

APRIL 1998

PROVINCIAL EXAMINATION

MINISTRY OF EDUCATION, SKILLS AND TRAINING

MATHEMATICS 12

GENERAL INSTRUCTIONS

1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above and on the **back** cover of this booklet. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this booklet.**
2. Ensure that in addition to this examination booklet, you have an **Examination Response Form**. Follow the directions on the front of the Response Form.
3. **Disqualification** from the examination will result if you bring books, paper, notes or unauthorized electronic devices into the examination room.
4. All multiple-choice answers must be entered on the Response Form using an **HB pencil**. Multiple-choice answers entered in this examination booklet will **not** be marked.
5. For each of the written-response questions, write your answer in the space provided in this booklet.
6. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

END OF EXAMINATION.

7. At the end of the examination, place your Response Form inside the front cover of this booklet and return the booklet and your Response Form to the supervisor.

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MATHEMATICS 12 PROVINCIAL EXAMINATION

- | | Value | Suggested Time |
|---|------------------------|--------------------|
| 1. This examination consists of two parts: | | |
| PART A: 50 multiple-choice questions | 50 | 75 |
| PART B: 7 written-response questions
2 questions worth two marks each,
4 questions worth three marks each, and
1 question worth four marks. | 20 | 45 |
| | Total: 70 marks | 120 minutes |
- Aside from an approved calculator, electronic devices, including dictionaries and pagers, are **not** permitted in the examination room.
 - The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae, Rough Work for Graphing, and Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.
 - You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.
 - An approved scientific calculator is essential for the examination. The calculator must be a hand-held device designed **only** for mathematical computations such as logarithmic and trigonometric functions. It **can be** programmable, but **must not** contain any graphing capabilities. You **must not** bring into the examination room any devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or keyboards.
 - You are permitted to use rulers, compasses and protractors.
 - The time allotted for this examination is **two hours**.

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PART A: MULTIPLE CHOICE

Value: 50 marks

Suggested Time: 75 minutes

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the Response Form provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. Which conic is described by the equation $3x^2 - 12x - 2y^2 + 16y - 21 = 0$?

- A. circle
- B. ellipse
- C. parabola
- D. hyperbola

2. Determine the vertex of the parabola $y = (x - 5)^2 - 6$.

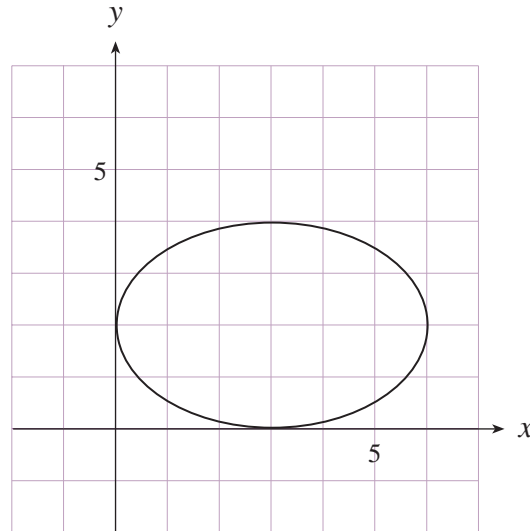
- A. $(-5, -6)$
- B. $(-5, 6)$
- C. $(5, -6)$
- D. $(5, 6)$

3. Which absolute value inequality describes the solution shown?



- A. $|x - 1| > 4$
- B. $|x + 1| > 4$
- C. $|x - 4| > 1$
- D. $|x + 4| > 1$

4. Determine an equation of the ellipse graphed below.



- A. $\frac{(x+3)^2}{3} + \frac{(y+2)^2}{2} = 1$
- B. $\frac{(x+3)^2}{9} + \frac{(y+2)^2}{4} = 1$
- C. $\frac{(x-3)^2}{3} + \frac{(y-2)^2}{2} = 1$
- D. $\frac{(x-3)^2}{9} + \frac{(y-2)^2}{4} = 1$

5. The diameter AB of a circle has endpoints A(1, 7) and B(-1, -7). Find the equation of the circle.

- A. $x^2 + y^2 = 5\sqrt{2}$
- B. $x^2 + y^2 = 10\sqrt{2}$
- C. $x^2 + y^2 = 50$
- D. $x^2 + y^2 = 200$

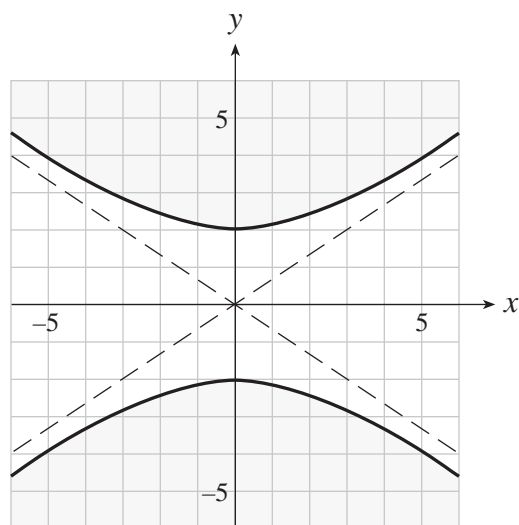
6. Find the range of $x^2 + 6x + y^2 = 0$.

- A. $-9 \leq y \leq 9$
- B. $-6 \leq y \leq 6$
- C. $-6 \leq y \leq 0$
- D. $-3 \leq y \leq 3$

7. A point $P(x, y)$ moves such that it is always three times as far from the point $A(2, 0)$ as it is from the point $B(0, -4)$. Find an equation of this locus.

