

AUGUST 1996

PROVINCIAL EXAMINATION

MINISTRY OF EDUCATION, SKILLS AND TRAINING

MATHEMATICS 12

GENERAL INSTRUCTIONS

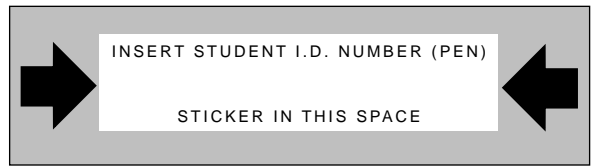
1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above. **Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this paper.**
2. Take the separate Answer Sheet and follow the directions on its front page.
3. Be sure you have an **HB pencil** and an eraser for completing your Answer Sheet. Follow the directions on the Answer Sheet when answering multiple-choice questions.
4. For each of the written-response questions, write your answer in the space provided.
5. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

END OF EXAMINATION .

6. At the end of the examination, place your Answer Sheet inside the front cover of this booklet and return the booklet and your Answer Sheet to the supervisor.

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FOR OFFICE USE ONLY



MATHEMATICS 12 AUGUST 1996 PROVINCIAL

Course Code = MA

Examination Type = P

1. $\frac{\quad}{(3)}$

2. $\frac{\quad}{(3)}$

3. $\frac{\quad}{(2)}$

4. $\frac{\quad}{(3)}$

5. $\frac{\quad}{(1)}$

6. $\frac{\quad}{(2)}$

7. $\frac{\quad}{(2)}$

8. $\frac{\quad}{(4)}$

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MATHEMATICS 12 PROVINCIAL EXAMINATION

	Value	Suggested Time
1. This examination consists of two parts:		
PART A: 50 multiple-choice questions	50	75
PART B: 7 written-response questions	20	45
2 questions worth two marks each, 4 questions worth three marks each, and 1 question worth four marks.		
	Total: 70 marks	120 minutes
2. The last three pages inside the back cover contain A Summary of Basic Identities and Formulae , Rough Work for Graphing , and Rough Work for Multiple-Choice . These pages may be detached for convenient reference prior to writing this examination.		
3. You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.		
4. An approved scientific calculator is essential for the examination. The calculator must be a hand-held device designed only for mathematical computations such as logarithmic and trigonometric functions. It can be programmable, but must not contain any graphing capabilities. You must not bring into the examination room any devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or keyboards.		
5. You are permitted to use rulers, compasses, and protractors.		
6. You have two hours to complete this examination.		

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PART A: MULTIPLE CHOICE

Value: 50 marks

Suggested Time: 75 minutes

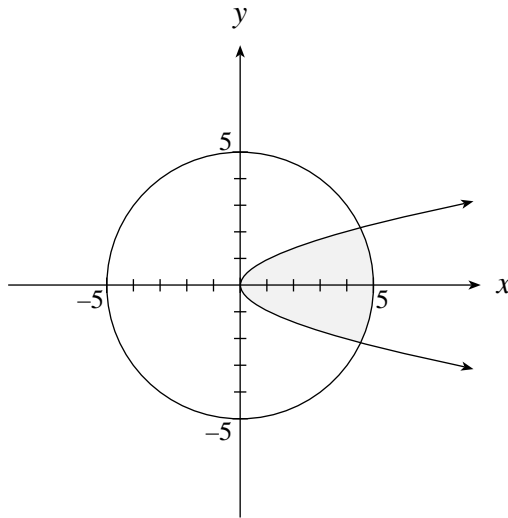
INSTRUCTIONS: For each question, select the **best** answer and record your choice on the Answer Sheet provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. What is the maximum number of solutions of a system involving a parabola and hyperbola?
 - A. 2
 - B. 3
 - C. 4
 - D. infinitely many

2. Identify the conic: $3x^2 + 4y^2 = 12$
 - A. circle
 - B. ellipse
 - C. parabola
 - D. hyperbola

3. Determine the centre of the circle that has $(2, 3)$ and $(-8, 7)$ as endpoints of a diameter.
 - A. $(-3, 5)$
 - B. $(3, -5)$
 - C. $(-5, 2)$
 - D. $(5, -2)$

4. Which system describes the shaded region in the diagram below?



A. $x \leq y^2$
 $x^2 + y^2 \leq 25$

B. $x \leq y^2$
 $x^2 + y^2 \geq 25$

C. $x \geq y^2$
 $x^2 + y^2 \leq 25$

D. $x \geq y^2$
 $x^2 + y^2 \geq 25$

5. Determine the slopes of the asymptotes of the hyperbola $\frac{x^2}{4} - \frac{(y-5)^2}{9} = 1$.

A. $\pm \frac{2}{3}$

B. $\pm \frac{3}{2}$

C. $\pm \frac{4}{9}$

D. $\pm \frac{9}{4}$

6. Solve the following system.

$$x^2 + y^2 = 12$$

$$y = x^2$$

- A. $(\sqrt{3}, 3), (-\sqrt{3}, 3)$
- B. $(2, 4), (-2, 4)$
- C. $(3, 3), (-3, 3)$
- D. $(3, 9), (-3, 9)$

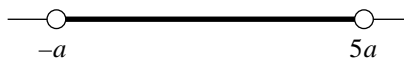
7. What is the sum of the lengths of the major and minor axes of the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$?

