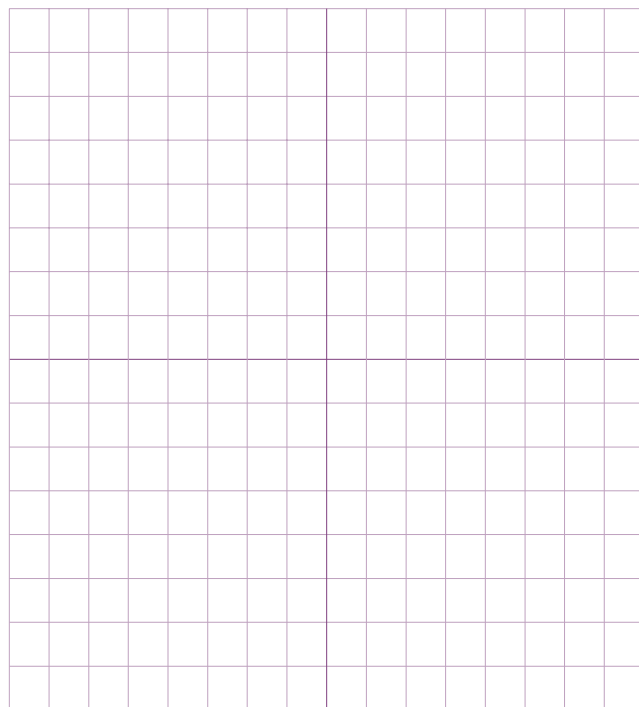
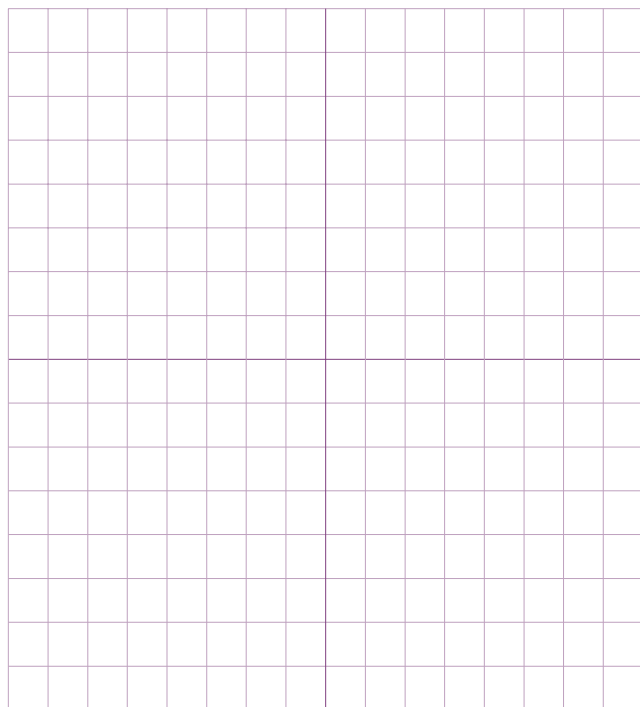
$$S = \frac{a}{1-r}$$

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Exercise care when tearing along perforations.**