- A. $\sqrt{(x-2)^2 + y^2} = 3\sqrt{x^2 + (y+4)^2}$
 B. $\sqrt{(x+2)^2 + y^2} = 3\sqrt{x^2 + (y-4)^2}$
 C. $3\sqrt{(x+2)^2 + y^2} = \sqrt{x^2 + (y-4)^2}$
 D. $3\sqrt{(x-2)^2 + y^2} = \sqrt{x^2 + (y+4)^2}$

8. Which inequality represents the shaded region?



- A. $\frac{x^2}{9} - \frac{y^2}{4} \geq 1$
 B. $\frac{x^2}{9} - \frac{y^2}{4} \leq 1$
 C. $\frac{x^2}{9} - \frac{y^2}{4} \geq -1$
 D. $\frac{x^2}{9} - \frac{y^2}{4} \leq -1$

9. For which value of b will the following system have exactly 3 different real solutions?

$$(y-3)^2 - (x+2)^2 = 4$$

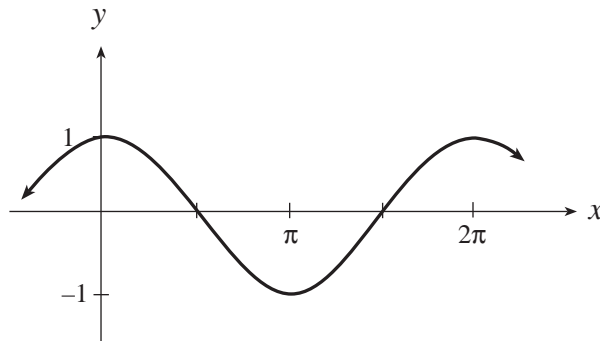
$$y = -(x+2)^2 + b$$

- A. -5
- B. -1
- C. 2
- D. 5

10. Determine the value of k so that the graph of $x = 2y^2 + ky + 33$ has $y = -3$ as its axis of symmetry.

- A. -12
- B. -6
- C. 6
- D. 12

11. Determine an equation of the trigonometric function graphed below.

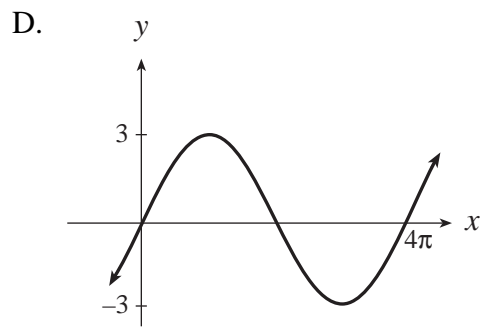
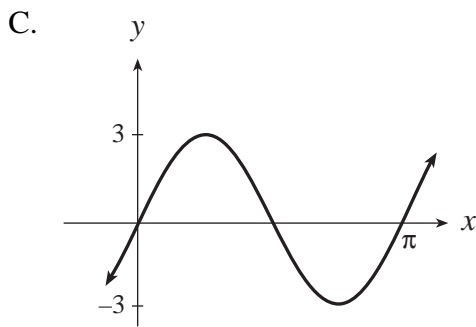
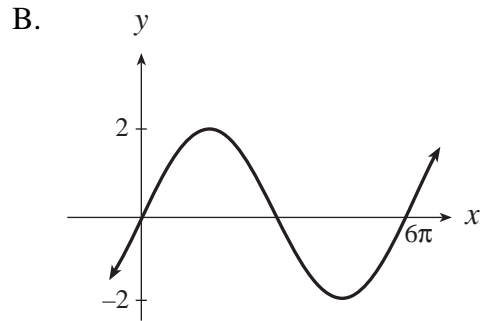
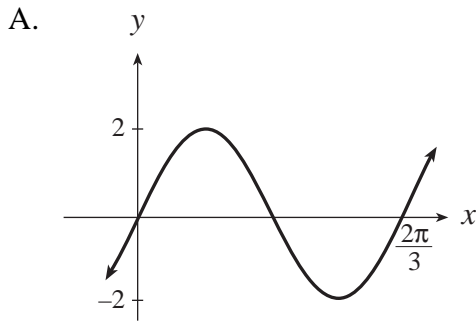


- A. $y = \cos x$
- B. $y = \sin x$
- C. $y = -\cos x$
- D. $y = -\sin x$

12. Convert x degrees to radians.

- A. $\frac{\pi}{180x}$
- B. $\frac{\pi x}{180}$
- C. $\frac{180}{\pi x}$
- D. $\frac{180x}{\pi}$

13. Which graph **best** represents $y = 3 \sin 2x$?



14. Determine the phase shift of $y = \cos(2x + 3)$.

- A. 3 units to the left
- B. 3 units to the right
- C. $\frac{3}{2}$ units to the left
- D. $\frac{3}{2}$ units to the right

15. The point $P(-12, -5)$ lies on the terminal arm of an angle θ in standard position. Determine the value of $\csc \theta$.
- A. $-\frac{13}{5}$
 - B. $-\frac{13}{12}$
 - C. $\frac{5}{12}$
 - D. $\frac{12}{5}$
16. Which expression is equivalent to $\frac{\cos 2\theta - 1}{\sin 2\theta}$?
- A. $\cot \theta$
 - B. $\tan \theta$
 - C. $-\cot \theta$
 - D. $-\tan \theta$
17. Determine an expression for x such that $\frac{2 - \cos^2 \theta}{\sin \theta} = x + \sin \theta$ is an identity.
- A. $\csc \theta$
 - B. $\sec \theta$
 - C. $\cos \theta$
 - D. $\cos^2 \theta$
18. How many solutions does $\cos 2x \cos x - \sin 2x \sin x = 0$ have over the interval $0 \leq x < 2\pi$?
- A. 2
 - B. 3
 - C. 4
 - D. 6