- A. 7
- B. 11
- C. 14
- D. 25

8. A point P (x, y) moves such that it is equidistant from the points A $(2, 5)$ and B $(3, -1)$. Determine an equation of the locus of P.

- A. $\sqrt{(x-2)^2 + (y-5)^2} = \sqrt{(x-3)^2 + (y+1)^2}$
- B. $\sqrt{(x-2)^2 - (y-5)^2} = \sqrt{(x-3)^2 - (y+1)^2}$
- C. $\sqrt{(x+2)^2 + (y+5)^2} = \sqrt{(x+3)^2 + (y-1)^2}$
- D. $\sqrt{(x+2)^2 - (y+5)^2} = \sqrt{(x+3)^2 - (y-1)^2}$

9. Given the constant $a > 0$, which absolute value statement has a solution whose graph is shown below?



- A. $|x - 2a| < 3a$
- B. $|x - 3a| < 2a$
- C. $|x + 2a| < 3a$
- D. $|x + 3a| < 2a$

OVER

10. Determine all values of the constant k such that the following hyperbola will have a horizontal transverse axis.

$$2x^2 - 3y^2 + 18y - k = 0$$

- A. $k < -27$
 - B. $k > -27$
 - C. $k < 27$
 - D. $k > 27$
11. Convert 120° to radians.
- A. $\frac{\pi}{3}$
 - B. $\frac{2\pi}{3}$
 - C. 2
 - D. 3
12. Evaluate: $\sec 2.3$ (Accurate to 4 decimal places.)
- A. -1.5009
 - B. -0.6663
 - C. 0.9070
 - D. 1.1210
13. Solve: $2 \cos \theta + \sqrt{3} = 0$, where $0^\circ \leq \theta < 360^\circ$
- A. $30^\circ, 330^\circ$
 - B. $150^\circ, 330^\circ$
 - C. $150^\circ, 210^\circ$
 - D. $210^\circ, 330^\circ$

14. Determine the range of $y = \tan x$.
- A. $-1 \leq y \leq 1$
 - B. $-\frac{\pi}{2} < x < \frac{\pi}{2}$
 - C. all real numbers
 - D. all real numbers x , such that $x \neq \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \dots$
15. The graph of $y = \sin bx$ has a period of $\frac{3\pi}{4}$. Determine a value of the constant b .
- A. $\frac{3}{8}$
 - B. $\frac{3}{4}$
 - C. $\frac{4}{3}$
 - D. $\frac{8}{3}$
16. Simplify: $\frac{\sec \theta - \cos \theta}{\csc \theta - \sin \theta}$
- A. $\tan^3 \theta$
 - B. $\tan^2 \theta$
 - C. $\cot \theta$
 - D. $\tan \theta - \cot \theta$
17. Write $2 \cos^2 4x - 2 \sin^2 4x$ in terms of a single trigonometric function.
- A. $\cos 2x$
 - B. $2 \cos 2x$
 - C. $\cos 8x$
 - D. $2 \cos 8x$

18. Write $a^b = c$ in logarithmic form.

- A. $\log_a b = c$
- B. $\log_a c = b$
- C. $\log_c a = b$
- D. $b \log a = c$

19. Solve: $\log_{2x} 16 = 2$

- A. 2
- B. $2\sqrt{2}$
- C. 4
- D. 8

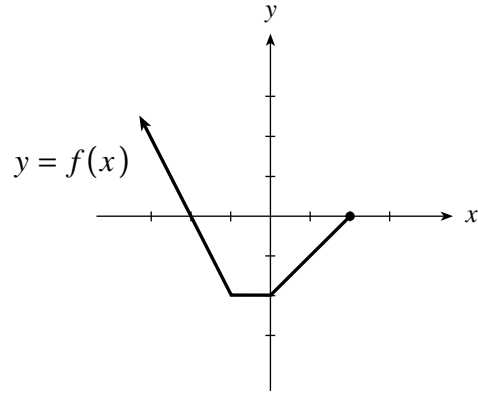
20. If $\log x = a$, write $\log\left(\frac{1}{x^2}\right)$ in terms of a .

