INSERT STUDENT I.D. NUMBER (PEN)

STICKER IN THIS SPACE

JUNE 1998

PROVINCIAL EXAMINATION

MINISTRY OF EDUCATION, SKILLS AND TRAINING

MATHEMATICS 12

GENERAL INSTRUCTIONS

1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above and on the **back** cover of this booklet. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this booklet.**

2. Ensure that in addition to this examination booklet, you have an **Examination Response Form**. Follow the directions on the front of the Response Form.

3. **Disqualification** from the examination will result if you bring books, paper, notes or unauthorized electronic devices into the examination room.

4. All multiple-choice answers must be entered on the Response Form using an **HB pencil**. Multiple-choice answers entered in this examination booklet will **not** be marked.

5. For each of the written-response questions, write your answer in the space provided in this booklet.

6. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

   END OF EXAMINATION.

7. At the end of the examination, place your Response Form inside the front cover of this booklet and return the booklet and your Response Form to the supervisor.

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MATHEMATICS 12 PROVINCIAL EXAMINATION

1. This examination consists of **two** parts:

		Value	Suggested Time
PART A:	50 multiple-choice questions	50	75
PART B:	7 written-response questions 2 questions worth **two** marks each, 4 questions worth **three** marks each, and 1 question worth **four** marks.	20	45
	Total:	**70 marks**	**120 minutes**

2. Aside from an approved calculator, electronic devices, including dictionaries and pagers, are **not** permitted in the examination room.

3. The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae**, **Rough Work for Graphing**, and **Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.

4. You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.

5. An approved scientific calculator is essential for the examination. The calculator must be a hand-held device designed **only** for mathematical computations such as logarithmic and trigonometric functions. It **can be** programmable, but **must not** contain any graphing capabilities. You **must not** bring into the examination room any devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or keyboards.

6. You are permitted to use rulers, compasses and protractors.

7. The time allotted for this examination is **two hours**.

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PART A: MULTIPLE CHOICE

Value: 50 marks **Suggested Time: 75 minutes**

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the Response Form provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. Which conic is described by the equation $4x^2 + 5x - 3y + 2 = 0$?

   A. circle
   B. ellipse
   C. parabola
   D. hyperbola

2. Determine the midpoint of the line segment joining $\mathrm{P}(7, -5)$ and $\mathrm{Q}(-3, 1)$.

   A. $(-5, 3)$
   B. $(5, -3)$
   C. $(-2, 2)$
   D. $(2, -2)$

3. Determine the length of the minor axis of the ellipse $\frac{(x-1)^2}{25} + \frac{(y+3)^2}{49} = 1$.

   A. 5
   B. 7
   C. 10
   D. 14

4. Change $x^2 + y^2 - 4x + 2y - 4 = 0$ to standard form.

   A. $(x-2)^2 + (y+1)^2 = 1$
   B. $(x-2)^2 + (y+1)^2 = 4$
   C. $(x-2)^2 + (y+1)^2 = 9$
   D. $(x-2)^2 + (y+1)^2 = 10$

OVER

5. Which equation below could be used to determine the coordinates of the point on the *x*-axis that is equidistant from $A(-2, 1)$ and $B(3, 4)$?

A. $\sqrt{(x-2)^2+(0+1)^2} = \sqrt{(x+3)^2+(0+4)^2}$

B. $\sqrt{(x+2)^2+(0-1)^2} = \sqrt{(x-3)^2+(0-4)^2}$

C. $\sqrt{(0+2)^2+(y-1)^2} = \sqrt{(0-3)^2+(y-4)^2}$

D. $\sqrt{(0-2)^2+(y+1)^2} = \sqrt{(0+3)^2+(y+4)^2}$

6. Which absolute value inequality describes the solution shown?

A. $|x+3| < 4$

B. $|x-4| < 3$

C. $|x+4| < 3$

D. $|x-3| < 4$

7. Solve the system:

$$2x^2 - y^2 = -1$$

$$3x^2 + y^2 = 21$$

A. $(2, 3)$

B. $(2, 3)$, $(-2, -3)$

C. $(2, 3)$, $(2, -3)$

D. $(2, 3)$, $(2, -3)$, $(-2, 3)$, $(-2, -3)$

8. Find an equation of the rectangular hyperbola with vertices $(4, -1)$ and $(4, 5)$.

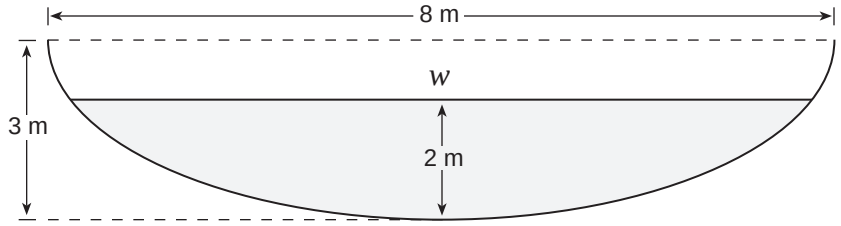
A. $(x+4)^2 - (y+2)^2 = -9$

B. $(x-4)^2 - (y-2)^2 = -9$

C. $(x+4)^2 - (y+2)^2 = 9$

D. $(x-4)^2 - (y-2)^2 = 9$

9. The cross section of an irrigation canal is in the shape of a semi-ellipse with a width of 8 metres and a depth of 3 metres. What is the width, w, of the water when its depth is 2 metres? (Accurate to 2 decimal places.)