19. Evaluate: $\log_3 81$

- A. $\frac{1}{4}$
- B. 2
- C. 4
- D. 27

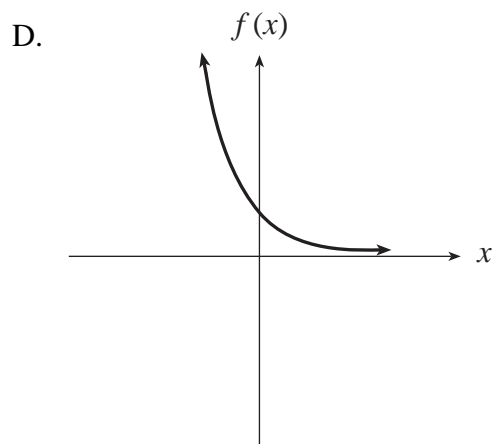
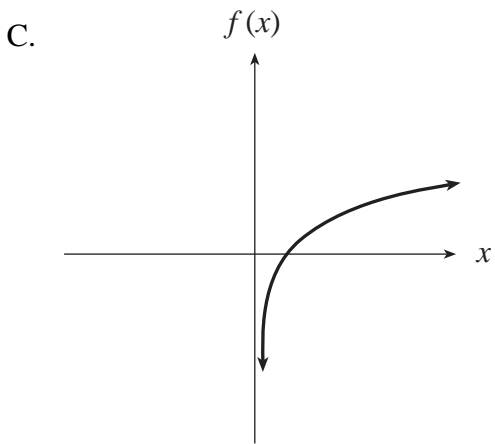
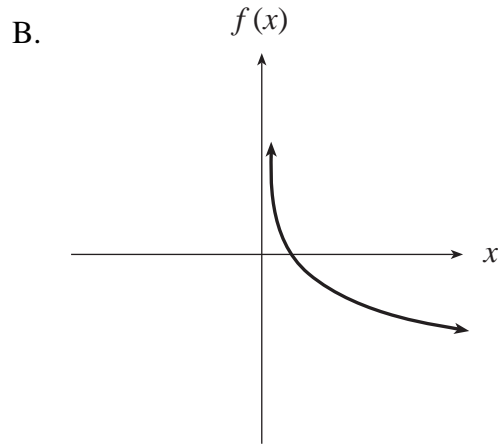
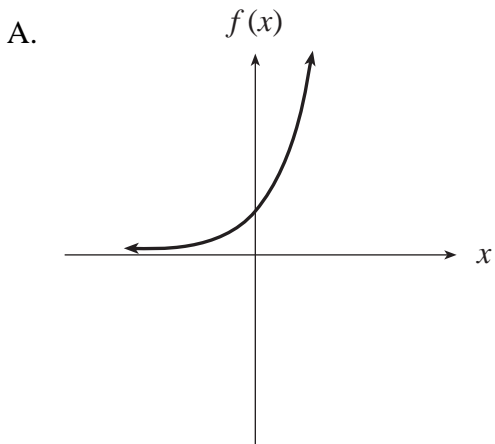
20. Which expression is equivalent to $\log\left(\frac{b}{c}\right)$?

- A. $\log b + \log c$
- B. $\log b - \log c$
- C. $(\log b)(\log c)$
- D. $\frac{\log b}{\log c}$

21. Solve for x : $6^{\log x} = \frac{1}{36}$

- A. -2
- B. $\frac{1}{100}$
- C. 2
- D. 100

22. Which graph **best** represents the function $f(x) = \log_{\frac{1}{2}} x$?



23. Given $f(x) = 7^x$, determine its inverse $f^{-1}(x)$.

A. $f^{-1}(x) = \log_x 7$

B. $f^{-1}(x) = \log_7 x$

C. $f^{-1}(x) = \frac{1}{7^x}$

D. $f^{-1}(x) = 7^{\frac{1}{x}}$

24. Simplify: $5^{(2\log_5 m - \log_5 n)}$

A. $\frac{m^2}{n}$

B. $m^2 - n$

C. $5^{\frac{m^2}{n}}$

D. 5^{m^2-n}

25. Solve: $\log_2 x = \log_4 x^2$

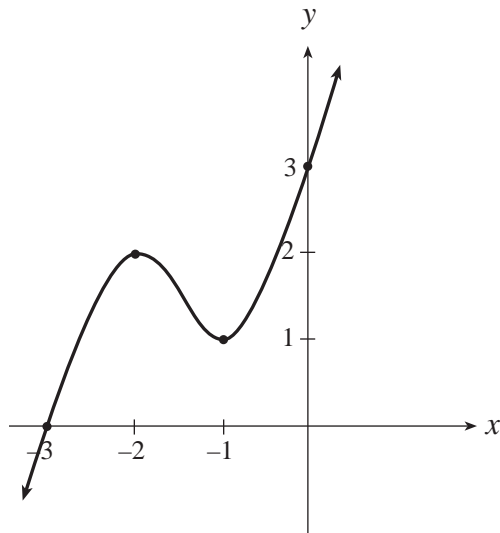
A. 1

B. 2

C. 1, 2, 4

D. all positive real numbers

26. Determine a real zero of the function graphed below.



- A. -3
- B. -2
- C. -1
- D. 3

27. Determine the quotient when $x^3 + x + 2$ is divided by $x + 1$.

- A. $x^2 + 2$
- B. $x^2 + 2x$
- C. $x^2 + x + 2$
- D. $x^2 - x + 2$

28. Which graph represents the solution of the inequality $(x - 2)(x + 1)(x + 2) < 0$?

- A.
- B.
- C.
- D.

