

APRIL 1997

## PROVINCIAL EXAMINATION

MINISTRY OF EDUCATION, SKILLS AND TRAINING

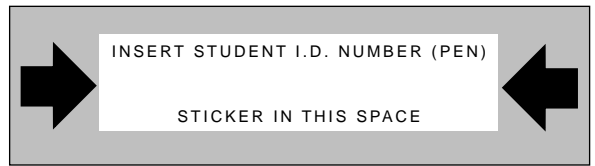
# MATHEMATICS 12

### GENERAL INSTRUCTIONS

1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this paper.**
2. Take the separate Answer Sheet and follow the directions on its front page.
3. Be sure you have an **HB pencil** and an eraser for completing your Answer Sheet. Follow the directions on the Answer Sheet when answering multiple-choice questions.
4. For each of the written-response questions, write your answer in the space provided.
5. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by **ENDOFEXAMINATION**.
6. At the end of the examination, place your Answer Sheet inside the front cover of this booklet and return the booklet and your Answer Sheet to the supervisor.

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**FOR OFFICE USE ONLY**



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**MATHEMATICS 12 APRIL 1997 PROVINCIAL**

**Course Code = MA      Examination Type = P**

1.  $\frac{\quad}{(3)}$

2.  $\frac{\quad}{(3)}$

3.  $\frac{\quad}{(2)}$

4.  $\frac{\quad}{(3)}$

5.  $\frac{\quad}{(1)}$

6.  $\frac{\quad}{(2)}$

7.  $\frac{\quad}{(2)}$

8.  $\frac{\quad}{(4)}$

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## MATHEMATICS 12 PROVINCIAL EXAMINATION

	Value	Suggested Time
1. This examination consists of <b>two</b> parts:		
PART A: 50 multiple-choice questions	50	75
PART B: 7 written-response questions 2 questions worth <b>two</b> marks each, 4 questions worth <b>three</b> marks each, and 1 question worth <b>four</b> marks.	20	45
	<b>Total: 70 marks</b>	<b>120 minutes</b>
2. The last <b>three</b> pages inside the back cover contain <b>A Summary of Basic Identities and Formulae</b> , <b>Rough Work for Graphing</b> , and <b>Rough Work for Multiple-Choice</b> . These pages may be detached for convenient reference prior to writing this examination.		
3. You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.		
4. An approved scientific calculator is essential for the examination. The calculator must be a hand-held device designed <b>only</b> for mathematical computations such as logarithmic and trigonometric functions. It <b>can be</b> programmable, but <b>must not</b> contain any graphing capabilities. You <b>must not</b> bring into the examination room any devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or keyboards.		
5. You are permitted to use rulers, compasses, and protractors.		
6. You have <b>two hours</b> to complete this examination.		

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**PART A: MULTIPLE CHOICE**

**Value: 50 marks**

**Suggested Time: 75 minutes**

**INSTRUCTIONS:** For each question, select the **best** answer and record your choice on the Answer Sheet provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. Determine the distance between  $(6, 4)$  and  $(-2, 5)$ .

- A.  $\sqrt{15}$
- B.  $\sqrt{17}$
- C.  $\sqrt{65}$
- D.  $\sqrt{66}$

2. Determine the length of the minor axis of the ellipse:  $\frac{x^2}{16} + \frac{y^2}{9} = 1$

- A. 3
- B. 4
- C. 6
- D. 8

3. Which absolute value inequality describes the solution shown?



- A.  $|x - 6| < 1$
- B.  $|x - 1| < 6$
- C.  $|x + 1| < 6$
- D.  $|x + 6| < 1$

4. Which equation represents a rectangular hyperbola?

- A.  $3x^2 + 2y^2 = 15$
- B.  $3x^2 + 3y^2 = 15$
- C.  $3x^2 - 2y^2 = 15$
- D.  $3x^2 - 3y^2 = 15$

5. Which ordered pair is a solution of the following system of inequalities?

$$x^2 - y^2 > 4$$

$$x^2 + y^2 \leq 16$$

- A. (1, 1)
- B. (2, 0)
- C. (3, 1)
- D. (5, 1)

6. What is the equation of the axis of symmetry of the parabola  $x = -2(y+1)^2 + 2$ ?

- A.  $y = 1$
- B.  $y = -1$
- C.  $x = 2$
- D.  $x = -2$

7. How many different real solutions does the following system have?

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$

$$y = -x^2 + 2$$

- A. 1
- B. 2
- C. 3
- D. 4

8. Determine the radius of the circle  $x^2 + y^2 - 2x + 6y - 15 = 0$ .

- A. 5
- B. 25
- C.  $\sqrt{15}$
- D.  $\sqrt{26}$



9. A point  $P(x, y)$  moves such that it is always twice as far from  $A(2, 0)$  as it is from  $B(-3, 0)$ . Which equation below represents this locus?

A.  $(x-2)^2 + y^2 = 2[(x+3)^2 + y^2]$

B.  $(x-2)^2 + y^2 = 4[(x+3)^2 + y^2]$

C.  $2[(x-2)^2 + y^2] = (x+3)^2 + y^2$

D.  $4[(x-2)^2 + y^2] = (x+3)^2 + y^2$

10. A hyperbola with vertices  $(8, 1)$  and  $(-4, 1)$  has asymptotes whose slopes are  $\pm \frac{2}{3}$ . Determine an equation for the hyperbola.

