INSERT STUDENT I.D. NUMBER (PEN)

STICKER IN THIS SPACE

AUGUST 1997

PROVINCIAL EXAMINATION

MINISTRY OF EDUCATION, SKILLS AND TRAINING

MATHEMATICS 12

GENERAL INSTRUCTIONS

1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this paper.**

2. Take the separate Answer Sheet and follow the directions on its front page.

3. Be sure you have an **HB pencil** and an eraser for completing your Answer Sheet. Follow the directions on the Answer Sheet when answering multiple-choice questions.

4. For each of the written-response questions, write your answer in the space provided.

5. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

   END OF EXAMINATION.

6. At the end of the examination, place your Answer Sheet inside the front cover of this booklet and return the booklet and your Answer Sheet to the supervisor.

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FOR OFFICE USE ONLY

INSERT STUDENT I.D. NUMBER (PEN)

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_____ – _____

MATHEMATICS 12 AUGUST 1997 PROVINCIAL

Course Code = MA Examination Type = P

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6. _____
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7. _____
(2)

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MATHEMATICS 12 PROVINCIAL EXAMINATION

		Value	Suggested Time
1.	This examination consists of **two** parts:		
	PART A: 50 multiple-choice questions	50	75
	PART B: 7 written-response questions 2 questions worth **two** marks each, 4 questions worth **three** marks each, and 1 question worth **four** marks.	20	45
	Total:	**70 marks**	**120 minutes**

2. The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae**, **Rough Work for Graphing**, and **Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.

3. You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.

4. An approved scientific calculator is essential for the examination. The calculator must be a hand-held device designed **only** for mathematical computations such as logarithmic and trigonometric functions. It **can be** programmable, but **must not** contain any graphing capabilities. You **must not** bring into the examination room any devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or keyboards.

5. You are permitted to use rulers, compasses, and protractors.

6. You have **two hours** to complete this examination.

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PART A: MULTIPLE CHOICE

Value: 50 marks **Suggested Time: 75 minutes**

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the Answer Sheet provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. Determine the distance between the points $(3,\ 9)$ and $(-1,\ 13)$.

   A. 6
   B. 8
   C. $2\sqrt{5}$
   D. $4\sqrt{2}$

2. Determine the midpoint of the line segment joining the points $(3,\ -5)$ and $(-2,\ 7)$.

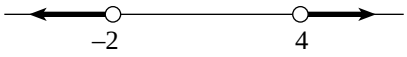
   A. $\left(-\frac{1}{2},\ -1\right)$
   B. $\left(-\frac{1}{2},\ 1\right)$
   C. $\left(\frac{1}{2},\ -1\right)$
   D. $\left(\frac{1}{2},\ 1\right)$

3. Give the centre of the circle $x^2 - 2x + y^2 = 11$.

   A. $(-2,\ 0)$
   B. $(-1,\ 0)$
   C. $(1,\ 0)$
   D. $(2,\ 0)$

OVER

4. Which absolute value inequality is represented by the solution shown below?



A. $|x-1|<3$
B. $|x-1|>3$
C. $|x+1|<3$
D. $|x+1|>3$

5. A point $P(x,\ y)$ moves such that it is always equidistant from the point $F(3,\ 2)$ and the line $y=-1$. Which equation represents this locus?

A. $(x-3)^2+(y-2)^2=(y+1)^2$
B. $(x-3)^2+(y-2)^2=(x+1)^2$
C. $(x+3)^2+(y+2)^2=(y-1)^2$
D. $(x+3)^2+(y+2)^2=(x-1)^2$

6. Determine the equation of the ellipse with vertices of $(3,\ 6)$ and $(3,\ -4)$ and minor axis of length 6.

A. $\dfrac{(x-3)^2}{9}+\dfrac{(y-1)^2}{25}=1$
B. $\dfrac{(x+3)^2}{9}+\dfrac{(y+1)^2}{25}=1$
C. $\dfrac{(x-3)^2}{25}+\dfrac{(y-1)^2}{9}=1$
D. $\dfrac{(x+3)^2}{25}+\dfrac{(y+1)^2}{9}=1$

7. Find all real solutions for the following system.

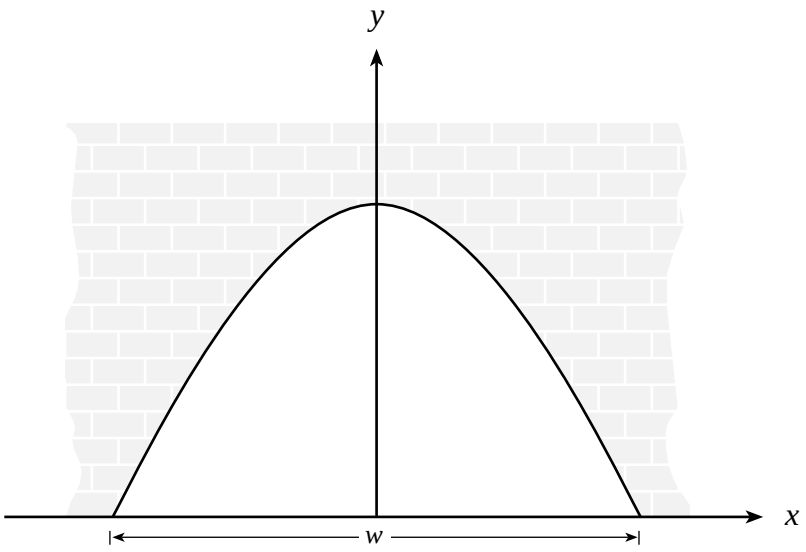
$$x^2 + y = 4$$
$$x^2 - y^2 = 4$$

A. $(-2,\ 0), (2,\ 0)$
B. $\left(-\sqrt{5},\ -1\right), \left(\sqrt{5},\ -1\right)$
C. $\left(-\sqrt{5},\ 1\right), \left(\sqrt{5},\ 1\right), (-2,\ 0), (2,\ 0)$
D. $\left(-\sqrt{5},\ -1\right), \left(\sqrt{5},\ -1\right), (-2,\ 0), (2,\ 0)$