A. 5.96 m
B. 6.83 m
C. 7.38 m
D. 7.54 m

10. Determine all values for p such that the following system has exactly 2 different real solutions.

$$\frac{(x-5)^2}{9} - \frac{(y-1)^2}{25} = 1$$

$$x - p = (y-1)^2$$

A. $p < 2$ or $p > 8$
B. $2 < p < 8$
C. $p < -8$ or $p > -2$
D. $-8 < p < -2$

11. Convert 4 radians to degrees. (Accurate to the nearest degree.)

A. 13°
B. 115°
C. 229°
D. 720°

12. Determine the amplitude of the graph of $y = -4\cos 2x$.

A. -4
B. 2
C. 4
D. 8

OVER

13. Solve: $3\cos x + 2 = 0$, $0 \le x < 2\pi$ (Accurate to 2 decimal places.)

A. 0.84 , 2.30
B. 0.84 , 5.44
C. 2.30 , 3.98
D. 2.36 , 3.93

14. Evaluate: $\csc\frac{3\pi}{8}$ (Accurate to 2 decimal places.)

A. 0.75
B. 1.08
C. 1.18
D. 2.61

15. Determine the number of asymptotes of the graph of $y = \tan x$ over the interval $-2\pi \le x \le 2\pi$.

A. 2
B. 4
C. 6
D. 8

16. Determine all restrictions for $\frac{\cot\theta}{1-\sin\theta}$.

A. $\sin\theta \ne 1$
B. $\sin\theta \ne 0$
C. $\sin\theta \ne 1$, $\cos\theta \ne 0$
D. $\sin\theta \ne 0$, $\sin\theta \ne 1$

17. Find the period of the sine function which has a minimum point at $\left(\frac{\pi}{3},\ 1\right)$ and its nearest maximum point to the right at $\left(\frac{2\pi}{3},\ 5\right)$.

A. $\frac{\pi}{3}$
B. $\frac{2\pi}{3}$
C. $\frac{4\pi}{3}$
D. 2π

18. Given $\csc^2\theta + \sin^2\theta = 5.34$, find the value of $\dfrac{1}{\csc^2\theta} + \dfrac{1}{\sin^2\theta}$.

A. 0.19
B. 2.27
C. 5.14
D. 5.34

19. Evaluate: $\log_2 8$

A. 2
B. 3
C. 4
D. 16

20. Which of the following is equivalent to $\log\left(\dfrac{a^3}{\sqrt{b}}\right)$?

A. $3\log a - \dfrac{1}{2}\log b$

B. $\dfrac{3\log a}{\frac{1}{2}\log b}$

C. $\log 3a - \log\dfrac{1}{2}b$

D. $\dfrac{\log 3a}{\log\frac{1}{2}b}$

21. Which of the following equations can be used to determine the number of years, t, that are needed for a \$300 deposit to increase to \$1 500 if it is invested at 9% compounded annually?

A. $1\,500 = 300(1.09)^t$

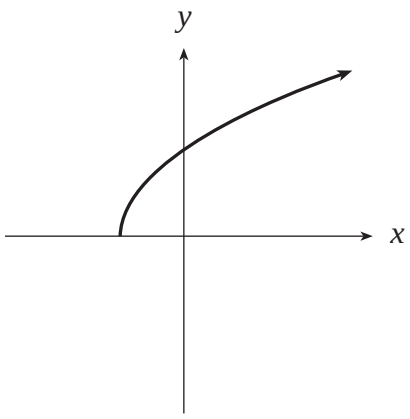
B. $300 = 1\,500(1.09)^t$

C. $1\,500 = 300(1.9)^t$

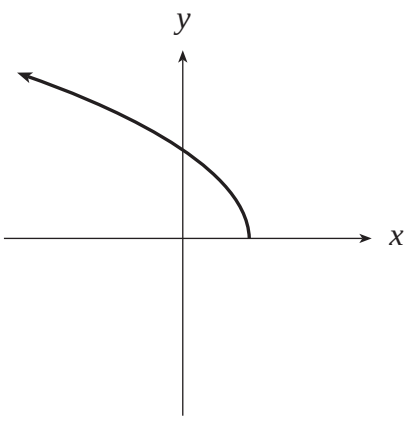
D. $300 = 1\,500(1.9)^t$

OVER

22. Which graph best represents the inverse relation of the graph shown below?



A.



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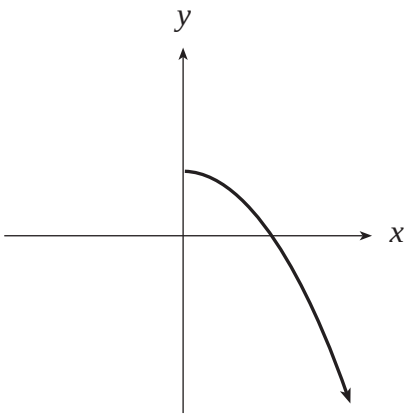

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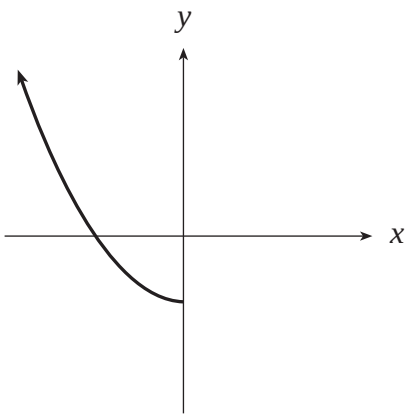
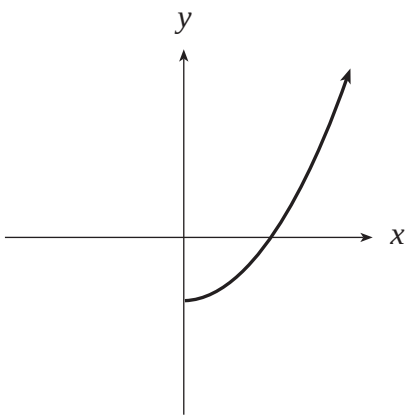


D.