29. Determine a factor of degree 2 of the polynomial $p(x)$ if $p(3) = 0$ and $p(-4) = 0$.

- A. $x^2 + x - 12$
- B. $x^2 - x + 12$
- C. $x^2 - x - 12$
- D. $x^2 + x + 12$

30. Determine the values of k for which $\frac{1}{3}$ is a zero of $p(x) = -9x^3 + 3x^2 - 3kx + k^3$.

- A. $-2, -1, 0$
- B. $-2, 0, 1$
- C. $-1, 0, 1$
- D. $-1, 0, 2$

31. Determine the 8th term of the geometric sequence $-4, 12, -36, \dots$

- A. $-26\,244$
- B. $-8\,748$
- C. $8\,748$
- D. $26\,244$

32. For the arithmetic sequence $-50, -32, -14, \dots$, determine t_{26} .

- A. 400
- B. 418
- C. 436
- D. 500

33. What is the 3rd term of the sequence given by the following recursive definition?

$$t_1 = 3$$

$$t_n = 2t_{n-1} - 1, n > 1$$

- A. 0
- B. 9
- C. 15
- D. 127

34. Which of the following infinite geometric series has a finite sum?

A. $\frac{1}{2} + 1 + 2 + \dots$

B. $\frac{1}{2} - 1 + 2 - \dots$

C. $1 + 2 + 4 + \dots$

D. $2 - 1 + \frac{1}{2} - \dots$

35. Given an arithmetic series where $a = 9$ and $d = 0.7$, find the sum of the first 15 terms.

A. 203.25

B. 208.5

C. 213.75

D. 219

36. In a geometric sequence, $t_3 = \frac{3}{2}$ and $t_6 = \frac{81}{16}$. Determine the value of the constant ratio r .

A. $\frac{4}{9}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. $\frac{27}{8}$

37. Evaluate: $\sum_{k=3}^{14} 2^{k-1}$

A. 8 188

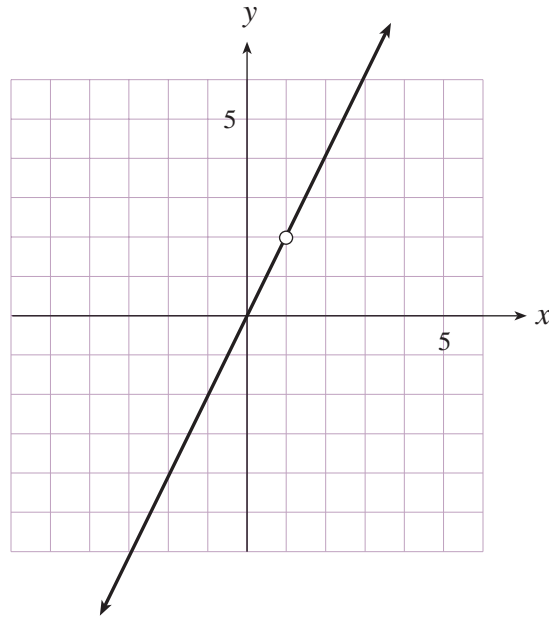
B. 16 380

C. 32 760

D. 65 532

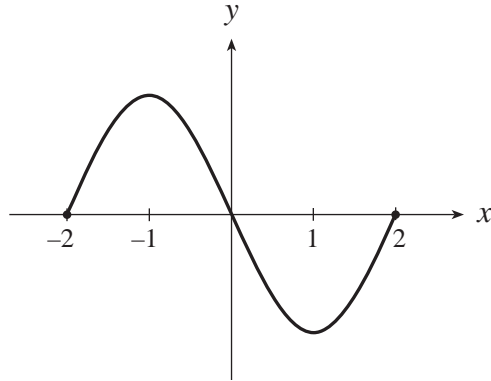
38. After inserting k arithmetic means between -8 and 55 , the sum of the resulting series is $1\,081$. Find k , the number of arithmetic means that were inserted.
- A. 32
 - B. 34
 - C. 44
 - D. 46
39. Find the derivative of $y = 4x^3 - 2x^2 + 3$.
- A. $4x^2 - 2x$
 - B. $12x^2 - 4x$
 - C. $12x^3 - 4x$
 - D. $12x^2 - 4x + 3$
40. Find the slope of the line tangent to $y = 7 - 4x^2$ at $x = 2$.
- A. -16
 - B. -9
 - C. -8
 - D. $-\frac{1}{4}$
41. Evaluate: $\lim_{x \rightarrow \infty} \frac{5x^2 + 4x - 1}{5x^2 - 3x + 2}$
- A. $-\frac{1}{2}$
 - B. $\frac{1}{2}$
 - C. 1
 - D. limit does not exist (no finite limit)

42. The graph of $y = f(x)$ is shown below. Determine $\lim_{x \rightarrow 1} f(x)$.

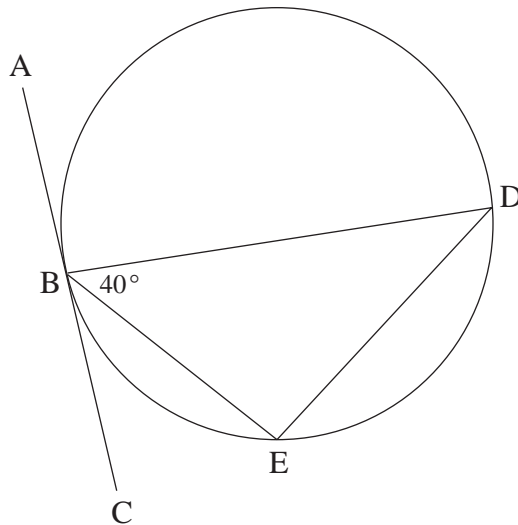


- A. 0
B. 1
C. 2
D. limit does not exist
43. A particle moves along the x -axis so that its position at time t is $x(t) = t^2 - 2t + 5$, where x is in centimetres and t is in seconds. At what time is the particle's velocity 4 cm/s?
- A. $t = 1$
B. $t = 3$
C. $t = 6$
D. $t = 13$
44. Given the function $f(x) = 4 - x^2$, determine the slope of the secant line intersecting the graph of f at the points where $x = -1$ and $x = 4$.
- A. -3
B. $-\frac{1}{3}$
C. $\frac{1}{3}$
D. 3