8. Determine the area of the rectangle formed by the horizontal and vertical tangents to the conic $\frac{(x-1)^2}{9} + \frac{(y+2)^2}{16} = 1.$

A. 12 square units
B. 24 square units
C. 48 square units
D. 144 square units

9. A parabolic arch supports a bridge over a canal, as shown in the diagram. If an equation of the arch is $y = -\frac{1}{30}x^2 + 5,$ determine the width w of the canal. (Accurate to 2 decimal places.)



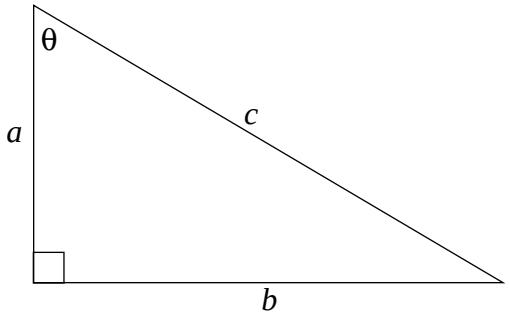
A. 10.00
B. 12.25
C. 17.32
D. 24.49

OVER

10. Which of the following values for the constants A and B will cause the equation $Axy + B = 0$ to represent a rectangular hyperbola with vertices on the line $y = -x$?

A. $A > 0$, $B < 0$
B. $A > 0$, $B > 0$
C. $A < 0$, $B > 0$
D. $A = 0$, $B < 0$

11. Use the diagram below to determine $\csc\theta$.



A. $\frac{a}{c}$
B. $\frac{b}{c}$
C. $\frac{c}{a}$
D. $\frac{c}{b}$

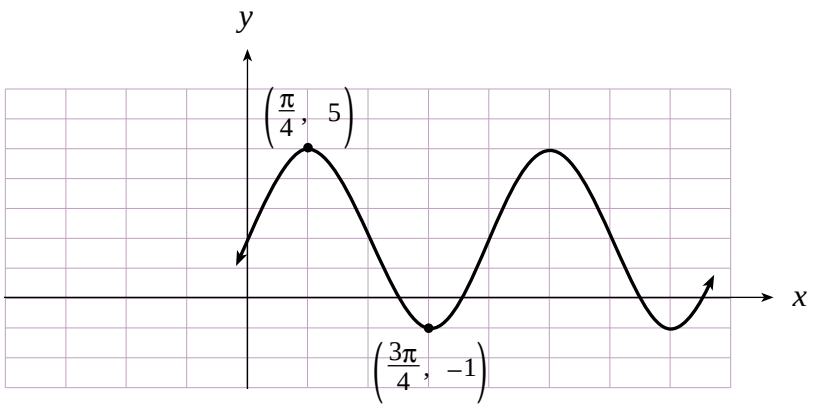
12. Convert $\frac{5\pi}{3}$ radians to degrees.

A. $60°$
B. $150°$
C. $300°$
D. $600°$

13. Determine the range of the function $f(x) = -2\cos 3x + 5$.

A. $-7 \le y \le -3$
B. $-2 \le y \le 5$
C. $-2 \le y \le 2$
D. $3 \le y \le 7$

14. Determine the period of the function graphed below.



A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. 2π

15. Which expression is equivalent to $\sin x \cos x$?

A. $\frac{1}{2}\sin 2x$

B. $2\sin 2x$

C. $2\sin x$

D. $\frac{1}{2}\sin x$

16. Simplify: $\frac{\sec\theta}{\tan\theta + \cot\theta}$

A. $\cos\theta$

B. $\sin\theta$

C. $\sec\theta$

D. $\csc\theta$

OVER

17. How many solutions are there in the interval $0 \le x < 2\pi$ for the equation $\sin^2 bx = 0.5$, where b is a positive integer?

A. 2
B. 4
C. $2b$
D. $4b$

18. Determine the smallest positive value for k such that $5\sin\pi(x-k) = -5\cos\pi x$.

A. $\frac{1}{2}$
B. 1
C. $\frac{\pi}{2}$
D. π

19. Determine an exponential equation equivalent to $y = \log_a\left(\frac{p}{q}\right)$.

A. $a^y = \frac{p}{q}$
B. $y^a = \frac{p}{q}$
C. $a^y = p - q$
D. $y^a = p - q$

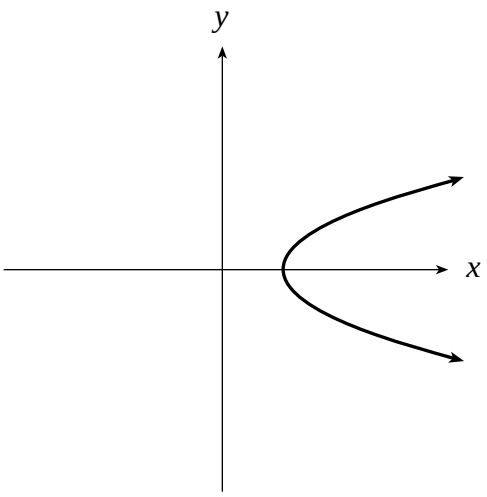
20. Determine the equation of the asymptote of the graph of $y = 3^x - 1$.