23. Determine the domain of the function $y = \log_3(x+1) - 2$.

A. $x > -1$
B. $x > 1$
C. $y > -2$
D. $y > 2$

24. Solve: $\frac{1}{\log_2 x} + \frac{1}{\log_8 x} = 2$

A. 2
B. 4
C. 8
D. 16

25. If $\log_3 7 = x$ and $\log_2 9 = y$, determine an expression for $\log_9 7 + \log_2 3$ in terms of x and y.

A. $\frac{1}{2}x + \frac{1}{2}y$
B. $2x + \frac{1}{2}y$
C. $\frac{1}{2}x + \sqrt{y}$
D. $2x + \sqrt{y}$

26. According to the Rational Root Theorem, determine all possible rational roots of $5x^3 - 4x^2 + 15 = 0$.

A. $\pm 1, \pm 5$
B. $\pm 1, \pm 3, \pm 5, \pm 15$
C. $\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{5}, \pm\frac{3}{5}$
D. $\pm 1, \pm 5, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{5}, \pm\frac{1}{15}$

OVER

27. If $3x-1$ is a factor of $p(x)$, which of the following must have a value of 0 ?

A. $p\left(\frac{1}{3}\right)$

B. $p\left(-\frac{1}{3}\right)$

C. $p(-1)$

D. $p(1)$

28. How many real roots are there for the polynomial equation $x\left(x^2-4\right)\left(x^2+9\right)=0$?

A. 1
B. 2
C. 3
D. 5

29. Factor: x^3-2x^2-5x+6

A. $(x+1)(x-2)(x+3)$
B. $(x+1)(x+2)(x-3)$
C. $(x-1)(x-2)(x+3)$
D. $(x-1)(x+2)(x-3)$

30. Determine the quotient when $2x^3-5x^2+7x+3$ is divided by $2x+1$.

A. x^2-3x+4
B. x^2-3x+5
C. x^2-2x-2
D. x^2-2x+2

31. If the cubic polynomial function $f(x) = k(x-1)(x+2)(x-3)$ passes through the point $(2,\ 6)$, determine the value of k.

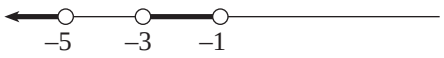
A. $-\frac{3}{2}$

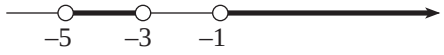
B. $-\frac{2}{3}$

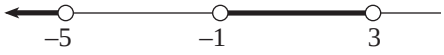
C. $\frac{2}{3}$

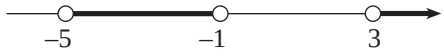
D. $\frac{3}{2}$

32. Solve: $(x+5)(x+1)(3-x) < 0$

A. 

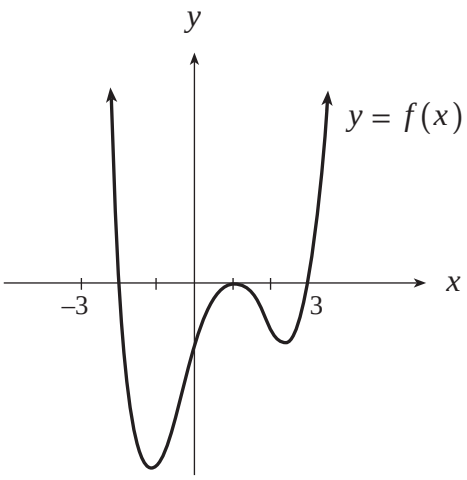
B. 

C. 

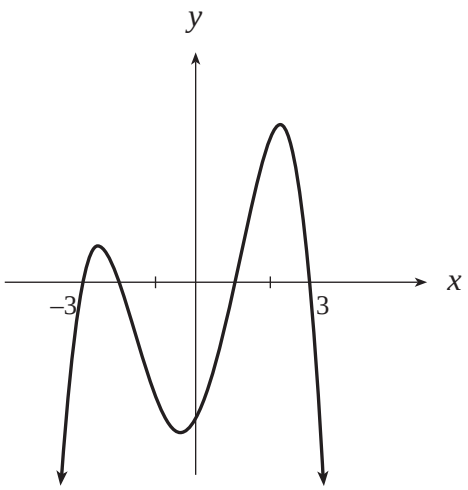
D. 

OVER

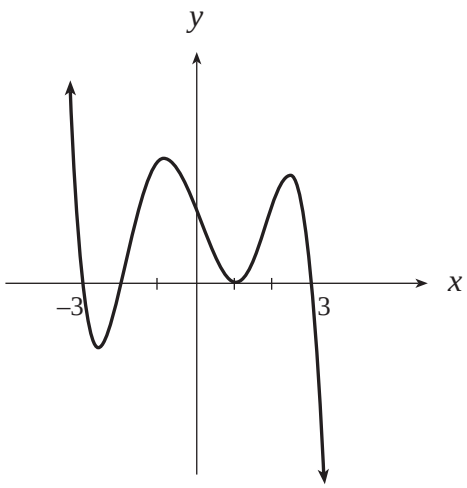
33. Given the graph of $y = f(x)$ below, which of the following **best** represents the graph of $y = (x+3)f(x)$?