45. The graph of $y = f(x)$ is shown below. For which interval is $f'(x) > 0$ and $f(x) < 0$?



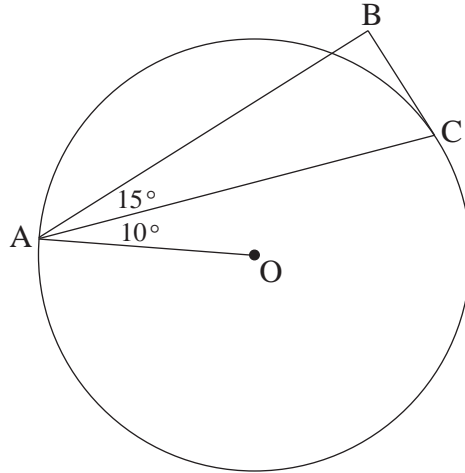
- A. $-2 < x < -1$
 B. $-1 < x < 0$
 C. $0 < x < 1$
 D. $1 < x < 2$
46. In the diagram below, AC is tangent to the circle at B and BE bisects $\angle DBC$.
 (Diagram not drawn to scale.)



Determine the measure of $\angle BED$.

- A. 40°
 B. 90°
 C. 100°
 D. 140°

47. In the diagram below, BC is tangent to the circle at C and O is the centre of the circle.
(Diagram not drawn to scale.)

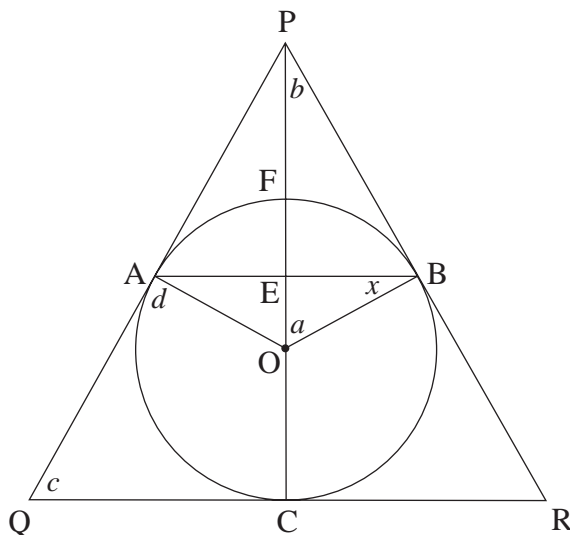


Determine the measure of $\angle ABC$.

- A. 75°
 - B. 85°
 - C. 90°
 - D. 95°
48. Given $\log 80 = a$, which of the following is an expression for $\log 2$ in terms of a ?
- A. $\frac{a}{30}$
 - B. $\frac{a-1}{3}$
 - C. $\frac{a-1}{4}$
 - D. $\frac{a-10}{3}$

Use the following diagram to answer question 49.

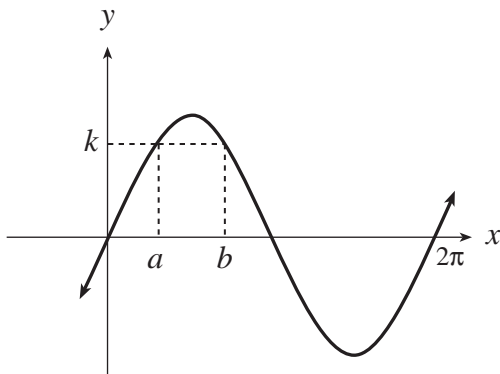
Given: Circle with centre O
 PQ, PR, QR are tangent
 at A, B, C respectively
 $AB \perp PC$



49. If $OE \neq EB$, which angle is equal to angle x ?

- A. a
- B. b
- C. c
- D. d

50. Given that $\sin x > k$ for $a < x < b$ as shown in the diagram, determine **all** real values of x for which $\sin x > k$.



- A. $a + \pi < x < b + \pi$
- B. $a + n\pi < x < b + n\pi$ (n is an integer)
- C. $a + 2\pi < x < b + 2\pi$
- D. $a + 2n\pi < x < b + 2n\pi$ (n is an integer)

This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.

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PART B: WRITTEN RESPONSE

Value: 20 marks

Suggested Time: 45 minutes

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

Full marks will NOT be given for the final answer only.

1. Solve for x : $3\sin^2 x - 8\sin x + 4 = 0$, where $0 \leq x < 2\pi$.
(Accurate to at least 2 decimal places.)

(2 marks)

ANSWER:

2. A colony of ants has a population of 250 000. It is growing at a rate of 3.7% per annum.
How long will it take for the population to reach 1 000 000? (Accurate to the nearest year.)
(3 marks)

ANSWER:

3. Determine the polynomial function of degree 3, with zeros of -2 , 0 , and 3 , that passes through the point $(2, 5)$. Answer may be left in factored form. **(3 marks)**

ANSWER:

4. Determine all ordered pairs that satisfy the following system.
(Answer exact or accurate to at least 2 decimal places.)

(3 marks)

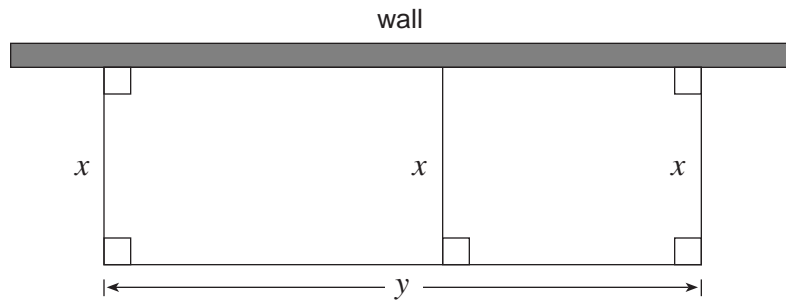
$$x - y^2 = -3$$

$$2x^2 + 3y^2 = 18$$

ANSWER:

OVER

5. A rectangular pigpen is to be constructed having one side along an existing wall. The pigpen is also to be divided into two parts as shown in the diagram.