A. $y = -1$
B. $y = 0$
C. $y = 1$
D. $y = 2$

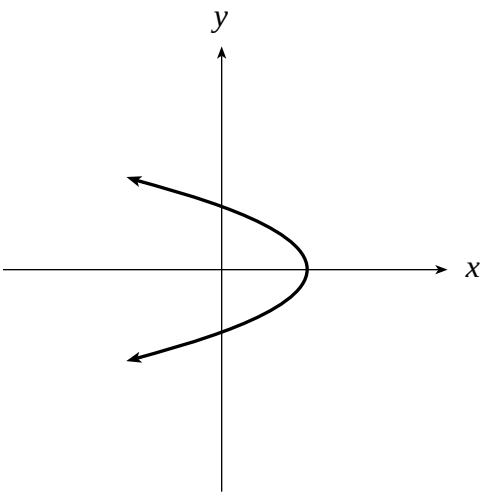
21. Which expression is equivalent to $\log_2 8x$?

A. $3\log_2 x$
B. $3 + \log_2 x$
C. $4 + \log_2 x$
D. $3 + 3\log_2 x$

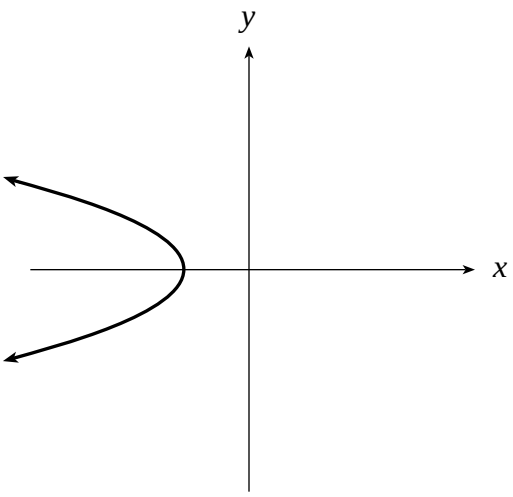
22. Which graph **best** represents the inverse relation of the graph shown?



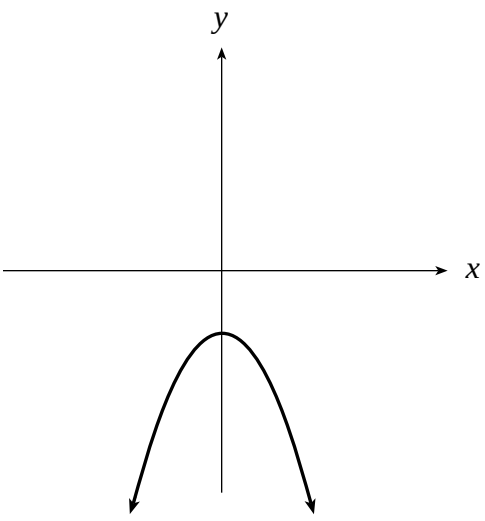
A.



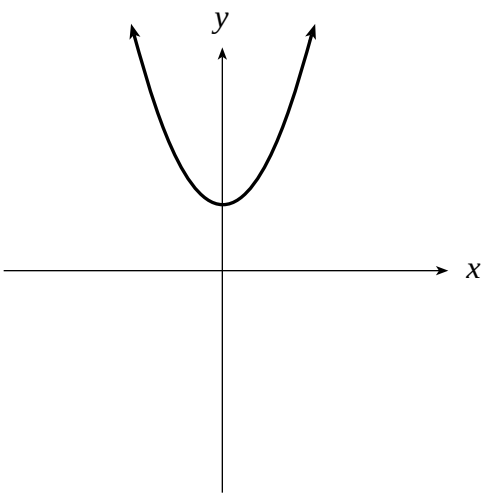
B.



C.



D.



OVER

23. A light filter reduces the intensity of light going through it by 10%. What percent of light passes through 8 such filters? (Accurate to the nearest percent.)

A. 20%
B. 30%
C. 43%
D. 48%

24. If $\log_9 4 = x$ and $\log_3 7 = y$, express $\log_3 28$ in terms of x and y.

A. xy
B. $2xy$
C. $x + y$
D. $2x + y$

25. Determine the domain of the function $y = \log\left(\frac{1}{4} - \frac{1}{x^2}\right)$.

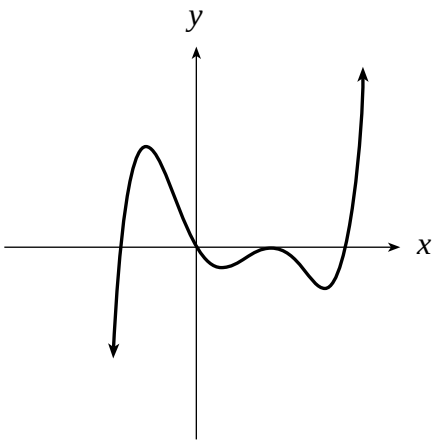
A. $x \neq 0$
B. $x > 2$
C. $x < -2$ or $x > 2$
D. $x < -2$ or $x > 0$

26. According to the Rational Root Theorem, which of the following is a possible root of $6x^4 - x^3 + 2x + 5 = 0$?

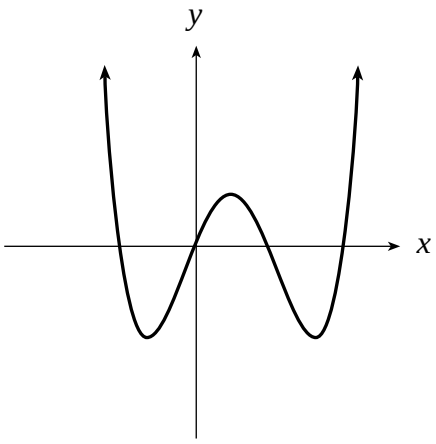
A. $\frac{2}{5}$
B. $\frac{5}{2}$
C. 2
D. 6