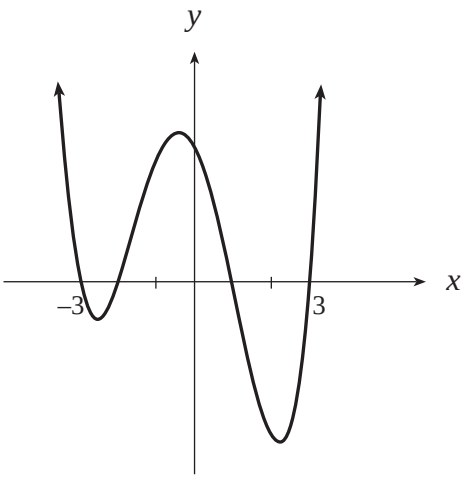
A.



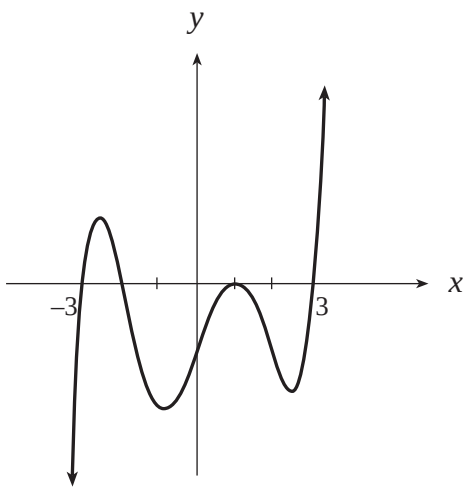
B.



C.



D.



34. Determine the common ratio of the geometric sequence: $-2,\ 6\sqrt{2},\ -36,\ \ldots$

A. $-3\sqrt{2}$

B. $-\frac{1}{3\sqrt{2}}$

C. $\frac{1}{3\sqrt{2}}$

D. $3\sqrt{2}$

35. Find the 14^{th} term of an arithmetic sequence whose first term is -3 and whose common difference is 5.

A. 62
B. 67
C. 68
D. 73

36. Give the second term of the series defined by $\sum_{j=0}^{5} 3\left(\frac{1}{2}\right)^{j-1}$.

A. $-\frac{3}{2}$

B. $\frac{3}{2}$

C. 3

D. 6

37. Find the sum of the infinite geometric series $20 - 4 + \frac{4}{5} - \frac{4}{25} + \ldots$

A. $\frac{10}{3}$

B. $\frac{50}{3}$

C. 24

D. 25

OVER

38. Determine the sum of the first 50 terms of the sequence given by the following recursive definition.

$$t_1 = 1$$

$$t_n = \left(2t_{n-1}\right)^2 - 5 \text{ , } n > 1$$

A. -50
B. -49
C. -48
D. -47

39. Find $f'(x)$ if $f(x) = 3$.

A. 0
B. 1
C. 3
D. $\frac{3}{x}$

40. Which expression represents the derivative of $f(x) = x^2 + 3x$?

A. $\lim\limits_{h \to 0} \frac{(x+h)^2 + 3(x+h) - \left(x^2 + 3x\right)}{h}$

B. $\lim\limits_{x \to 0} \frac{(x+h)^2 + 3(x+h) - \left(x^2 + 3x\right)}{h}$

C. $\lim\limits_{h \to 0} \frac{(x+h)^2 + 3(x+h) + \left(x^2 + 3x\right)}{h}$

D. $\lim\limits_{x \to 0} \frac{(x+h)^2 + 3(x+h) + \left(x^2 + 3x\right)}{h}$

41. Evaluate: $\lim\limits_{x\to\infty} \dfrac{4x-5x^2}{2x^2+3x-1}$

A. $-\dfrac{5}{2}$

B. 0

C. 2

D. limit does not exist (no finite limit)

42. A particle moves along the *x*-axis so that its position at time t is $x(t)=3t^3+2t^2+7$, where x is in metres and t is in seconds. Find the velocity at $t=2$ seconds.

A. 26 m/s

B. 39 m/s

C. 42 m/s

D. 44 m/s

43. Find the *x*-value of the point on the graph of $y=x^2-x$ where the slope of the tangent line is 2.

A. 0.5

B. 1.5

C. 2

D. 3

44. Given the function $f(x)=x^2+4x+9$, determine the slope of the secant line intersecting the graph of $f(x)$ at the points where $x=2$ and $x=4$.

A. 2

B. 8

C. 10

D. 12

45. If $y=-3x+1$ is tangent to the curve $f(x)$ at $x=a$, which must be true?

A. $f(a)=-3$

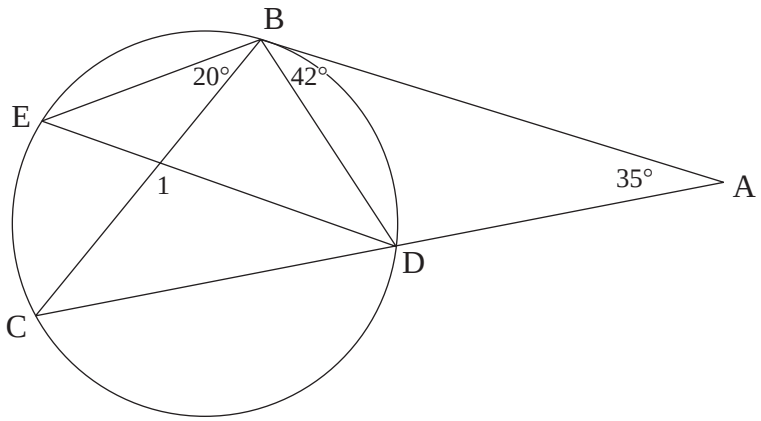
B. $f(a)=1$

C. $f'(a)=-3$

D. $f'(a)=1$

OVER

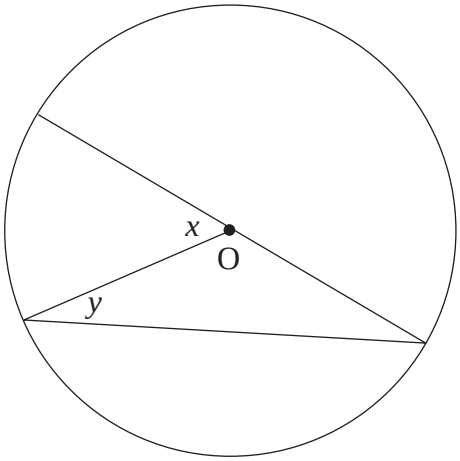
46. In the diagram below, AB is tangent to the circle at B and C, D, A are collinear.
(Diagram is not drawn to scale.)