A.  $\frac{(x-2)^2}{36} - \frac{(y-1)^2}{16} = 1$

B.  $\frac{(x-2)^2}{36} - \frac{(y-1)^2}{24} = 1$

C.  $\frac{(x-2)^2}{36} - \frac{(y-1)^2}{4} = 1$

D.  $\frac{(x-2)^2}{36} - \frac{(y-1)^2}{8} = 1$

11. Determine the amplitude of  $y = 6\sin 2x + 3$ .

- A. 2  
B. 3  
C. 6  
D. 12

12. Evaluate:  $\cot 1.5$  (Accurate to 2 decimal places.)

- A. 0.07  
B. 0.67  
C. 0.79  
D. 14.10

13. Solve:  $\cos x = 0.286$ ,  $0 \leq x < 2\pi$  (Accurate to 2 decimal places.)

- A. 0.96, 2.18
- B. 0.96, 5.32
- C. 1.28, 1.86
- D. 1.28, 5.00

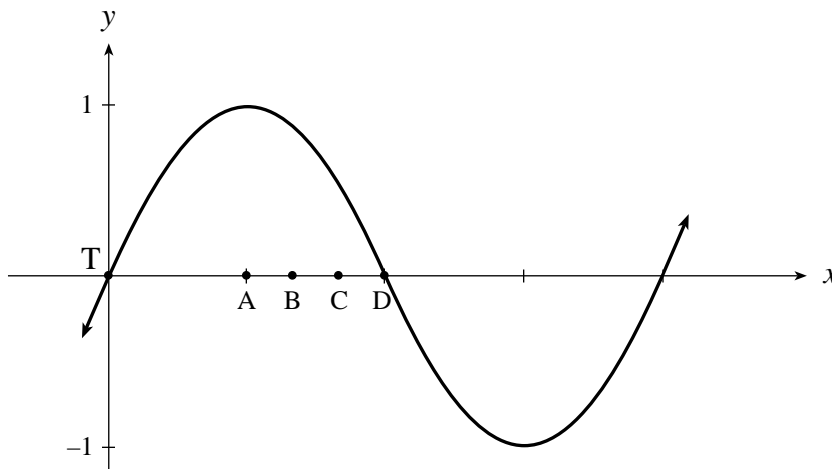
14. Determine the reference angle of 5.2 radians. (Accurate to 1 decimal place.)

- A. 0.5
- B. 0.8
- C. 1.1
- D. 2.1

15. Simplify:  $\cot^2 \theta \sin^2 \theta + \cos^2 \theta$

- A.  $\cos^4 \theta$
- B.  $2\cos^2 \theta$
- C.  $\sin^4 \theta$
- D.  $\cot^2 \theta$

16. The graph of  $y = \sin 5x$  is shown below. To obtain the graph of  $y = \sin 5\left(x - \frac{1}{2}\right)$  requires the graph to be shifted. Which point **best** represents the location of point T after the shift?



- A. A
- B. B
- C. C
- D. D

17. The point  $(\pi, \log m)$  lies on the graph of  $y = \sin\left(x + \frac{\pi}{2}\right)$ . Find  $m$ .
- A.  $-1$
  - B.  $\frac{1}{10}$
  - C.  $1$
  - D. no solution
18. Solve:  $\log_2 x = 1$
- A.  $0$
  - B.  $\frac{1}{2}$
  - C.  $1$
  - D.  $2$
19. Solve:  $\log 4x + \log x = \log 100$
- A.  $\frac{2}{5}$
  - B.  $\frac{5}{2}$
  - C.  $5$
  - D.  $20$
20. Which expression is equivalent to  $\log \frac{a^2}{bc}$  ?
- A.  $2 \log a - \log b + \log c$
  - B.  $2 \log a - \log b - \log c$
  - C.  $\log 2 + \log a - \log b - \log c$
  - D.  $\frac{2 \log a}{\log b + \log c}$

21. If  $\log_a n = 16$ , find  $\log_a \sqrt{n}$ .

- A. 4
- B. 8
- C. 32
- D. 256

22. If  $f(x) = \frac{1}{2}x + 4$ , find  $f^{-1}(x)$ , the inverse of  $f(x)$ .

- A.  $f^{-1}(x) = 2x + 8$
- B.  $f^{-1}(x) = 2x - 8$
- C.  $f^{-1}(x) = 2x + 4$
- D.  $f^{-1}(x) = 2x - 4$

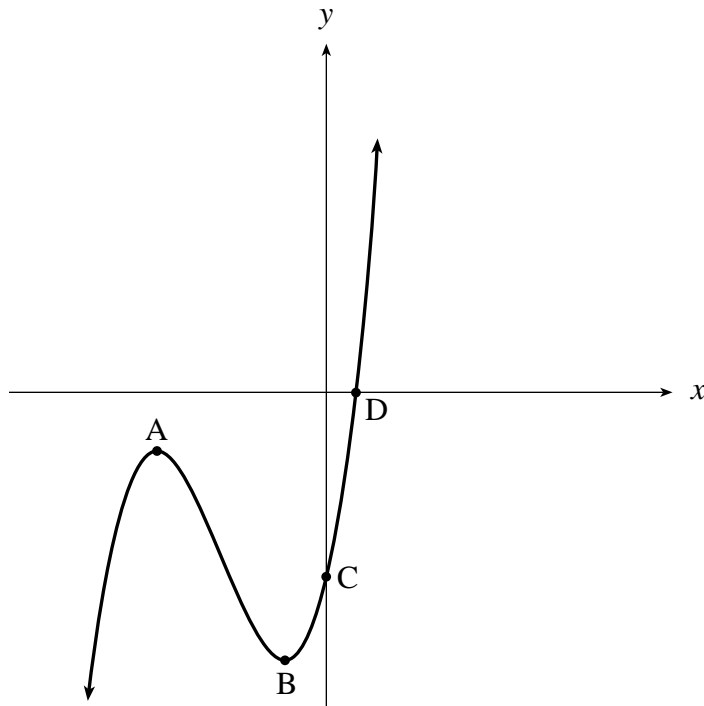
23. Simplify:  $a^{\log_a 12 - \log_a 4}$