27. Which of the following could represent the graph of a 5th degree polynomial function?

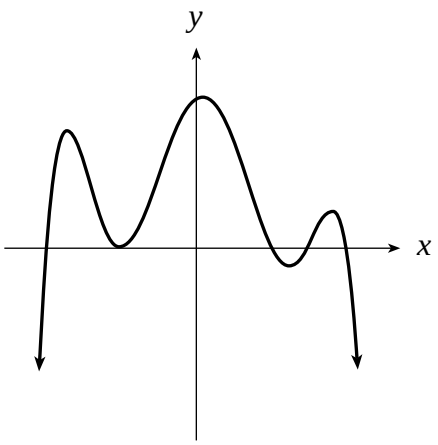
A.



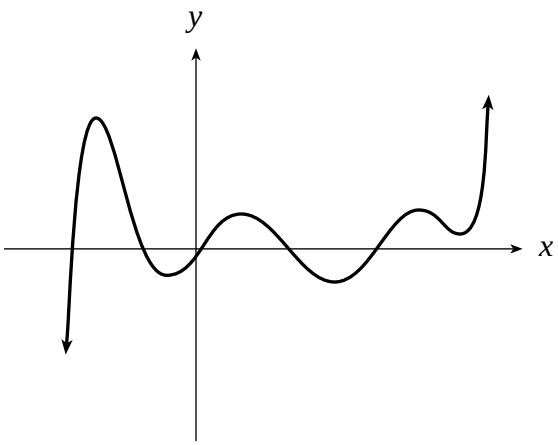
B.



C.



D.



28. Solve: $x^3 + x^2 - 17x + 15 = 0$

A. $-1, -3, 5$
B. $-1, 3, -5$
C. $1, 3, -5$
D. $1, -3, 5$

29. Determine the coefficient of x in the quotient for the following division.

$$x^2 + 5x + 1 \overline{)2x^4 - 3x^3 + 4x^2 - 3}$$

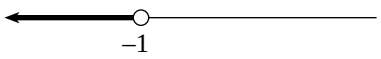
A. -13
B. -8
C. 7
D. 13

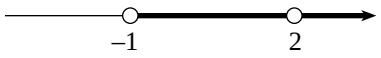
OVER

30. If $p(x) = (x-2)q(x) + r$, determine $p(2)$.

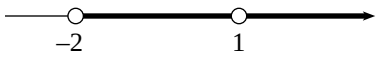
A. $q(2)$
B. $q(-2)$
C. $-r$
D. r

31. For what values of x does the graph of $y = -(x-1)^2(x+2)$ lie below the *x*-axis?

A. 

B. 

C. 

D. 

32. Which of the following is a geometric sequence?

A. 4, 2, 0
B. 4, 2, 0.5
C. 4, 2, 1
D. 4, 2, $\sqrt{2}$

33. Find the 35th term of the arithmetic sequence 103, 99, 95, . . .

A. –37
B. –33
C. –29
D. –25

34. Find the sum of the infinite geometric series whose terms are given by:

$$t_1 = 6$$

$$t_n = \frac{2}{3}t_{n-1},\ n > 1$$

A. 4
B. 12
C. 18
D. no finite sum

35. Determine n such that the sum of the first n terms of the geometric sequence $3, -6, 12, \ldots$ is $-1\,023$.

A. 9
B. 10
C. 11
D. 12

36. Which sigma notation represents the series $0 + 6 + 12 + 18$?

A. $\sum_{k=0}^{4} 6k$

B. $\sum_{k=0}^{3} (6k - 6)$

C. $\sum_{k=1}^{4} 6k$

D. $\sum_{k=1}^{4} (6k - 6)$

37. If $S_n = n^2 + 3n$, determine an expression for t_n.

A. $n + 3$
B. $2n + 1$
C. $2n + 2$
D. $2n - 2$

OVER

38. What is the 3rd term of the sequence defined as $t_n = n\sin^n\left(\frac{\pi}{2}(2n+1)\right)$?

A. –27
B. –3
C. 3
D. 27

39. Find y' if $y = x^{\frac{3}{2}}$.

A. $\frac{2}{3}x^{\frac{1}{2}}$
B. $\frac{3}{2}x^{\frac{1}{2}}$
C. $\frac{2}{3}x^{\frac{5}{2}}$
D. $\frac{3}{2}x^{\frac{5}{2}}$

40. Evaluate: $\lim_{x\to -2} \frac{x^2-4}{x^2+x-2}$

A. 1
B. $\frac{4}{3}$
C. 2
D. limit does not exist

41. Evaluate: $\lim_{x\to \infty} \frac{x^3-1}{3-5x^3}$

A. $-\frac{1}{3}$
B. $-\frac{1}{5}$
C. $\frac{1}{5}$
D. $\frac{1}{3}$

42. A particle moves along the *x*-axis according to the position function $x(t) = t^3 - 4t^2 + 3$ (*x* in metres, *t* in seconds). Determine the velocity in m/s at $t = -2$.