Determine the measure of $\angle 1$.

A. 62°
B. 96°
C. 108°
D. 118°

47. In the diagram below, O is the centre of the circle. Determine an expression for $\angle y$ in terms of x.



A. x
B. $\frac{1}{2}x$
C. $2x$
D. $90° - \frac{1}{2}x$

48. Points $A(1, 4)$ and $B(7, 6)$ are the endpoints of line segment AB. Point $P(x, y)$ divides the line segment, such that $\frac{AP}{PB} = \frac{3}{2}$. Find the *x*-coordinate of point P.

A. $\frac{17}{5}$

B. $\frac{21}{5}$

C. $\frac{23}{5}$

D. 5

49. Find the difference between 10^{299} and 10^{297}.

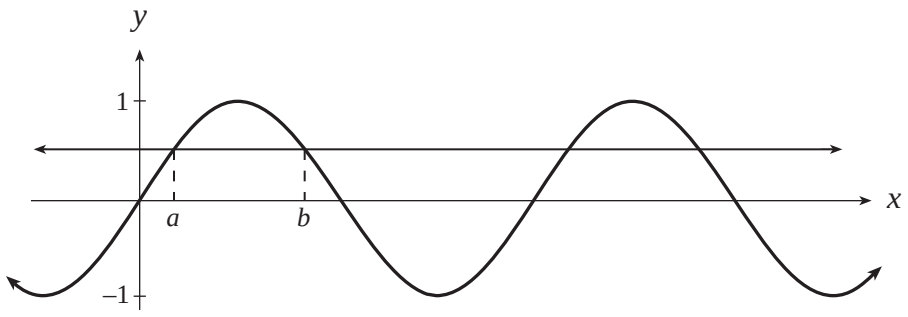
A. 10^{297}

B. $10^{297}(9)$

C. 10^{298}

D. $10^{297}(99)$

50. Solve $\sin \pi x > \frac{1}{2}$ over the real numbers, using the graphs of $y = \sin \pi x$ and $y = \frac{1}{2}$ shown below. Express the answer in terms of a and b.



A. $a + n < x < b + n$

B. $a + 2n < x < b + 2n$

C. $a + \pi < x < b + \pi$

D. $a + 2\pi < x < b + 2\pi$

This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.

OVER

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PART B: WRITTEN RESPONSE

Value: 20 marks **Suggested Time: 45 minutes**

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

Full marks will NOT be given for the final answer only.

OVER

1. Solve for x: $\log(3x-5)+\log(2x-1)=1$ **(3 marks)**

ANSWER:

OVER

2. Prove the identity: **(2 marks)**

$$\frac{\csc\theta}{\tan\theta + \cot\theta} = \cos\theta$$

Left Side	Right Side

OVER

3. The 780 students of a high school form a pyramid shape by sitting in rows on the bleachers. The top row has one student, and each row below has one more student than the previous row. How many rows are required to seat all 780 students? **(3 marks)**

ANSWER:

OVER

4. Determine all values of x such that the function $f(x) = x^4 - 18x^2 + 8$ is **decreasing**.

(3 marks)

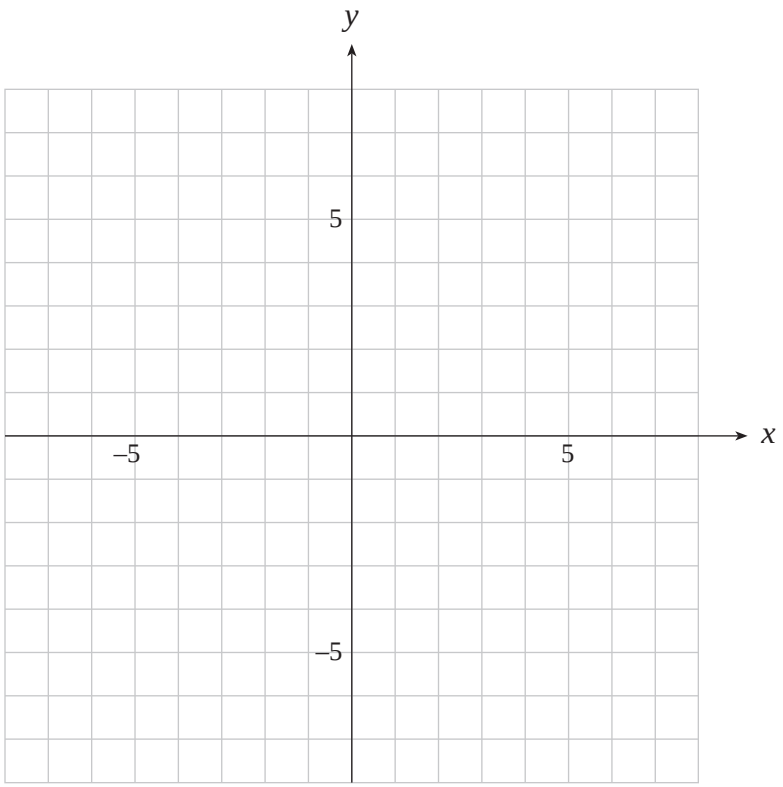
ANSWER:

OVER

5. Graph the solution of the following system of inequalities on the grid provided. **(3 marks)**

$$\frac{x^2}{9} - \frac{y^2}{4} \geq 1$$

$$(x-3)^2 + y^2 < 9$$

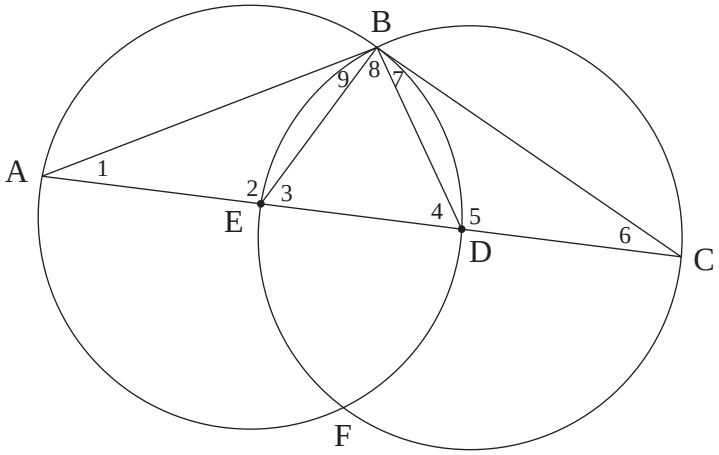


OVER

6. Complete the proof. **(4 marks)**

Given: 2 circles with centres E and D
BE = BD
A, E, D, C are collinear

Prove: ΔABC is isosceles



PROOF

Statement	Reason

OVER

7. Given: Circle with centre O **(2 marks)**

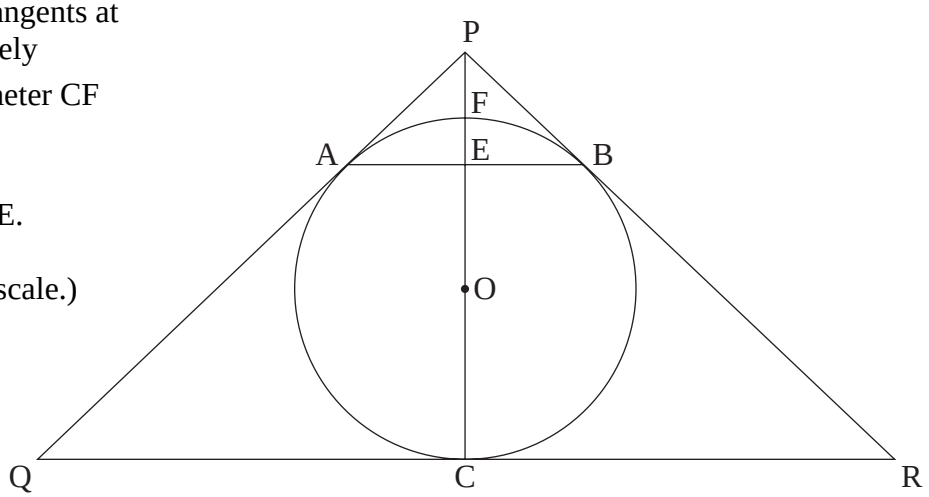
PQ, PR, QR are tangents at A, B, C respectively

Chord AB $\perp$ diameter CF

$CF = 8,\ EF = 1$

Determine the length of PE.

(Diagram is not drawn to scale.)



ANSWER:

END OF EXAMINATION

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A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\qquad 1 + \tan^2\theta = \sec^2\theta$ $\qquad 1 + \cot^2\theta = \csc^2\theta$

Reciprocal and Quotient Identities

$\sec\theta = \dfrac{1}{\cos\theta}$ $\qquad \csc\theta = \dfrac{1}{\sin\theta}$ $\qquad \cot\theta = \dfrac{1}{\tan\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Addition Identities

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\qquad \tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\qquad \tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

Double-Angle Identities

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\qquad \sin 2\theta = 2\sin\theta\cos\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$ $\qquad \tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Formulae

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad t_n = a + (n-1)d$ $\qquad t_n = ar^{n-1}$

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ $\qquad S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$ $\qquad S_n = \dfrac{a\left(1 - r^n\right)}{1 - r}$

$c^2 = a^2 + b^2 - 2ab\cos C$ $\qquad S_n = \dfrac{n}{2}(a + \ell)$ $\qquad S_n = \dfrac{a - \ell r}{1 - r}$

$S = \dfrac{a}{1 - r}$

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ROUGH WORK FOR GRAPHING

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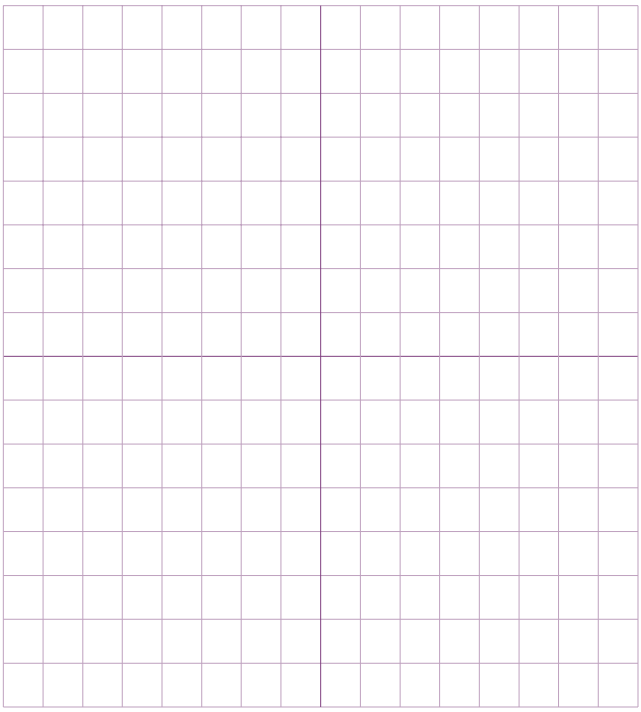