- A. 3
- B. 8
- C.  $a^3$
- D.  $a^8$

24. Simplify:  $\left(\frac{a}{b}\right)^{\log \frac{1}{2}} \left(\frac{a}{b}\right)^{\log \frac{1}{5}}$

- A.  $\frac{b}{a}$
- B.  $\frac{a}{b}$
- C.  $\frac{1}{ab}$
- D.  $\left(\frac{a}{b}\right)^7$

25. At which point does a zero of the function occur?



- A. A
- B. B
- C. C
- D. D

26. If  $p(x) = 3x^3 + 6x^2 + 12x - 8$  is divided by  $x + 1$ , find the remainder.

- A. -29
- B. -17
- C. 1
- D. 13

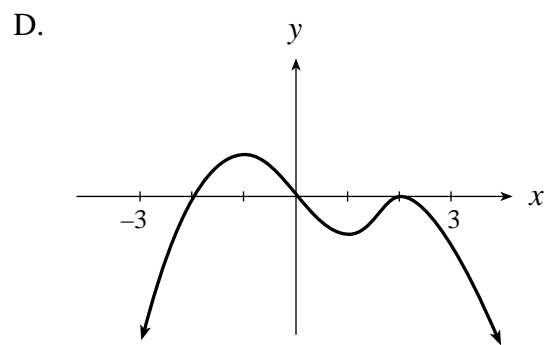
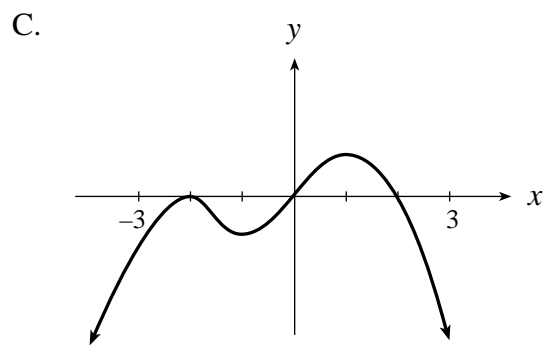
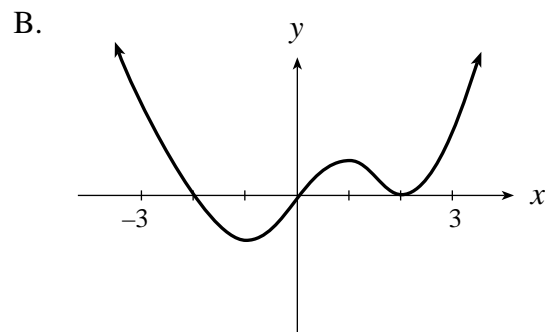
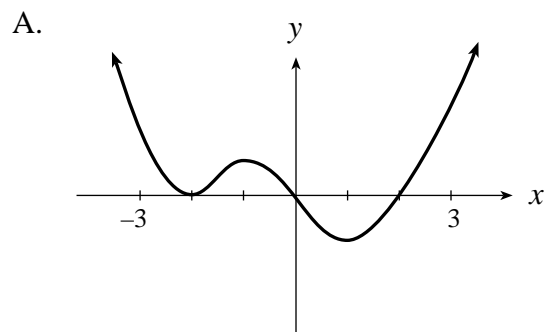
27. Determine the number of real roots for the equation  $6(x^2 - 4)(x^2 + 25)(2x - 3)(x + 3) = 0$ .

- A. 3
- B. 4
- C. 5
- D. 6

28. According to the Rational Root Theorem, determine all possible rational roots of  $4x^3 + 7x^2 - 5x + 2 = 0$ .

- A.  $\pm 1, \pm 2$
- B.  $\pm 1, \pm 2, \pm 4$
- C.  $\pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{4}$
- D.  $\pm 1, \pm 2, \pm 4, \pm \frac{1}{4}, \pm \frac{1}{2}$

29. Which graph **best** represents  $y = ax(x+2)(x-2)^2$ , if  $a < 0$  ?



30. Which of the following is a factor of  $x^3 - 5x^2 + 9x - 5$  ?

- A.  $x + 1$
- B.  $x - 5$
- C.  $x^2 + 4x + 5$
- D.  $x^2 - 4x + 5$

31. Determine the cubic polynomial function which has zeros of  $-1$ ,  $2$  and  $3$ , and goes through the point  $(4, 6)$ .
- A.  $f(x) = (x+1)(x-2)(x-3)$   
B.  $f(x) = \frac{3}{5}(x+1)(x-2)(x-3)$   
C.  $f(x) = (x-1)(x+2)(x+3)$   
D.  $f(x) = \frac{1}{21}(x-1)(x+2)(x+3)$
32. Solve:  $x^2 + 2x < x^3$
- A.  $-1 < x < 2$   
B.  $x < -1$  or  $x > 2$   
C.  $x < -1$  or  $0 < x < 2$   
D.  $-1 < x < 0$  or  $x > 2$
33. Which of the following sequences is **neither** arithmetic nor geometric?
- A.  $1, 4, 9$   
B.  $1, 4, 16$   
C.  $1, -4, -9$   
D.  $1, -4, 16$
34. Find the 120<sup>th</sup> term for the arithmetic sequence  $5, 9, 13, 17, \dots$
- A. 362  
B. 476  
C. 481  
D. 485
35. Determine the sum of the infinite geometric series  $6 - \frac{6}{5} + \frac{6}{25} - \frac{6}{125} + \dots$
- A. 4.80  
B. 5.00  
C. 5.04  
D. no finite sum

36. Which general term describes the sequence  $\frac{5}{2}, \frac{7}{3}, \frac{9}{4}, \frac{11}{5}, \frac{13}{6}, \dots$  ?