- A. $-a^2$
- B. $-2a$
- C. $1 - a^2$
- D. $1 - 2a$

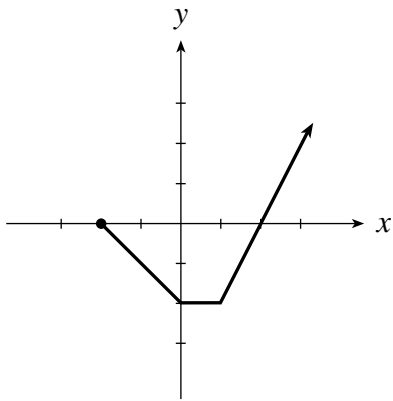
21. Solve: $\log_3 x + \log_3(x + 2) = 1$

- A. -3, 1
- B. 0.5
- C. 0.12
- D. 1

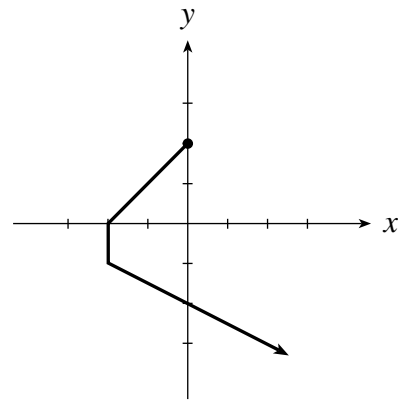
22. Given the graph of $y = f(x)$, which graph below represents its inverse, $f^{-1}(x)$?



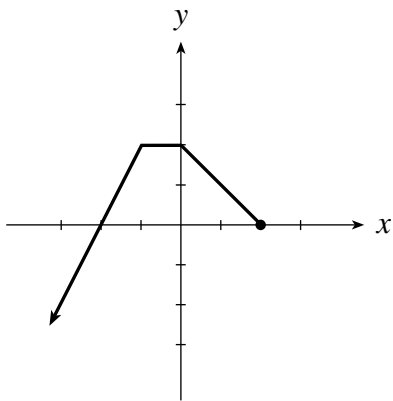
A.



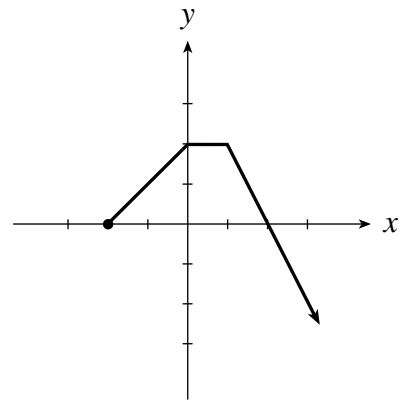
B.



C.



D.

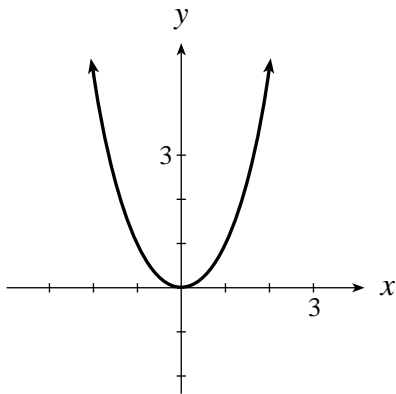


23. Simplify: $\log_a x^{\log_x b}$

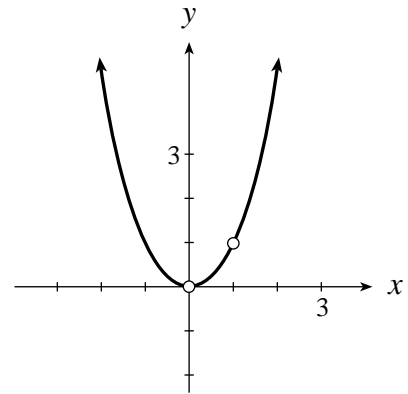
- A. $\frac{b}{a}$
- B. a^b
- C. $\log_a b$
- D. $b \log a$

24. Graph: $\log_x y = 2$

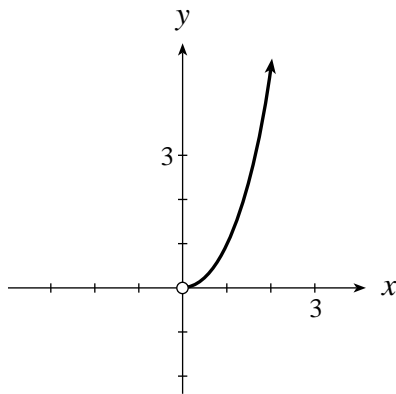
A.