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ROUGH WORK FOR GRAPHING

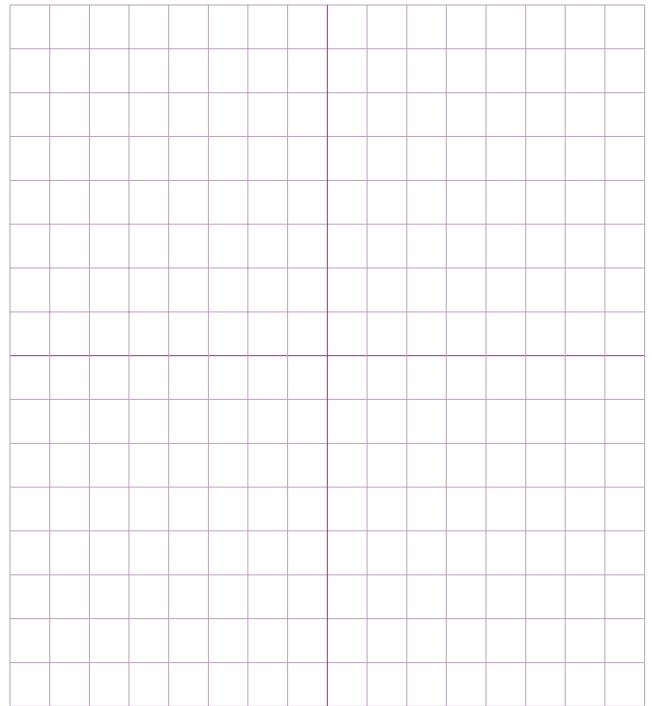
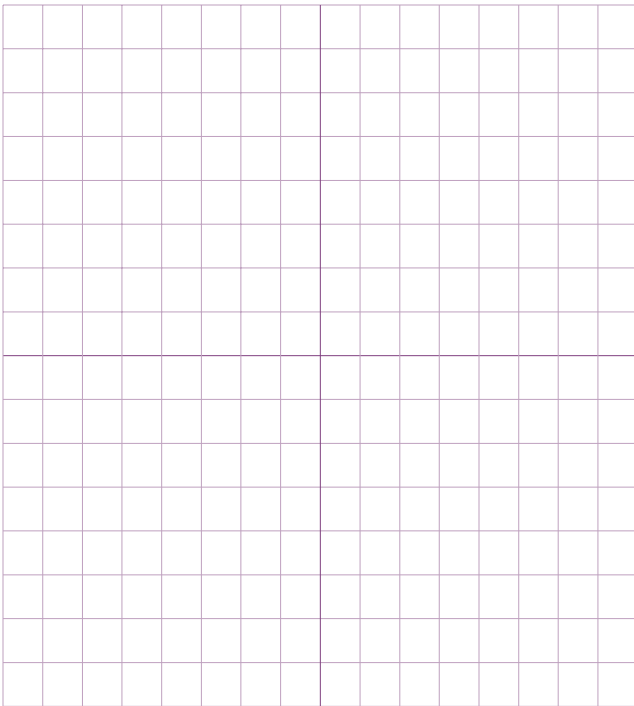
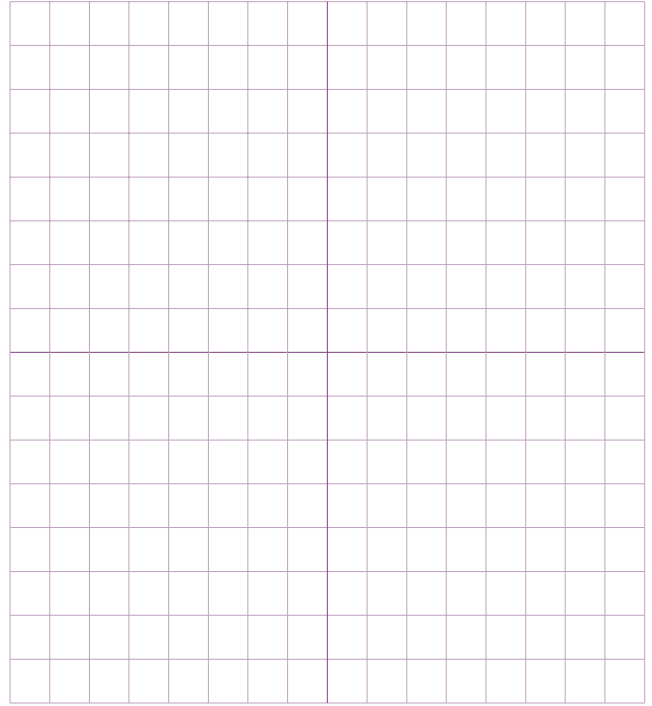
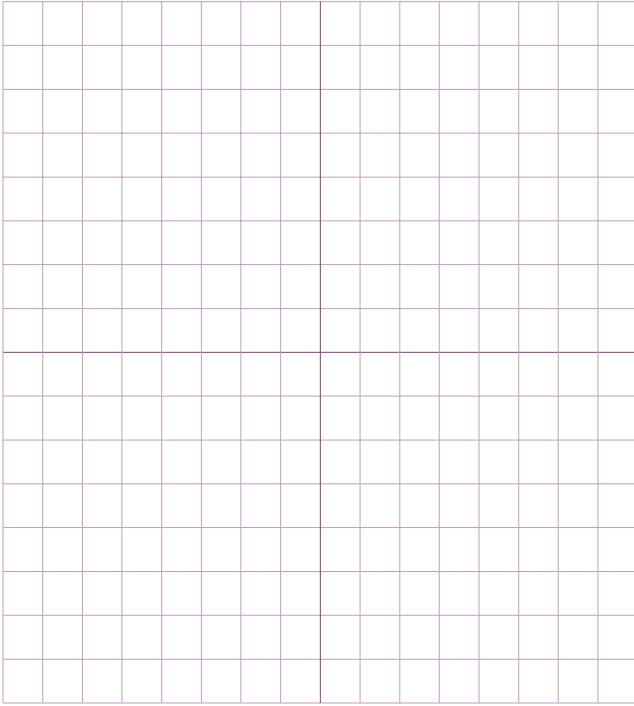
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ROUGH WORK FOR GRAPHING

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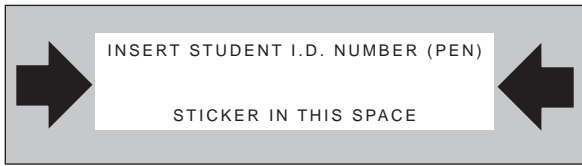


ROUGH WORK FOR MULTIPLE-CHOICE

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ROUGH WORK FOR MULTIPLE-CHOICE





PRINCIPLES OF MATHEMATICS 12

January 1999

Course Code = MA

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**PRINCIPLES OF
MATHEMATICS 12**

January 1999

Course Code = MA

Score for
Question 1:

1. $\frac{\quad}{(3)}$

Score for
Question 8:

8. $\frac{\quad}{(4)}$

Score for
Question 2:

2. $\frac{\quad}{(3)}$

Score for
Question 3:

3. $\frac{\quad}{(3)}$

Score for
Question 4:

4. $\frac{\quad}{(3)}$

Score for
Question 5:

5. $\frac{\quad}{(3)}$

Score for
Question 6:

6. $\frac{\quad}{(3)}$

Score for
Question 7:

7. $\frac{\quad}{(3)}$