A.  $t_n = \frac{n+4}{n+1}$

B.  $t_n = \frac{2n+3}{n+1}$

C.  $t_n = \frac{3n+2}{2n}$

D.  $t_n = \frac{5n}{3n-1}$

37. The sum of the first  $n$  terms of an arithmetic sequence is given by  $S_n = 5n^2 - 3n$ . Find the common difference of this arithmetic sequence.

A. 10

B. 11

C. 12

D. 13

38. Given  $\sum_{k=1}^{11} (kx + k) = 90$ , find  $x$ .

A.  $\frac{4}{11}$

B.  $\frac{1}{2}$

C.  $\frac{13}{2}$

D.  $\frac{79}{11}$

39. Which of the following represents the derivative of  $f(x) = x^3$  ?

A.  $\lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$

B.  $\lim_{h \rightarrow 0} \frac{(x-h)^3 + x^3}{h}$

C.  $\lim_{x \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$

D.  $\lim_{x \rightarrow 0} \frac{(x-h)^3 + x^3}{h}$



40. Given  $f(x) = 3x^2 - 4x + 5$ , find  $f'(x)$ .

- A.  $5x + 1$
- B.  $6x + 1$
- C.  $5x - 4$
- D.  $6x - 4$

41. Evaluate:  $\lim_{n \rightarrow \infty} \frac{6n^2 - 4n}{3n^2 + 5n}$

- A.  $-\frac{4}{5}$
- B.  $0$
- C.  $2$
- D. limit does not exist (no finite limit)

42. Determine the slope of the line tangent to  $y = \frac{6}{x}$  at  $(2, 3)$ .

- A.  $-3$
- B.  $-2$
- C.  $-\frac{3}{2}$
- D.  $-\frac{2}{3}$

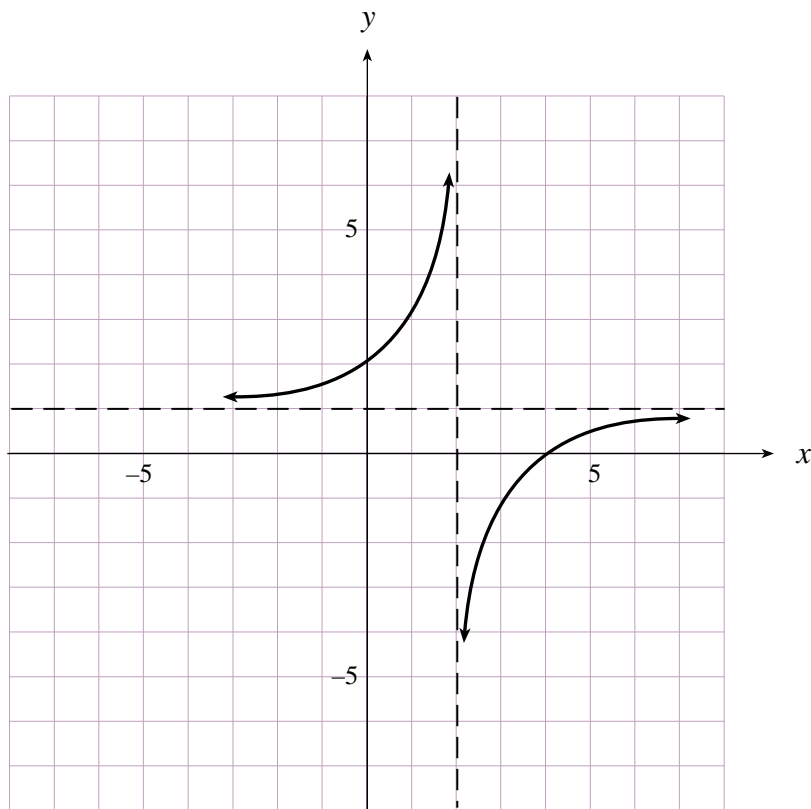
43. Find the maximum value of the function  $y = -13 - 6x - x^2$ .

- A.  $-40$
- B.  $-13$
- C.  $-4$
- D.  $-3$

44. Choose the most appropriate function to solve the following problem:  
“The product of two numbers is 10. What is the smallest possible value for their sum?”

- A.  $S = x + \frac{10}{x}$
- B.  $S = x - \frac{10}{x}$
- C.  $S = x(10 - x)$
- D.  $S = x(10 + x)$

45. The graph of the function  $y = f(x)$  is given below. Determine  $\lim_{x \rightarrow 2} f(x)$ .

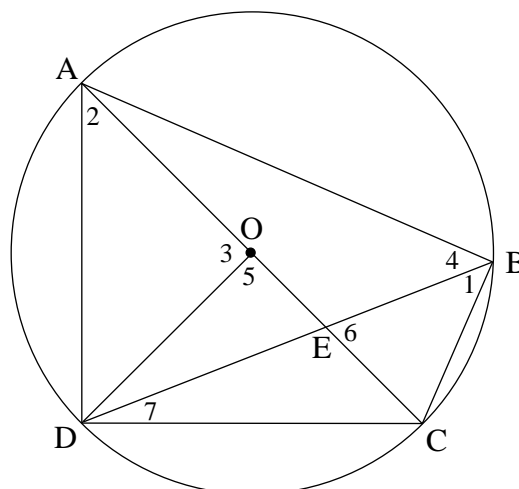


- A. 1
- B. 2
- C. 4
- D. limit does not exist

Use the following diagram and proof to answer questions 46 and 47.