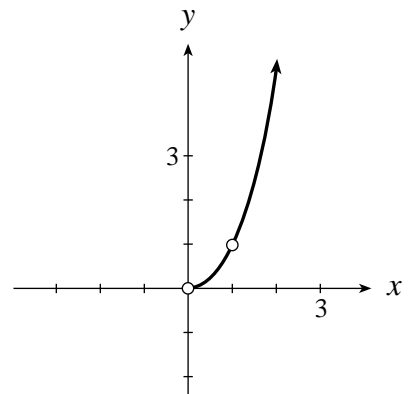
B.



C.



D.

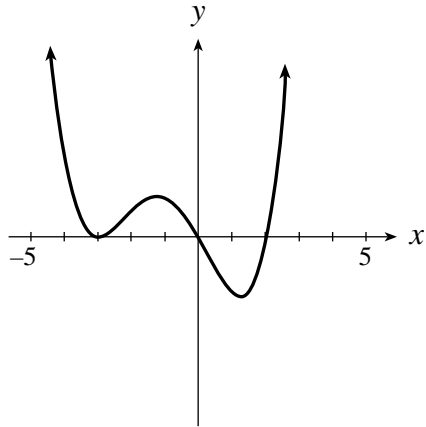


25. Using the Rational Root Theorem, determine all possible rational roots of $3x^3 + 2x^2 - x + 6 = 0$.

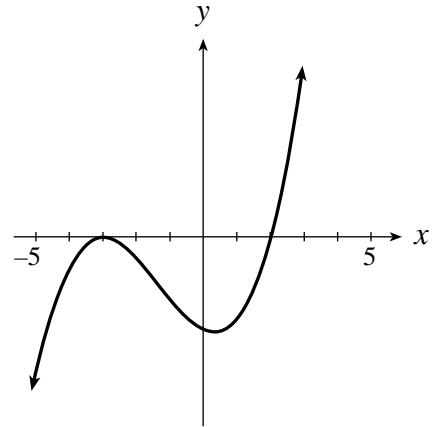
- A. $\pm 1, \pm 3$
- B. $\pm 1, \pm 2, \pm 3, \pm 6$
- C. $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{3}, \pm \frac{2}{3}$
- D. $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm \frac{3}{2}$

26. Which sketch **best** represents the graph of $y = x(x-2)(x+3)^2$?

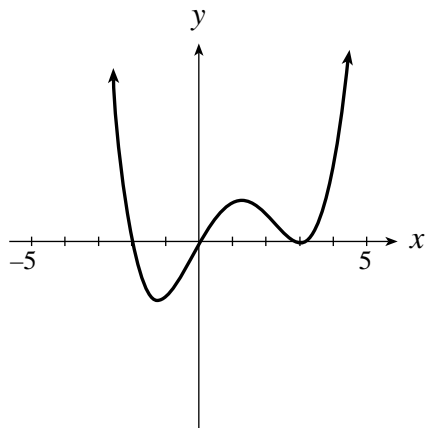
A.



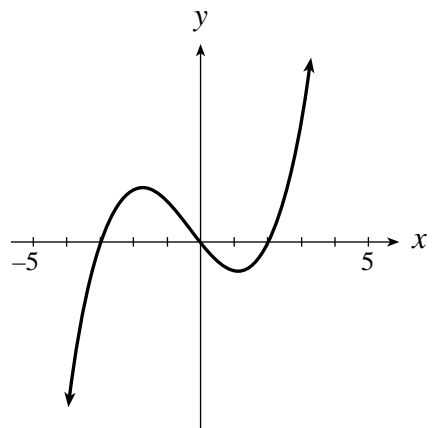
B.



C.



D.



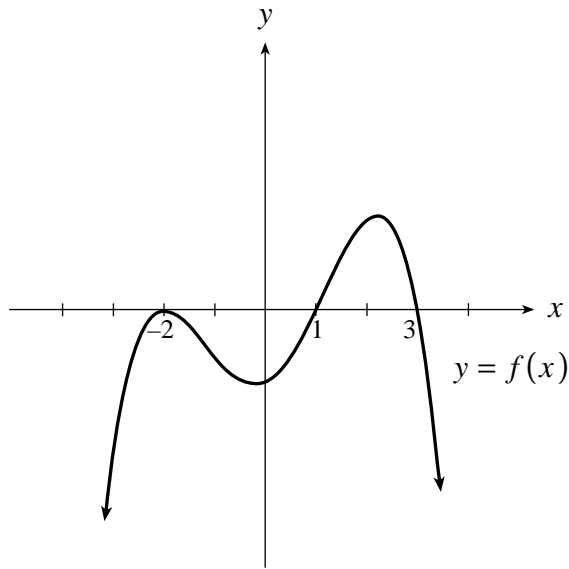
27. If $x+2$ is a factor of $x^3 + 2x^2 + kx + 5$, find k .