A. −21
B. 4
C. 28
D. 31

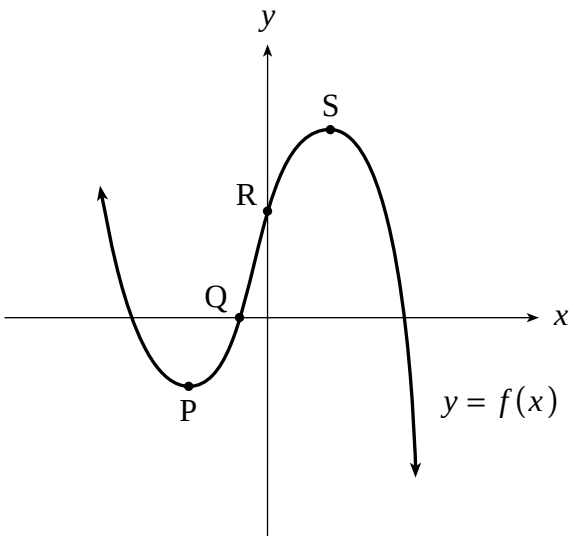
43. Determine the *x*-values of the critical points for the function $f(x) = x^3 + 3x^2 - 24x$.

A. $x = -4$, $x = 2$
B. $x = 4$, $x = -2$
C. $x = 0$, $x = 3.62$, $x = -6.62$
D. $x = 0$, $x = 3.62$, $x = 6.62$

44. Given the function $f(x) = x^3 - 3x + 2$, determine the slope of the secant line intersecting the graph of $f(x)$ at the points where $x = -3$ and $x = 2$.

A. −6
B. −3
C. 4
D. 16.5

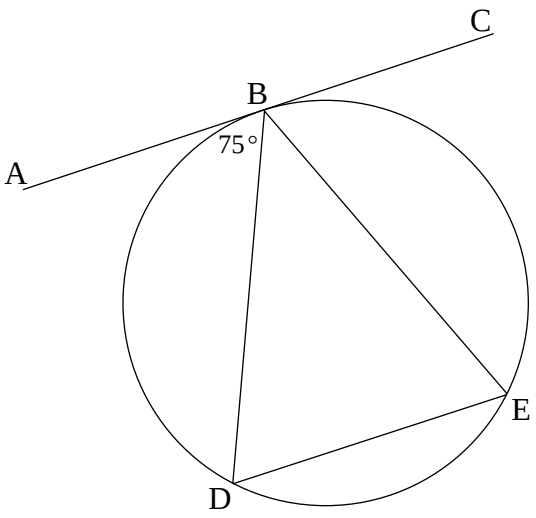
45. Given the graph of the function $y = f(x)$, at which point is $f'(x) = 0$ and $f(x) > 0$?



A. P
B. Q
C. R
D. S

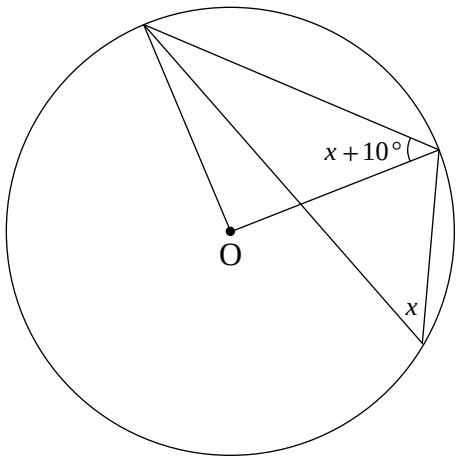
OVER

46. AC is a tangent to the circle at B, $\angle ABD = 75°$, and $AC \,||\, DE$ in the diagram below. Determine the measure of $\angle DBE$. (Diagram is not drawn to scale.)



A. 15°
B. 25°
C. 30°
D. 40°

47. Point O is the centre of the circle in the diagram below. Determine the value of x.



A. 40°
B. $42\frac{1}{2}°$
C. 50°
D. $53\frac{1}{3}°$

48. What is the difference between 10^{1997} and 10^{1995}?

A. 99

B. 100

C. $99\left(10^{1995}\right)$

D. $100\left(10^{1995}\right)$

49. A backyard patio area is made up of square patio tiles, all the same size, laid side by side to form a rectangle 35 tiles by 14 tiles. If a straight line is drawn diagonally from one corner of the patio to the opposite corner, how many tiles will the diagonal intersect?

A. 40
B. 42
C. 45
D. 49

50. If a function is defined by the equation $f(t) = t^2 - 4t$, then which conic is represented by $f(x) + 2f(y) = 0$?

A. circle
B. ellipse
C. parabola
D. hyperbola

This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.

OVER

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PART B: WRITTEN RESPONSE

Value: 20 marks **Suggested Time: 45 minutes**

INSTRUCTIONS:	Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided. **Full marks will NOT be given for the final answer only.**

OVER

1. Solve: $\log(x+4) = 1 - \log 2x$ **(3 marks)**

ANSWER:	Score for Question 1: 1. _____ (3)

OVER

2. Determine an equation of the line tangent to the curve $y = x^3 - 3x^2 - 5x$ at the point where $x = -1$. **(3 marks)**