Given: Circle with centre O  
 $DO \perp AC$

Prove: BD bisects  $\angle ABC$



Statement	Proof	Reason
O is the centre of the circle		given
(a) $\angle 1 = \angle 2$		inscribed $\angle$ s on the same chord are =
$DO \perp AC$		given
(b) $\angle 3 = 90^\circ$		definition of perpendicular
(c) $\angle 4 = 45^\circ$		inscribed $\angle = \frac{1}{2}$ central $\angle$
(d) $\angle 5 = 90^\circ$		definition of perpendicular
$\angle 1 = 45^\circ$		inscribed $\angle = \frac{1}{2}$ central $\angle$
$\angle 1 = \angle 4$		both = $45^\circ$
BD bisects $\angle ABC$		definition of $\angle$ bisector

46. Which line is **not** necessary in the given proof?

- A. a
- B. b
- C. c
- D. d

47. Determine the measure of  $\angle 6$  if  $\angle 7 = 25^\circ$ . (Diagram is not drawn to scale.)

- A.  $50^\circ$
- B.  $60^\circ$
- C.  $70^\circ$
- D.  $80^\circ$

OVER

48. If  $\log 2 = a$ ,  $\log 3 = b$ , express  $\log 18$  in terms of  $a$  and  $b$ .
- A.  $ab^2$
  - B.  $2ab$
  - C.  $a + b^2$
  - D.  $a + 2b$
49. Solve:  $5 \sin x - 1 > 3$ ,  $0 \leq x < 2\pi$  (Accurate to 2 decimal places.)
- A.  $0.93 < x < 2.21$
  - B.  $x < 0.93, x > 2.21$
  - C.  $0.41 < x < 2.73$
  - D.  $x < 0.41, x > 2.73$
50. Which statement below **best** describes the graph of  $x^2 - y^2 = 0$ ?
- A. no graph exists
  - B. a single line
  - C. a single point
  - D. two intersecting lines

**This is the end of the multiple-choice section.**  
**Answer the remaining questions directly in this examination booklet.**

**PART B: WRITTEN RESPONSE**

**Value: 20 marks**

**Suggested Time: 45 minutes**

**INSTRUCTIONS:** Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

**Full marks will NOT be given for the final answer only.**

1. Determine all ordered pairs that satisfy the following system.

**(3 marks)**

$$x^2 + y^2 = 5$$

$$y = 2x - 4$$

ANSWER:

Score for  
Question 1:

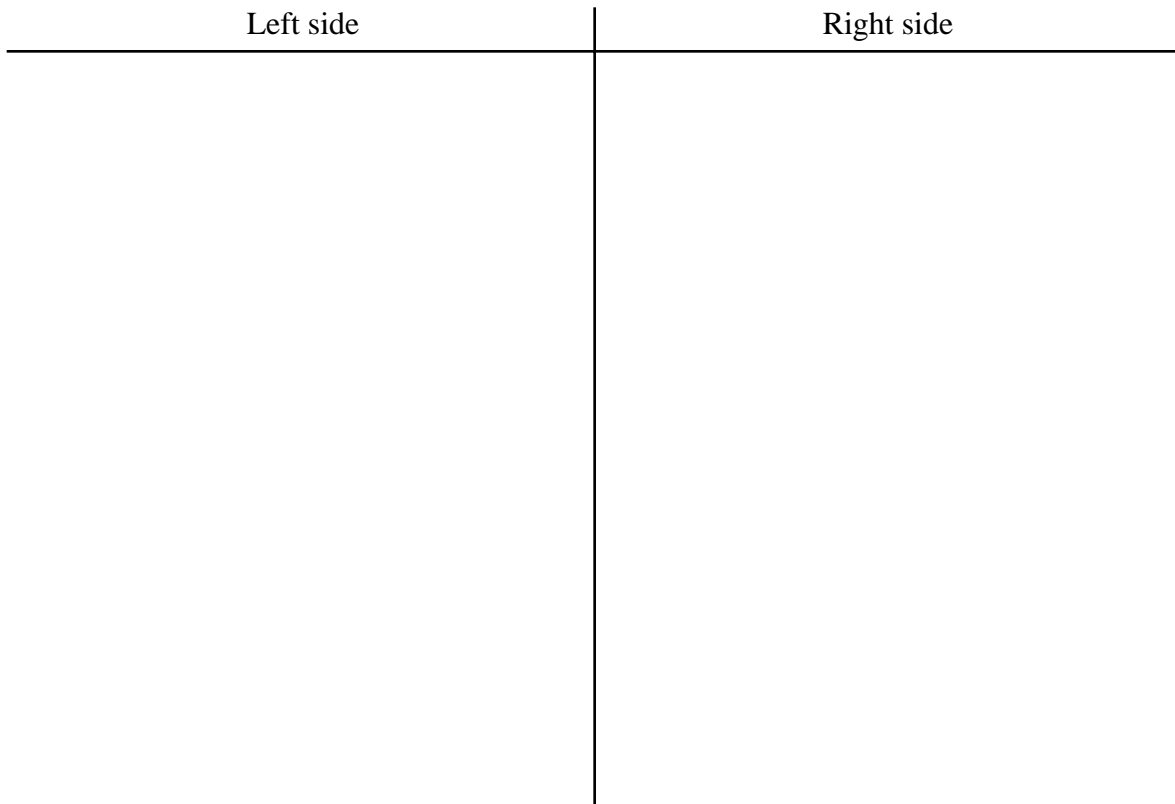
1.           
(3)

**OVER**

2. Prove:

$$\frac{\sin 2\theta}{2 - 2 \cos^2 \theta} = \cot \theta$$

**(3 marks)**





Score for  
Question 2:

2.  $\frac{\quad}{(3)}$

**OVER**

3. In a geometric sequence,  $t_3 = 22\,500$  and  $t_6 = 38\,880$ . Find the first term.

**(2 marks)**

ANSWER:

Score for  
Question 3:

3.           
(2)

**OVER**

4. Determine all values of  $x$  such that the function  $f(x) = x^4 - 8x^2 - 9$  is increasing. **(3 marks)**

ANSWER:

Score for  
Question 4:

4.           
(3)

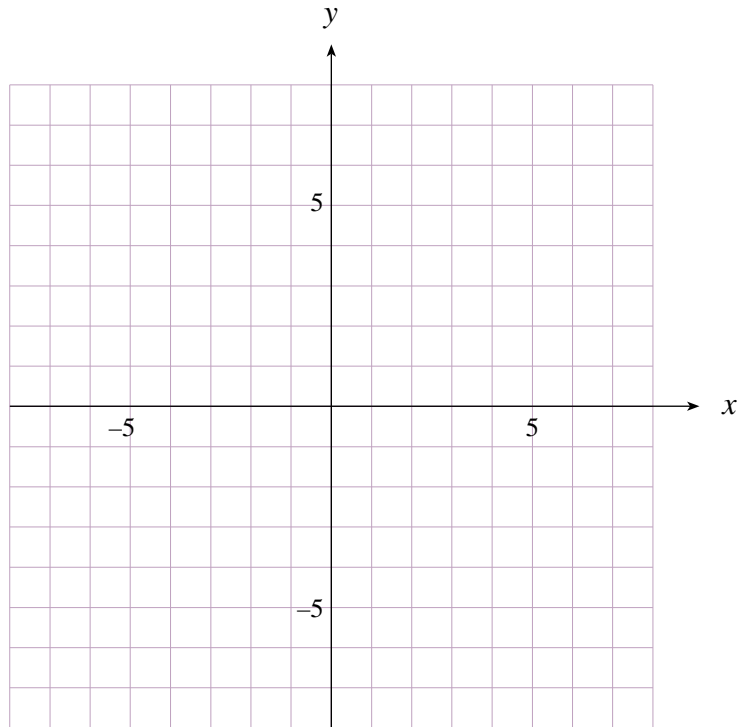
**OVER**

5. a) Determine the y-intercept of  $y = \log_2(x + 3) + 1$ . (Accurate to at least 2 decimal places.)

**(1 mark)**

ANSWER:	Score for Question 5a:  5. $\frac{\quad}{(1)}$
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- b) Graph  $y = \log_2(x + 3) + 1$ . Indicate the asymptote with a dotted or broken line and label it with its equation. **(2 marks)**



Score for  
Question 5b:

6.           
(2)

**OVER**

6. A river system currently has 4 million fish. Each year the population declines by 5%. After how many years will the population fall below 700 000 fish? (Accurate to the nearest year.) **(2 marks)**



ANSWER:

Score for  
Question 6:

7.           
(2)

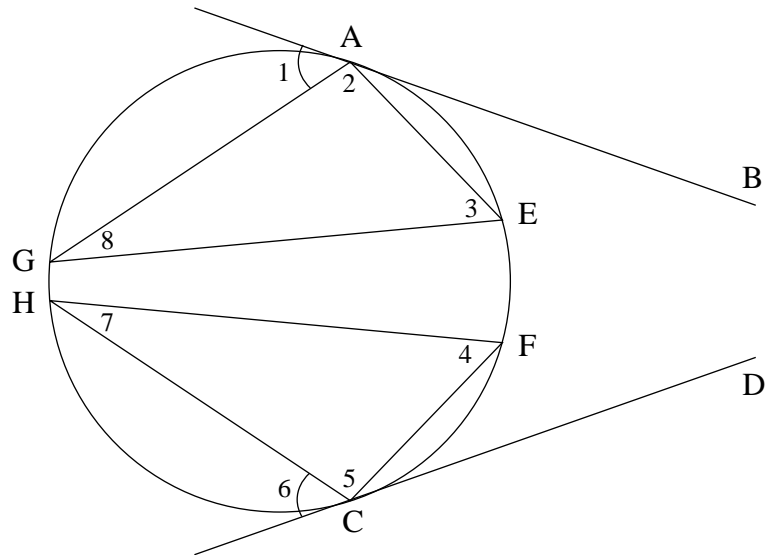
**OVER**

7. Complete the proof.

(4 marks)

Given: AB and CD are tangents  
 $AE = CF$   
 $\angle 1 = \angle 6$

Prove:  $GE = HF$



Proof	
Statement	Reason

Score for  
Question 7:  
8. \_\_\_\_\_  
(4)