- A. -10.5
- B. -2.5
- C. 2.5
- D. 3.5

28. Determine all real roots of: $7x(x^2 - 4)(x^2 + 9) = 0$

- A. $0, \pm 2$
- B. $7, \pm 2$
- C. $0, \pm 2, \pm 3$
- D. $7, \pm 2, \pm 3$

29. Given the graph of $y = f(x)$, determine where $f(x) \leq 0$.



- A. $x \leq -2$ or $x \geq 3$
- B. $x \leq 1$ or $x \geq 3$
- C. $-2 \leq x \leq 1$
- D. $1 \leq x \leq 3$

30. When $x^4 + kx^2 - 5$ is divided by $x^2 + 1$, the remainder is -6 . Find the value of k .

- A. -2
- B. 0
- C. 1
- D. 2

31. Determine the next term in the geometric sequence $4, 20, \dots$

- A. 24
- B. 36
- C. 80
- D. 100

32. Find the 27th term of an arithmetic sequence whose first term is 3 and whose common difference is -5 .
- A. -132
 - B. -127
 - C. 133
 - D. 138
33. What is the single positive geometric mean between 2 and 18?
- A. 6
 - B. 8
 - C. 10
 - D. 12
34. Determine the sum of the first 300 terms in the arithmetic series $2 + 4 + 6 + 8 + \dots$
- A. 90 000
 - B. 90 300
 - C. 90 600
 - D. 90 900
35. A ball is dropped from a height of 2 m. On each bounce, the ball rises to 60% of the height from which it fell. Calculate the total vertical distance the ball travels before coming to rest.
- A. 5 m
 - B. 7 m
 - C. 8 m
 - D. 10 m
36. Determine all values of a ($a \neq 0$) such that $a + 2a^2 + 4a^3 + 8a^4 + \dots$ is an infinite geometric series with a finite sum.
- A. $-3 < a < -1$
 - B. $-2 < a < 2$
 - C. $-1 < a < 1$
 - D. $-\frac{1}{2} < a < \frac{1}{2}$

37. Evaluate: $\sum_{k=2}^4 \log_2 k$ (Accurate to 2 decimal places.)

- A. 1.08
- B. 1.38
- C. 3.17
- D. 4.58

38. A sequence is defined recursively as follows:

$$t_1 = 0$$
$$t_n = t_{n-1} + 2n - 3, \quad n > 1$$

Which formula describes the same sequence?

- A. $t_n = (n-1)^2$
- B. $t_n = n-1$
- C. $t_n = n^2$
- D. $t_n = n$

39. If $y = 3x^3 - 4x^2 + 5$, find $\frac{dy}{dx}$.

- A. $3x^2 - 4x$
- B. $3x^2 - 4x + 5$
- C. $9x^2 - 8x$
- D. $9x^2 - 8x + 5$

40. Which of the following is the derivative of $f(x)$?

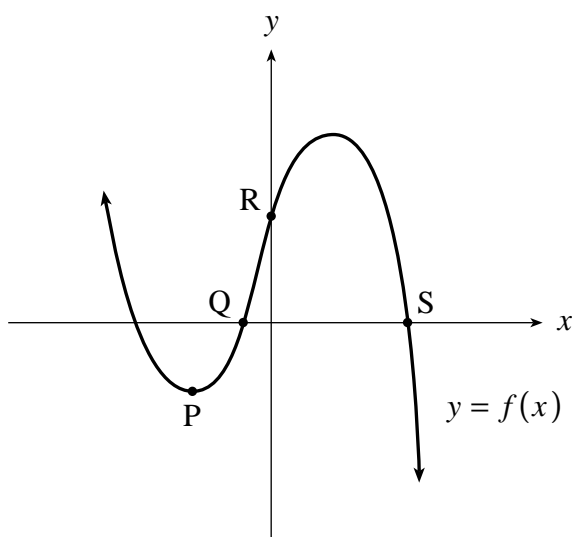
- A. $\lim_{x \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
- B. $\lim_{x \rightarrow h} \frac{f(x-h) + f(x)}{h}$
- C. $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
- D. $\lim_{h \rightarrow 0} \frac{f(x-h) + f(x)}{h}$