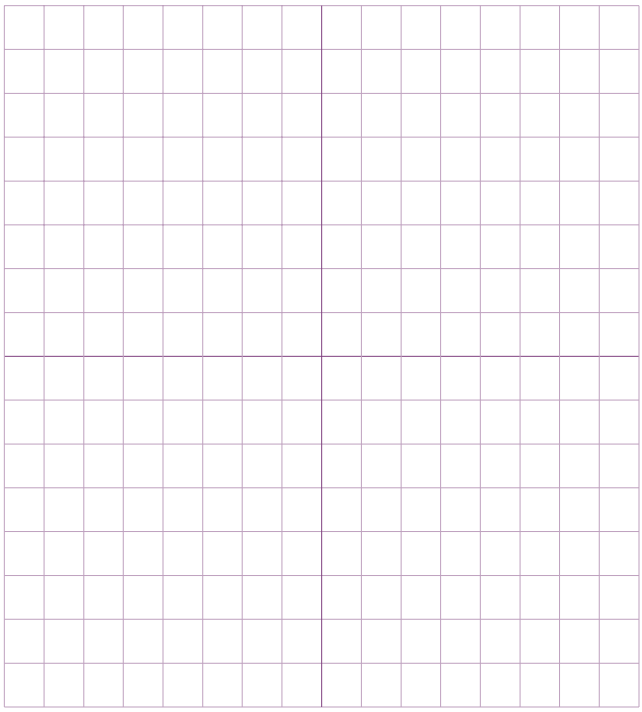

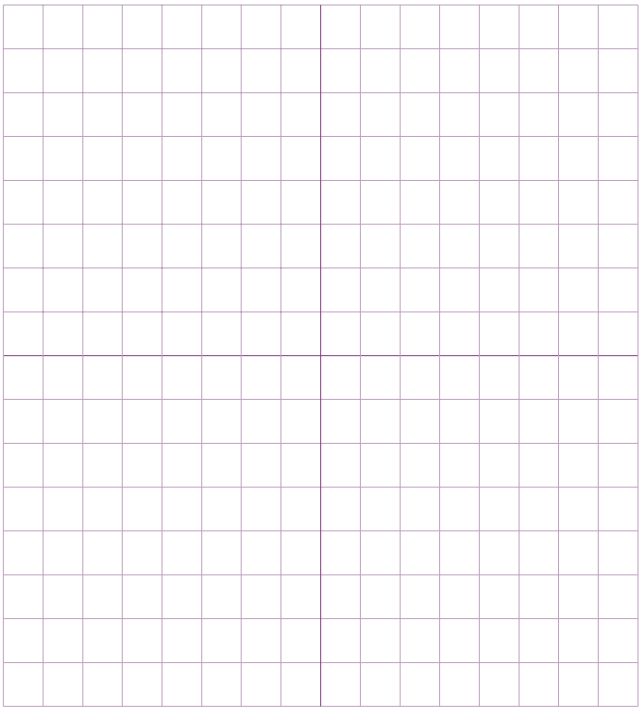

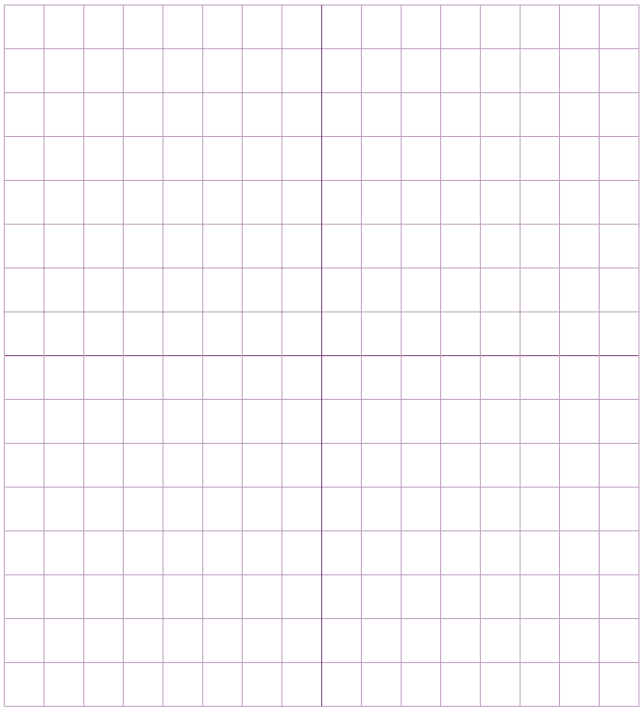

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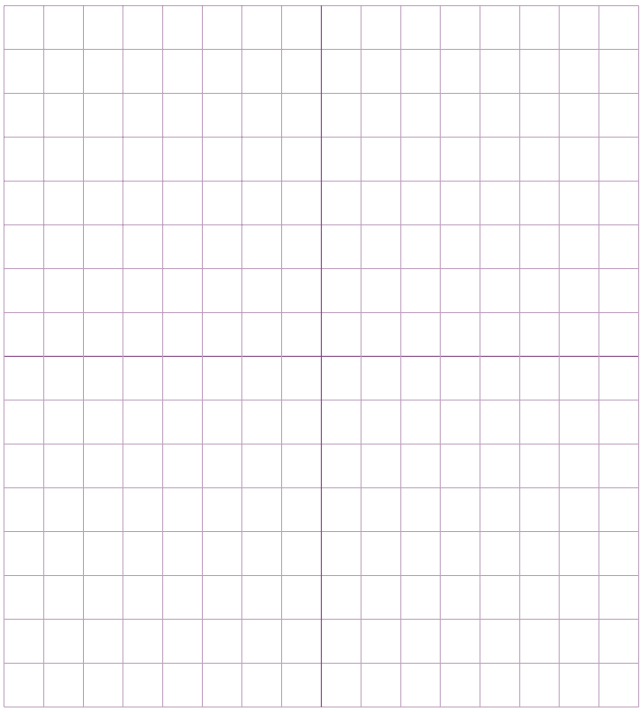