**END OF EXAMINATION**

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## A SUMMARY OF BASIC IDENTITIES AND FORMULAE

### Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

### Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

### Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

### Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

### Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a+1)$$

$$S_n = \frac{a-1r}{1-r}$$

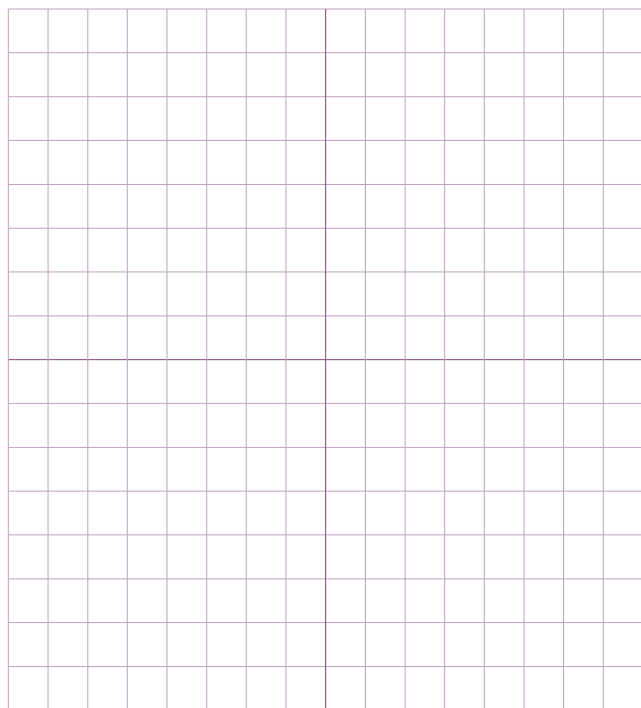
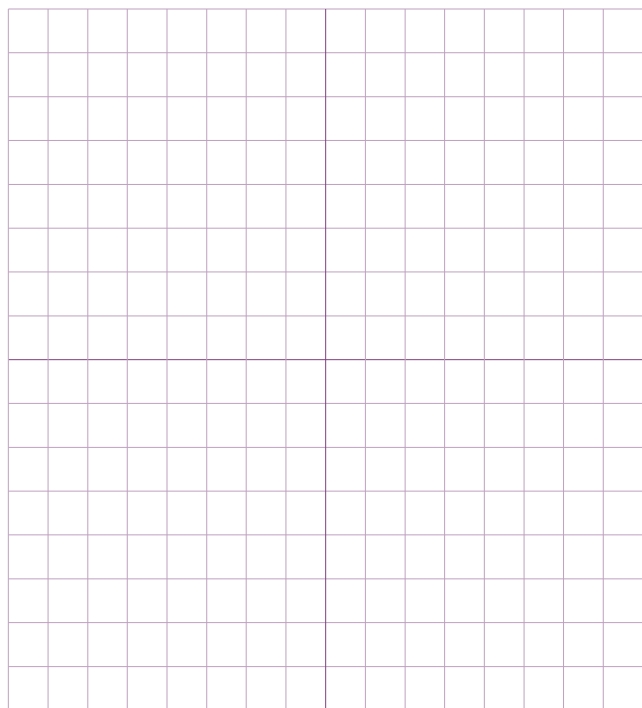
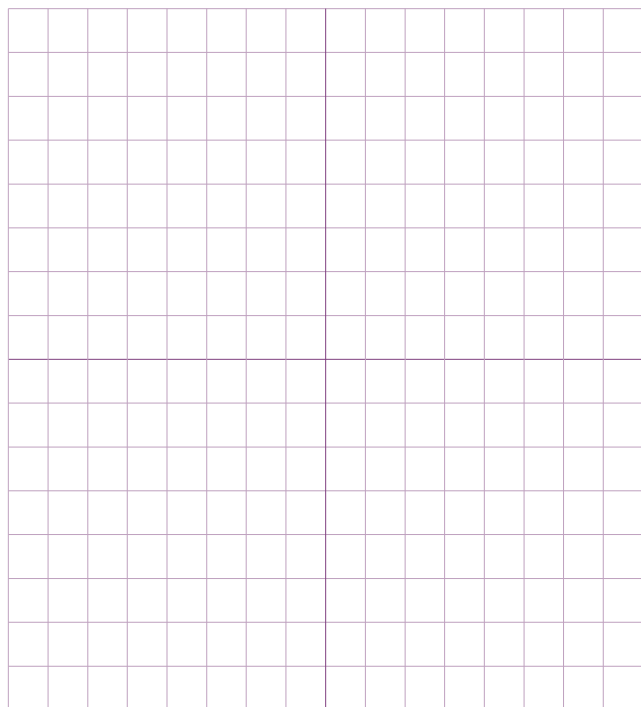
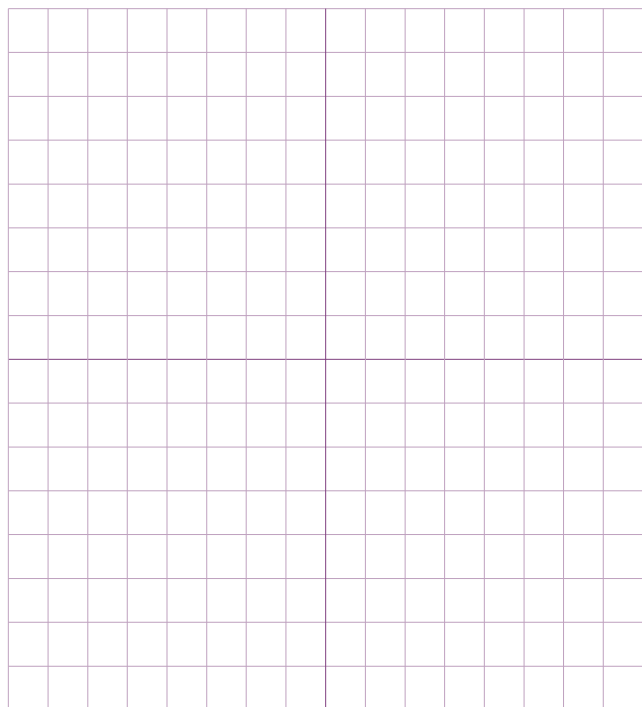
$$S = \frac{a}{1-r}$$

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Exercise care when tearing along perforations.**