41. Find the point on $y = 2x^2 + 6x - 1$ where the slope of the tangent line is 2.
- A. $(-1, -5)$
 - B. $(-1, 2)$
 - C. $(1, 7)$
 - D. $(2, 19)$
42. If $f(x) = \sqrt{x}$, determine the value of $f'(x)$ at $(16, 4)$.
- A. $-\frac{1}{4}$
 - B. $-\frac{1}{8}$
 - C. $\frac{1}{8}$
 - D. $\frac{1}{4}$
43. A particle moves along the x -axis according to the position function $x(t) = 2t^3 - 6t^2 + 9$, where x is in metres and t is in seconds. Find the value(s) of t when the particle is stationary.
- A. $t = 0$
 - B. $t = 2$
 - C. $t = 0, t = -2$
 - D. $t = 0, t = 2$

44. Evaluate: $\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x^2 - 9}$

- A. -1
- B. $\frac{5}{6}$
- C. 1
- D. no finite limit

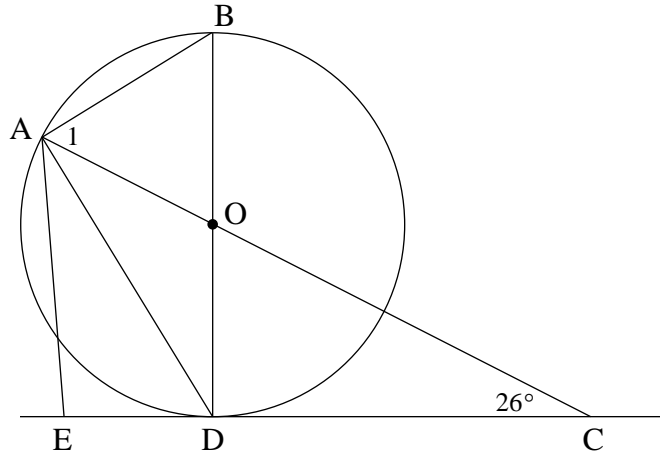
45. Given the graph of the function $y = f(x)$, at which point is $f'(x) > 0$ and $f(x) = 0$?



- A. P
- B. Q
- C. R
- D. S

Use the following diagram to answer questions 46 and 47.

Given: Circle with centre O
CE is tangent at D



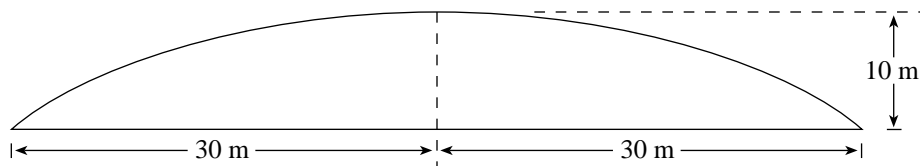
46. Determine the measure of $\angle 1$. (Diagram is not drawn to scale.)

- A. 53°
- B. 58°
- C. 60°
- D. 64°

47. Select the reason for the statement $\angle ADE = \angle ABD$.

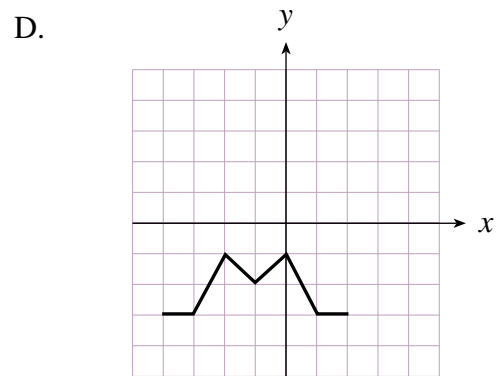
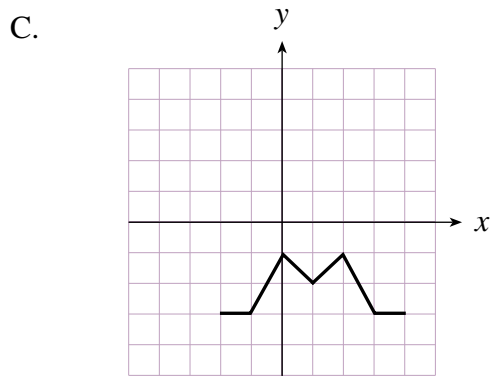
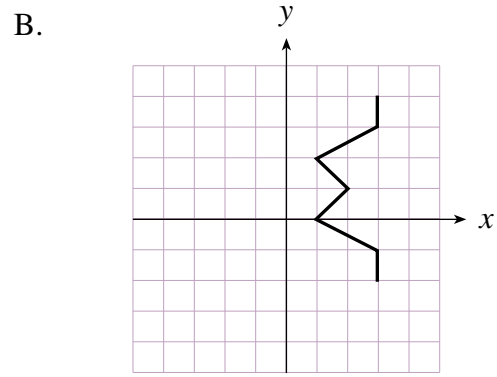
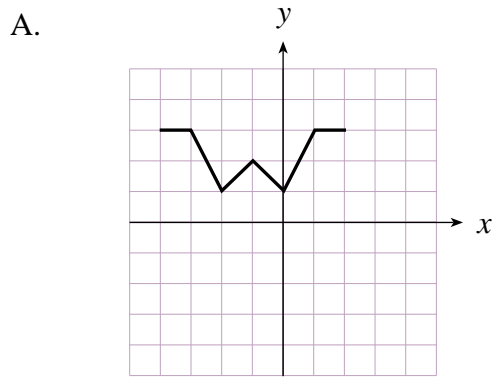
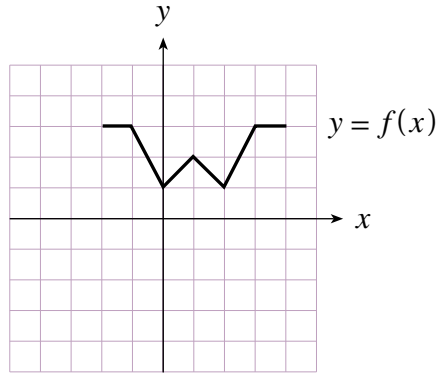
- A. corresponding \angle s are =
- B. inscribed \angle s on same chord are equal
- C. opposite \angle s in a cyclic quadrilateral are supplementary
- D. \angle between tangent and chord is = to the inscribed \angle on opposite side of chord