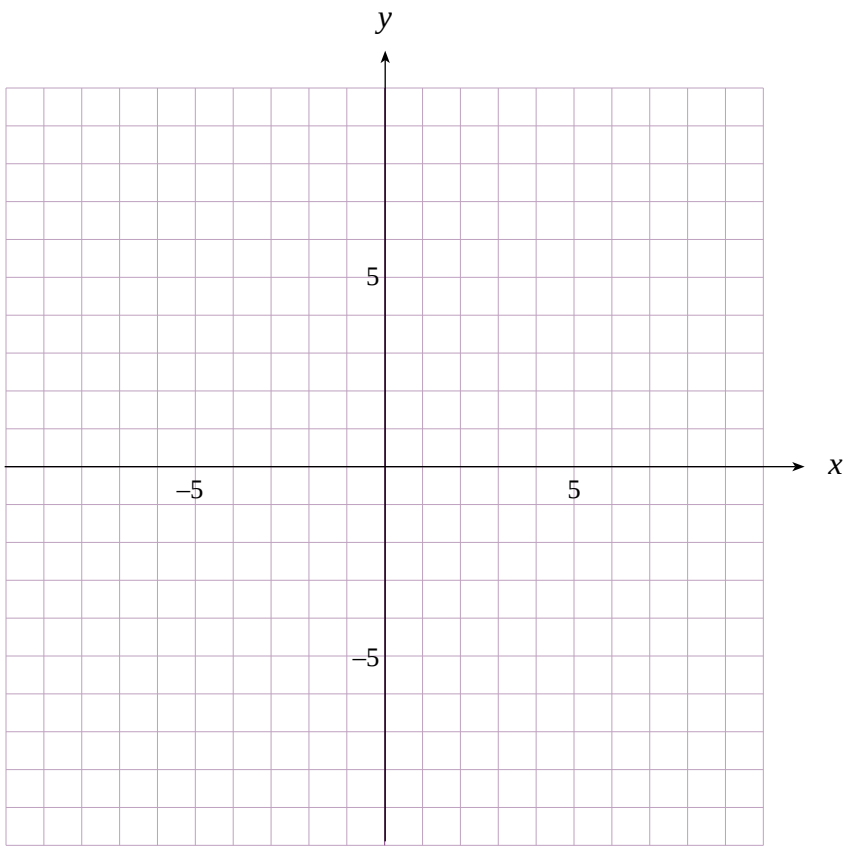
ANSWER:	Score for Question 2: 2. _____ (3)

OVER

3. Graph the solution of the following system of inequalities on the grid provided. **(3 marks)**

$$(x-2)^2+(y+3)^2>25$$

$$x\le -(y+3)^2+4$$



Score for Question 3:
3. _____ (3)

OVER

4. Solve for x: $\sec^2 x - \sec x - 6 = 0$, where $0 \le x < 2\pi$. (Accurate to at least 2 decimal places.)

(3 marks)

ANSWER:	Score for Question 4: 4. _____ (3)

OVER

5. A polynomial function of degree 3 has zeros $5,\ 3,\ -1,$ and passes through the point $(2,\ -6)$. Determine an equation of this function. (Answer may be left in factored form.) **(2 marks)**

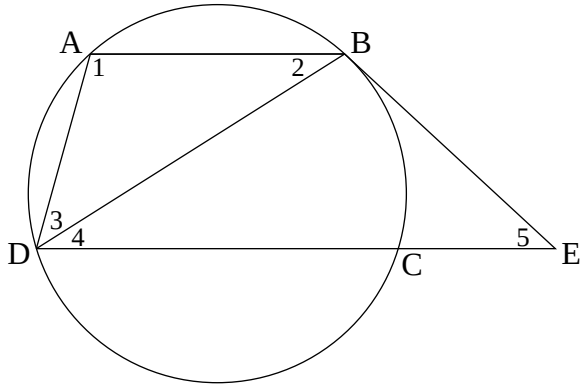
ANSWER:	Score for Question 5: 5. _____ (2)

OVER

6. Complete the proof. **(4 marks)**

Given: BE is a tangent
D, C, E are collinear
$\angle 3 = \angle 5$

Prove: AB $||$ DE



Proof

Statement	Reason

Score for Question 6:
6. _____ (4)

OVER

7. The line segment AB has endpoints $A(13,\ 6)$, and $B(1,\ 2)$. Find the coordinates of point P which divides the line segment AB in a ratio of 3:5, that is $\frac{PA}{PB} = \frac{3}{5}$. **(2 marks)**

ANSWER:	Score for Question 7: 7. _____ (2)

END OF EXAMINATION

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A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\qquad 1 + \tan^2\theta = \sec^2\theta$ $\qquad 1 + \cot^2\theta = \csc^2\theta$

Reciprocal and Quotient Identities

$\sec\theta = \dfrac{1}{\cos\theta}$ $\qquad \csc\theta = \dfrac{1}{\sin\theta}$ $\qquad \cot\theta = \dfrac{1}{\tan\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Addition Identities

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\qquad \tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\qquad \tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

Double-Angle Identities

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\qquad \sin 2\theta = 2\sin\theta\cos\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$ $\qquad \tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Formulae

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad t_n = a + (n-1)d$ $\qquad t_n = ar^{n-1}$

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ $\qquad S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$ $\qquad S_n = \dfrac{a\left(1 - r^n\right)}{1 - r}$

$c^2 = a^2 + b^2 - 2ab\cos C$ $\qquad S_n = \dfrac{n}{2}(a + \ell)$ $\qquad S_n = \dfrac{a - \ell r}{1 - r}$

$S = \dfrac{a}{1 - r}$

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ROUGH WORK FOR GRAPHING

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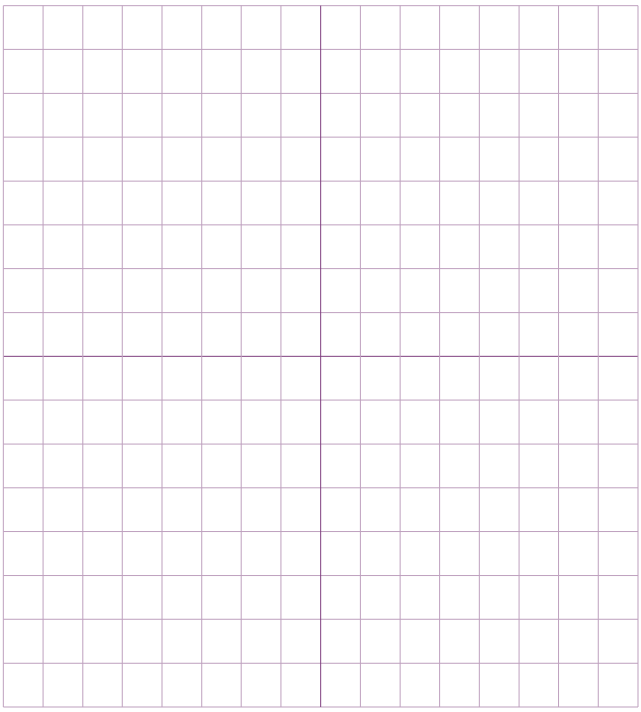