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**ROUGH WORK FOR GRAPHING**

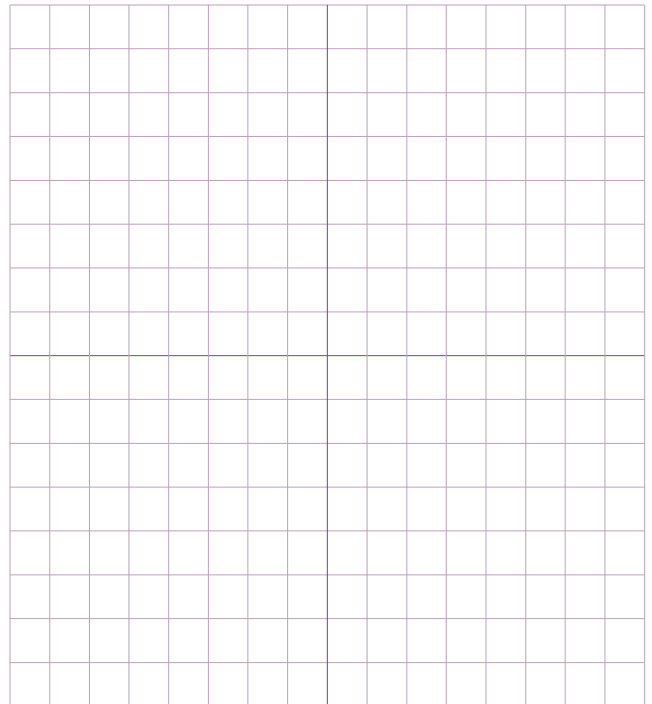
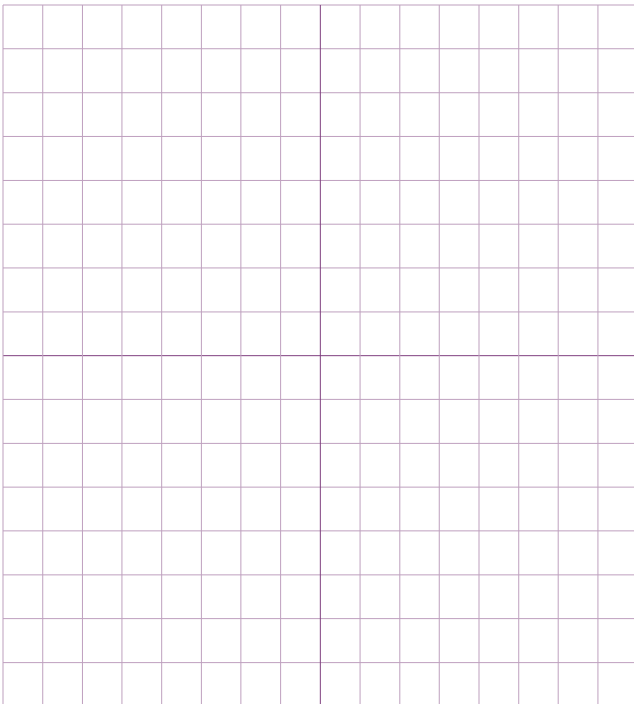
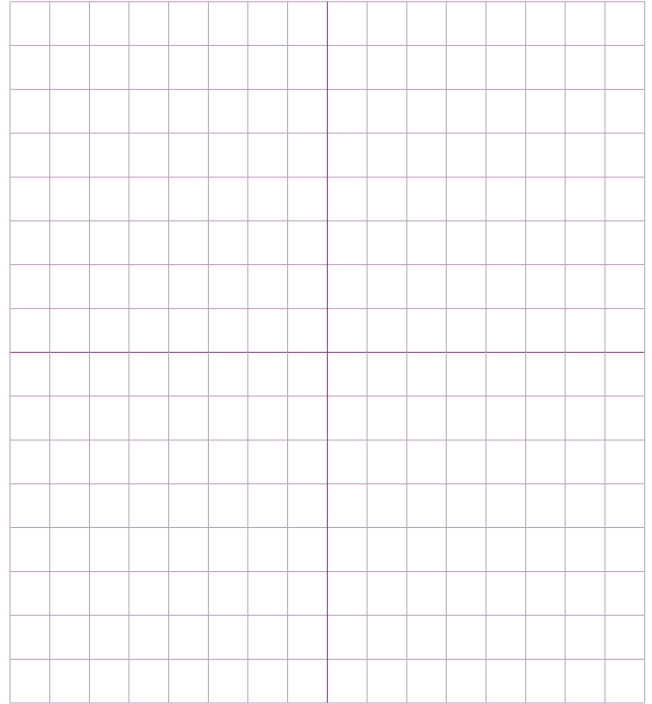
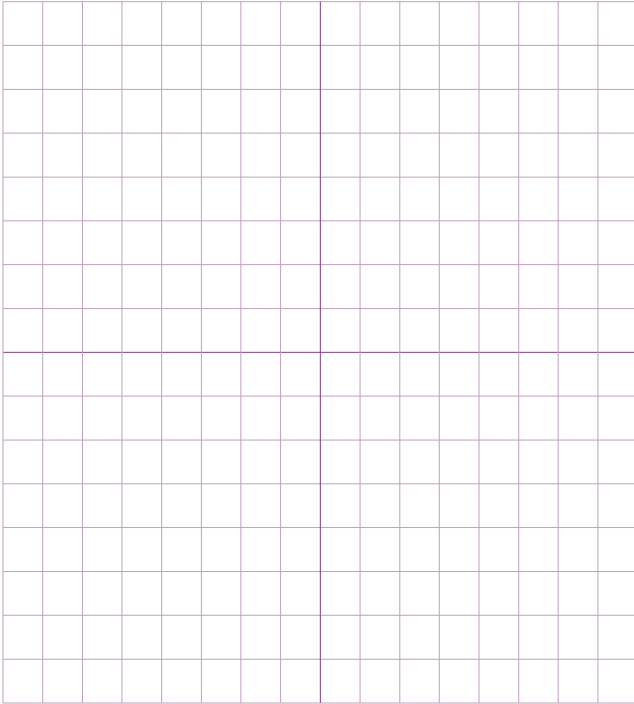
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# ROUGH WORK FOR GRAPHING

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**ROUGH WORK FOR MULTIPLE-CHOICE**

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## ROUGH WORK FOR MULTIPLE-CHOICE