48. A circular arch of a foot bridge spans a horizontal distance of 60 m and rises 10 m in the centre. Determine the radius of the circle.



- A. 30 m
- B. 40 m
- C. 50 m
- D. 60 m

49. Given the graph of the function $f(x)$ as shown, which graph represents $g(x) = -f(-x)$?



50. A fixed point of a function is a real number x for which $f(x) = x$. How many fixed points does $f(x) = \log(x + 3)$ have?

- A. 0
- B. 1
- C. 2
- D. 3

**This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.**

PART B: WRITTEN RESPONSE

Value: 20 marks

Suggested Time: 45 minutes

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

Full marks will NOT be given for the final answer only.

1. Solve: $3\sin^2\theta - 7\sin\theta + 2 = 0$, $0 \leq \theta < 2\pi$ (Accurate to at least 2 decimal places.) **(3 marks)**

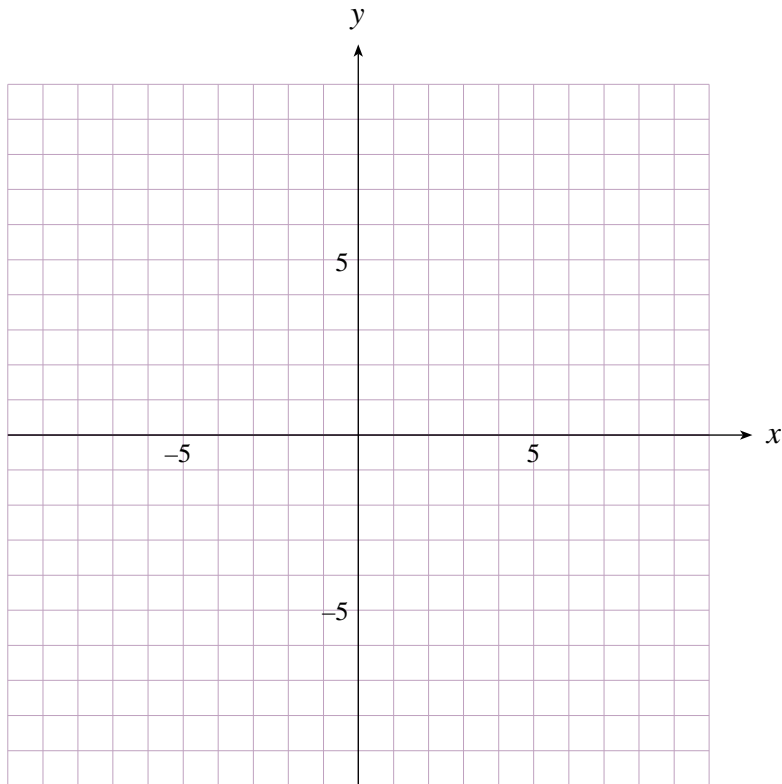
ANSWER:

Score for
Question 1:

1.
(3)

OVER

2. Graph $y = 3^{x+2} - 1$. Indicate the asymptote with a dotted or broken line and clearly show at least three points on the graph. **(3 marks)**



Score for
Question 2:

2.
(3)

OVER

3. A polynomial function of degree 3 has zeros -2 , 2 , 4 , and passes through the point $(3, -25)$. Determine an equation of the function. (Answer may be left in factored form.)