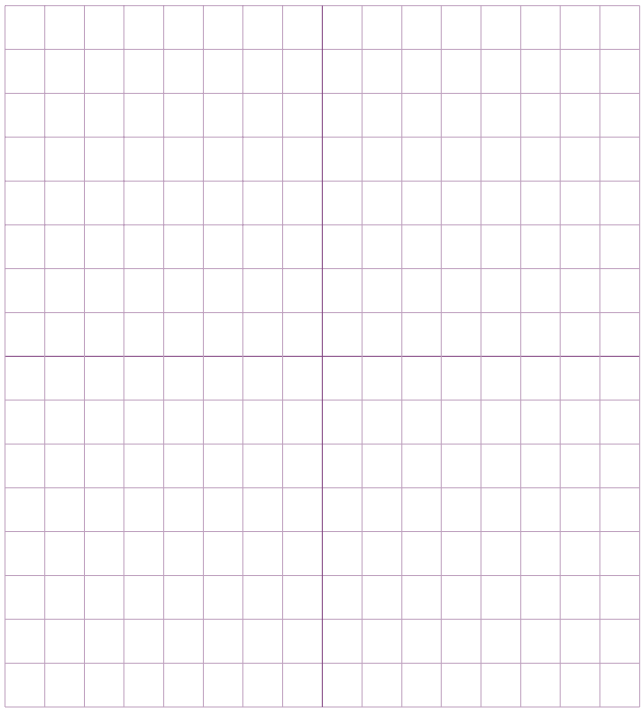

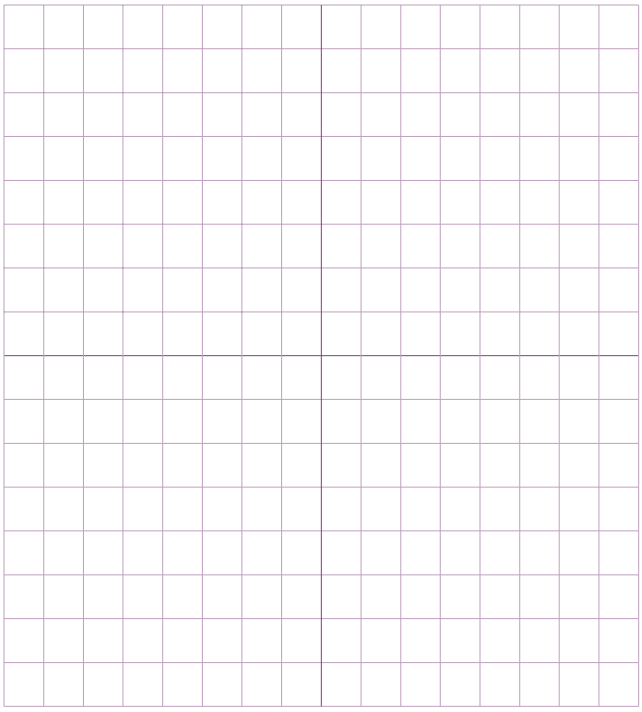

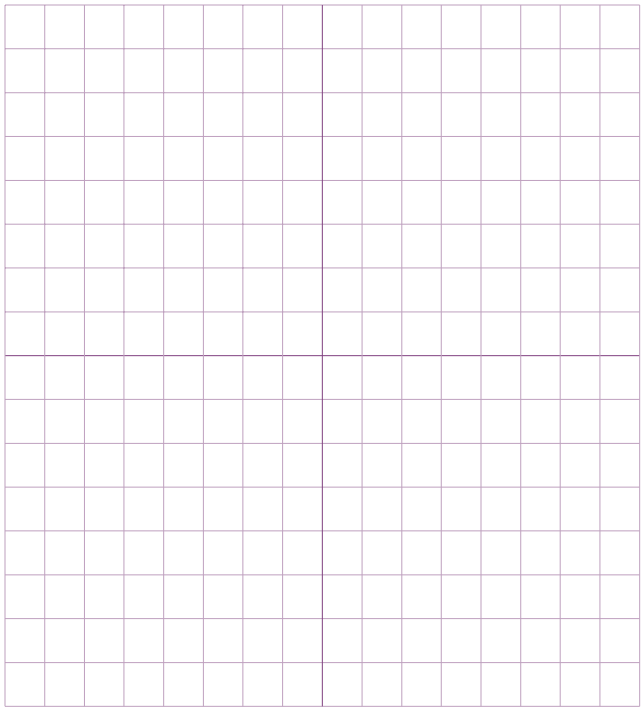

ROUGH WORK FOR MULTIPLE-CHOICE

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