If a total of 300 metres of fencing is used, determine the maximum area that the pigpen can have. **(3 marks)**

ANSWER:

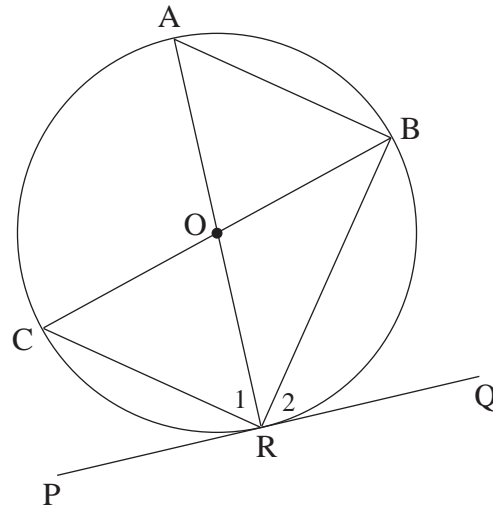
6. Complete the proof.

(4 marks)

Given: AR and BC are diameters.
PQ is tangent to the circle at R

Prove: $\angle 1 = \angle 2$

Note: Students are encouraged to number angles.



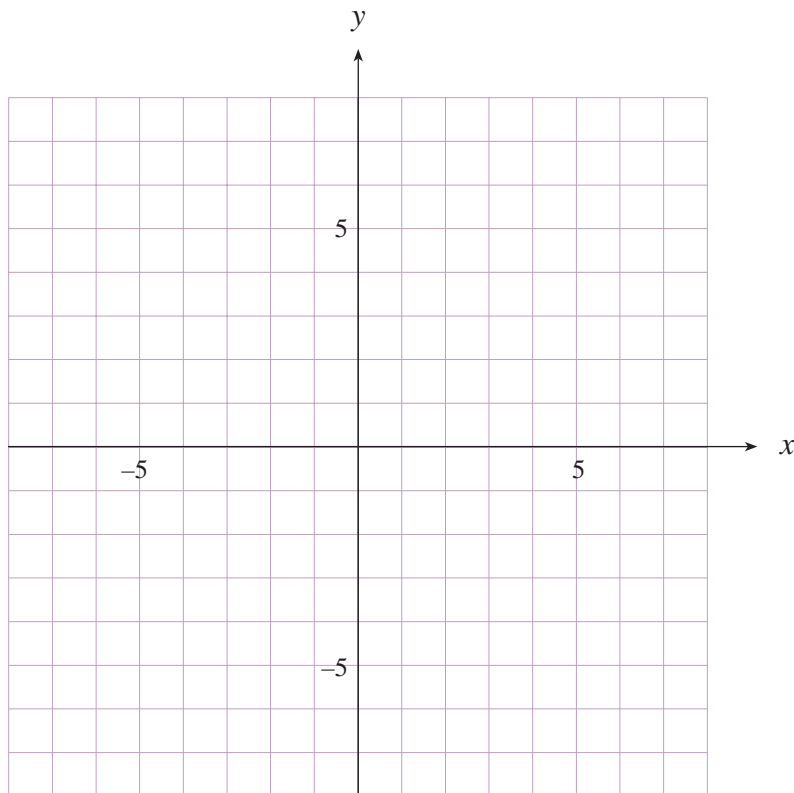
PROOF	
Statement	Reason

7. A function is defined by the equation:

$$f(t) = t^2 - 6t$$

Sketch the graph given by the solution of $f(x) - f(y) = 0$.

(2 marks)



END OF EXAMINATION

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A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_n = \frac{a - \ell r}{1-r}$$