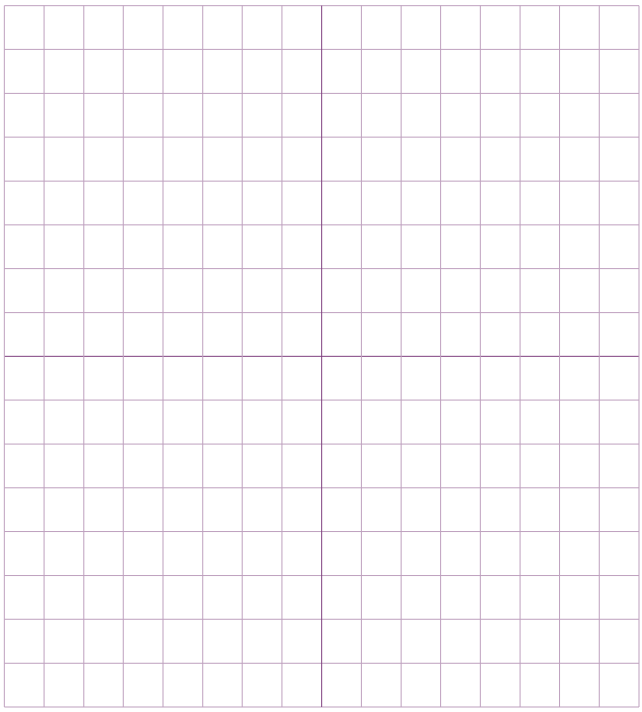

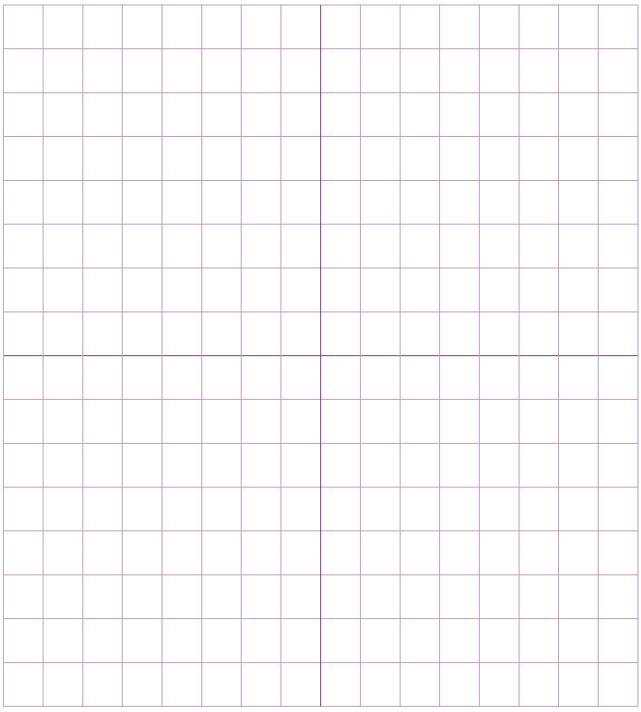

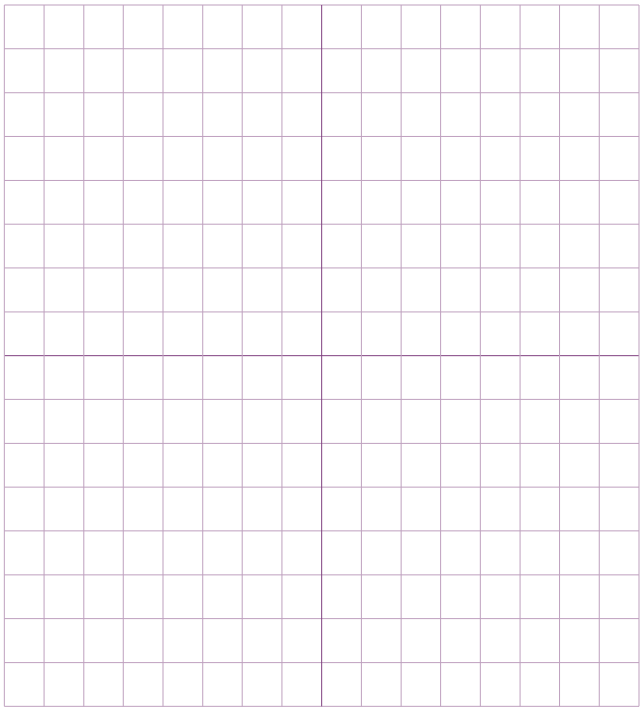

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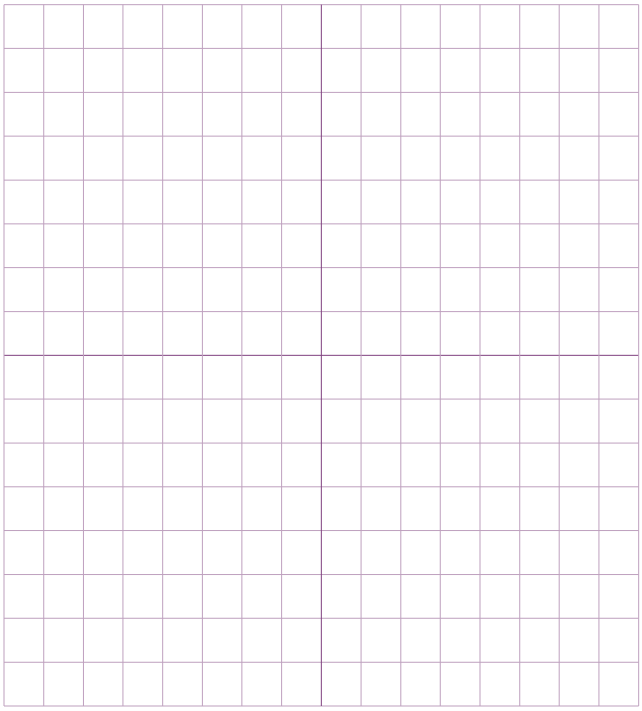