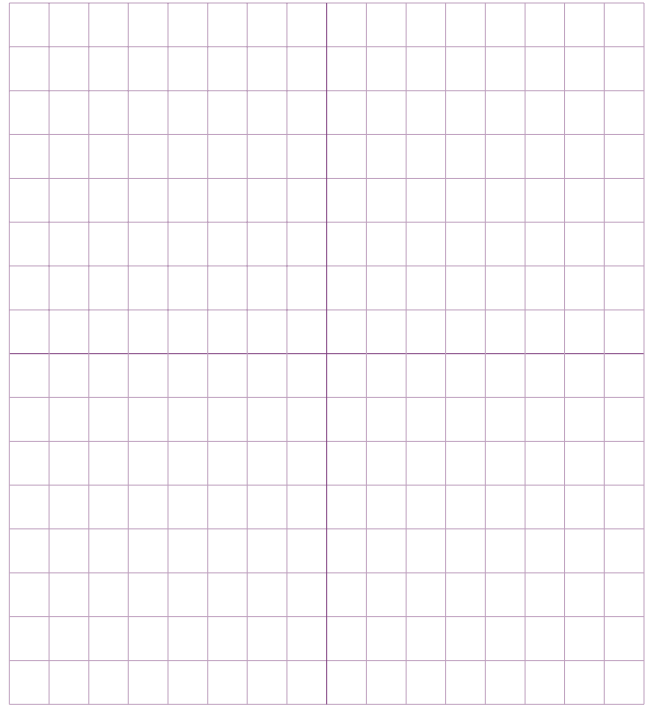
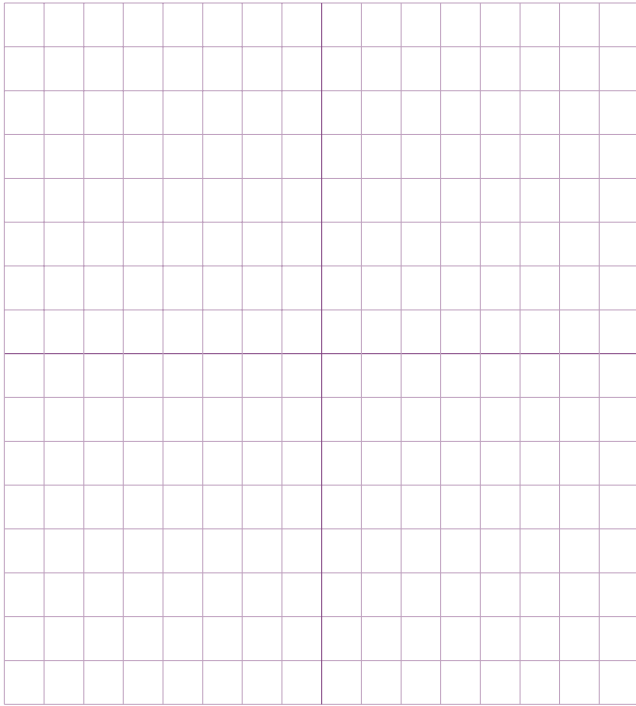
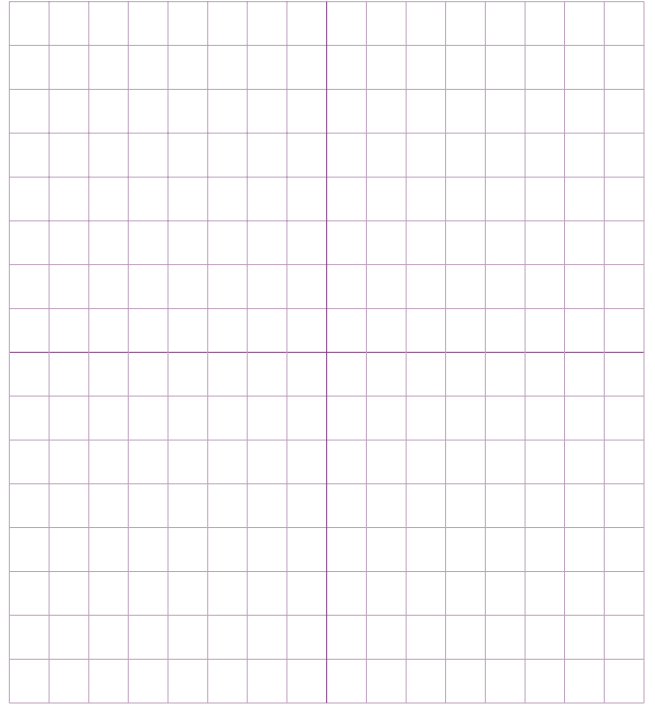
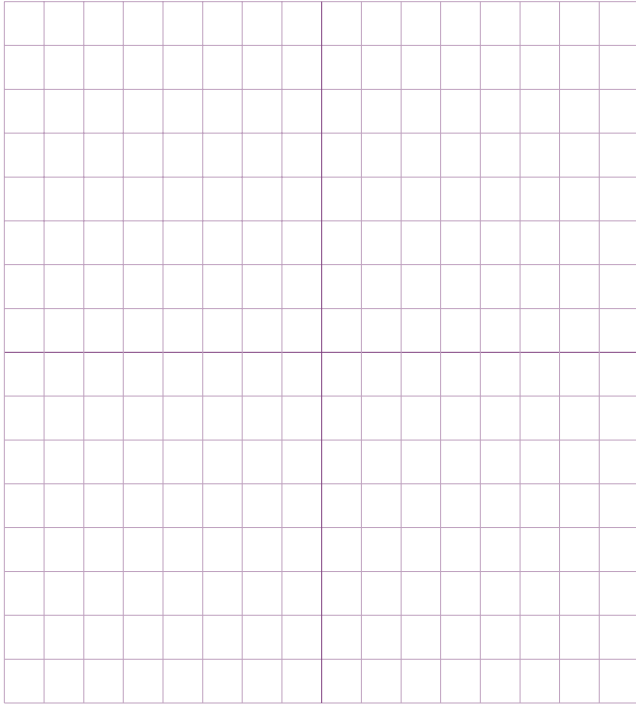
$$S = \frac{a}{1-r}$$