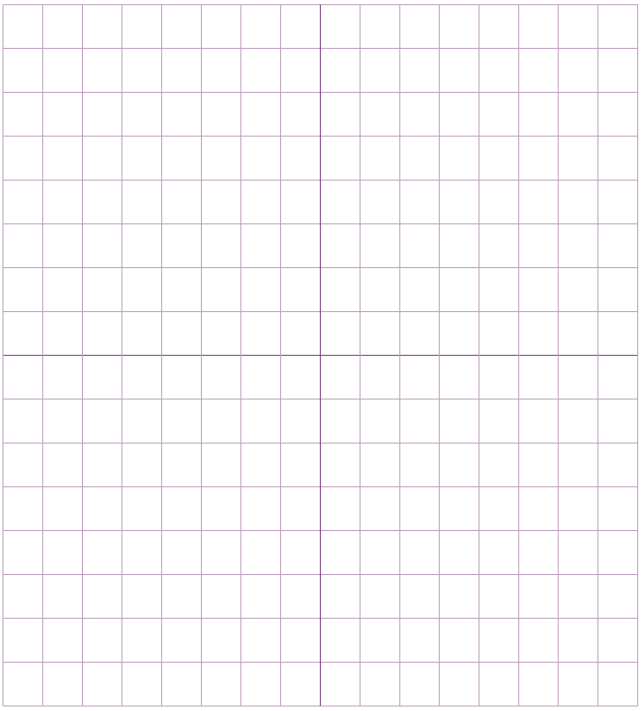

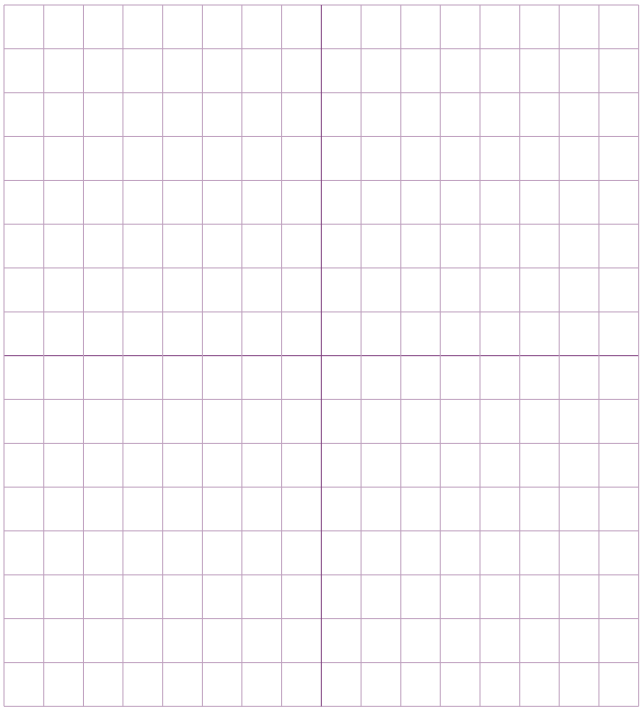

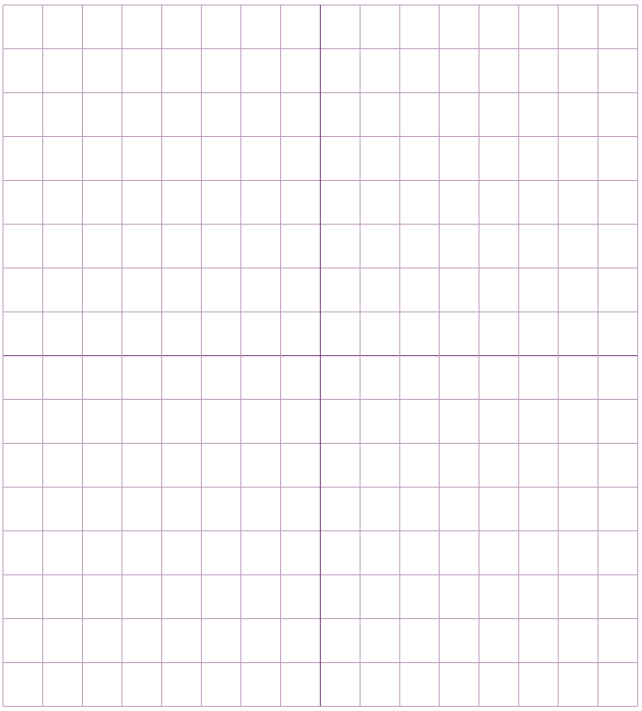

ROUGH WORK FOR MULTIPLE-CHOICE

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ROUGH WORK FOR MULTIPLE-CHOICE