**You may detach this page for convenient reference.
Exercise care when tearing along perforations.**

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ROUGH WORK FOR GRAPHING

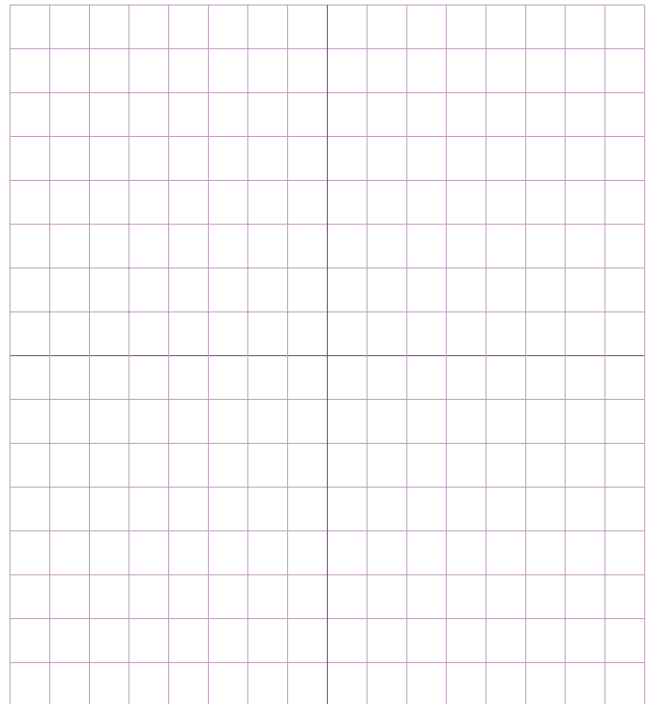
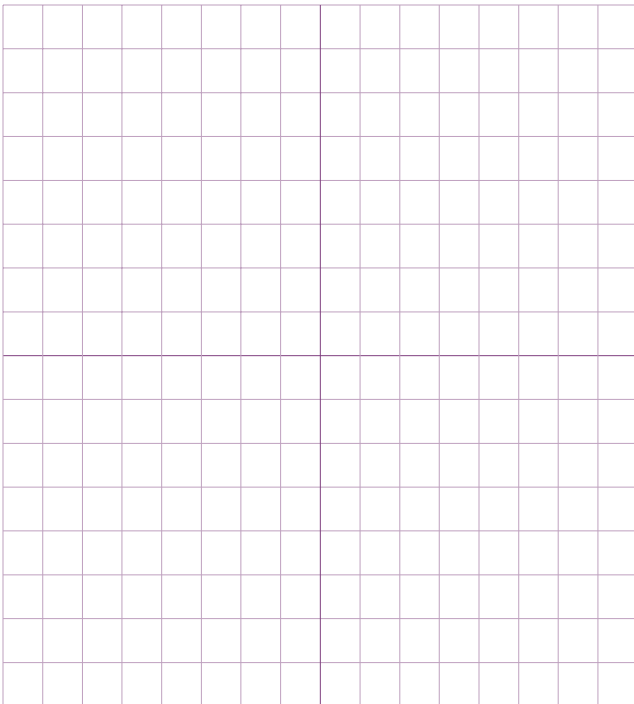
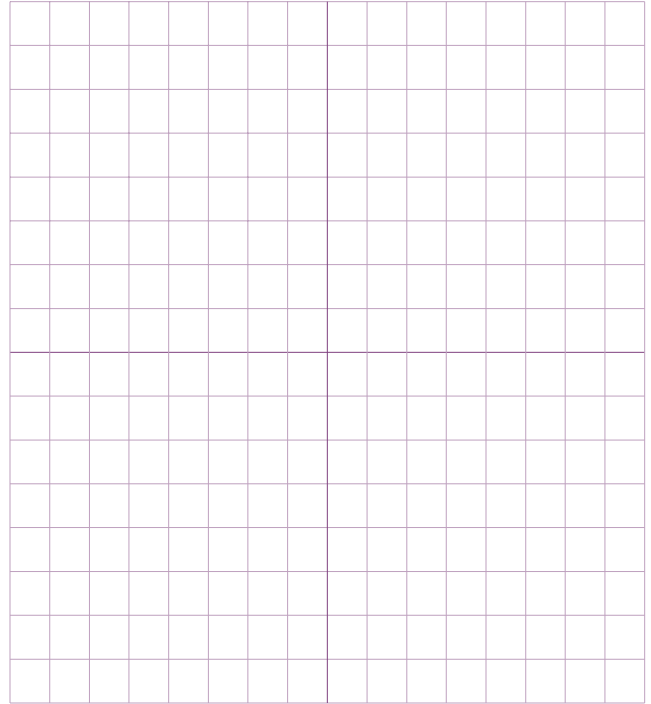
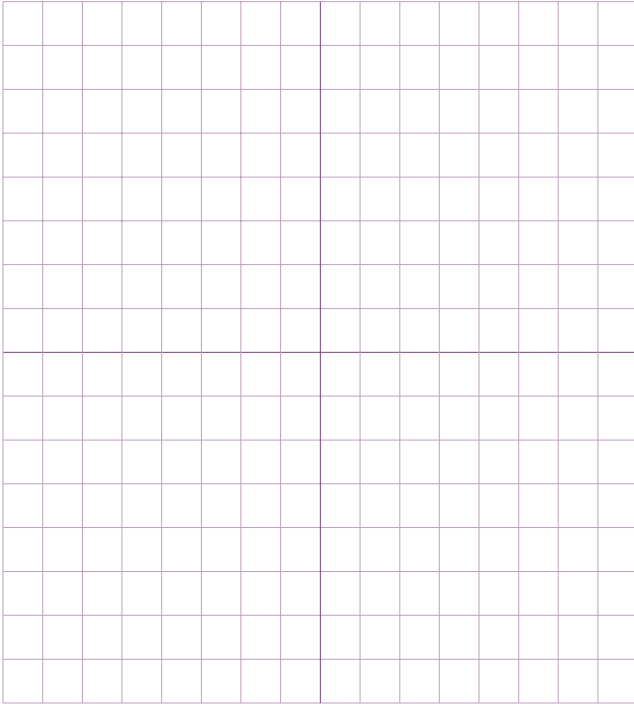
(No marks will be given for work done on this page.)



**You may detach this page for convenient reference.
Exercise care when tearing along perforations.**

ROUGH WORK FOR GRAPHING

(No marks will be given for work done on this page.)

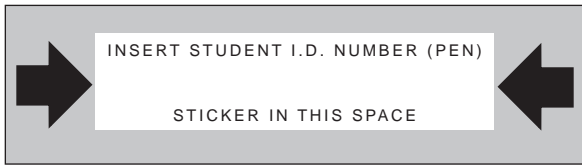


ROUGH WORK FOR MULTIPLE-CHOICE

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ROUGH WORK FOR MULTIPLE-CHOICE





MATHEMATICS 12

April 1998

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MATHEMATICS 12

April 1998

Course Code = MA

Score for
Question 1:

1. $\frac{\quad}{(2)}$

Score for
Question 2:

2. $\frac{\quad}{(3)}$

Score for
Question 3:

3. $\frac{\quad}{(3)}$

Score for
Question 4:

4. $\frac{\quad}{(3)}$

Score for
Question 5:

5. $\frac{\quad}{(3)}$

Score for
Question 6:

6. $\frac{\quad}{(4)}$

Score for
Question 7:

7. $\frac{\quad}{(2)}$