(2 marks)

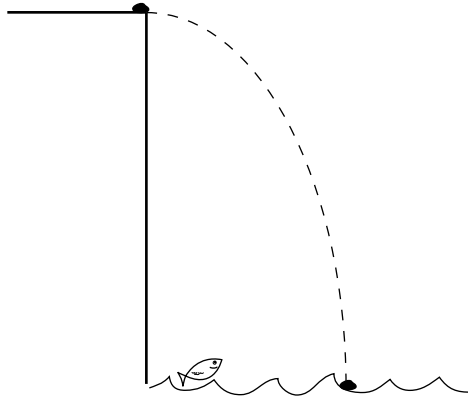
ANSWER:

Score for
Question 3:

3.
(2)

OVER

4. A rock is kicked off a vertical cliff and falls in a parabolic path to the water below. The cliff is 50 m high and the rock hits the water 20 m from the base of the cliff. What is the horizontal distance of the rock from the cliff face when the rock is at a height of 10 m above the water? (Accurate to at least 2 decimal places.) **(3 marks)**



ANSWER:

Score for
Question 4:

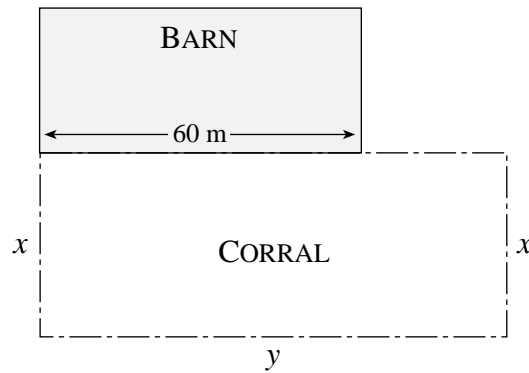
4.
(3)

OVER

5. A rancher wishes to build a rectangular corral, using the entire length of the barn as part of one side. The barn has a length of 60 m and the rancher has 220 m of fencing available.

a) Show that an expression for the area of the corral is $A = 140x - x^2$.

(1 mark)



Score for
Question 5a:

5.
(1)

b) Using $A = 140x - x^2$, determine the dimensions of the corral that will maximize its area.

(2 marks)

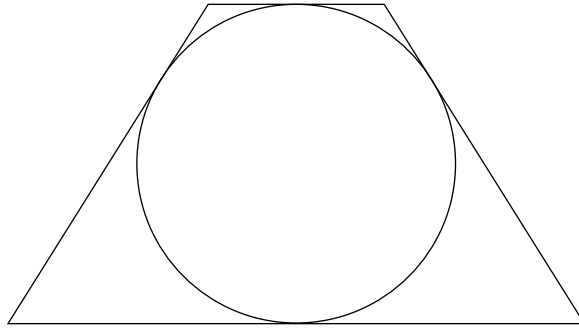
ANSWER:

Score for
Question 5b:

6.
(2)

OVER

6. A circle is inscribed in an isosceles trapezoid having bases of 18 cm and 50 cm. Find the diameter of the circle. **(2 marks)**



ANSWER:

Score for
Question 6:

7.
(2)

OVER

7. Complete the proof.

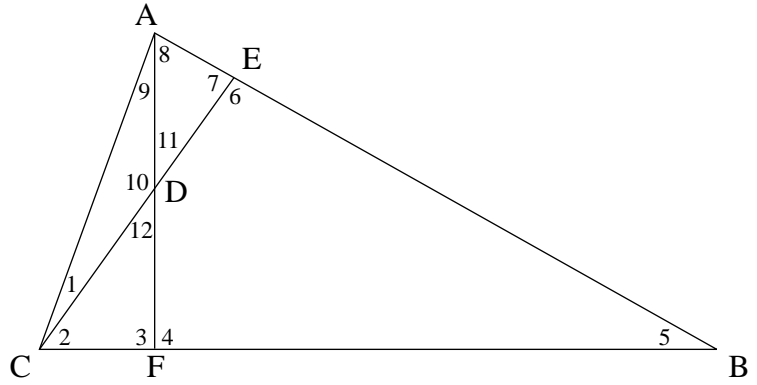
(4 marks)

Given: $CE \perp AB$

$AF \perp CB$

$DE = DF$

Prove: $\angle BCA = \angle BAC$



Proof	
Statement	Reason

Score for
Question 7:

8.
(4)

END OF EXAMINATION

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A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_n = \frac{a - \ell r}{1-r}$$

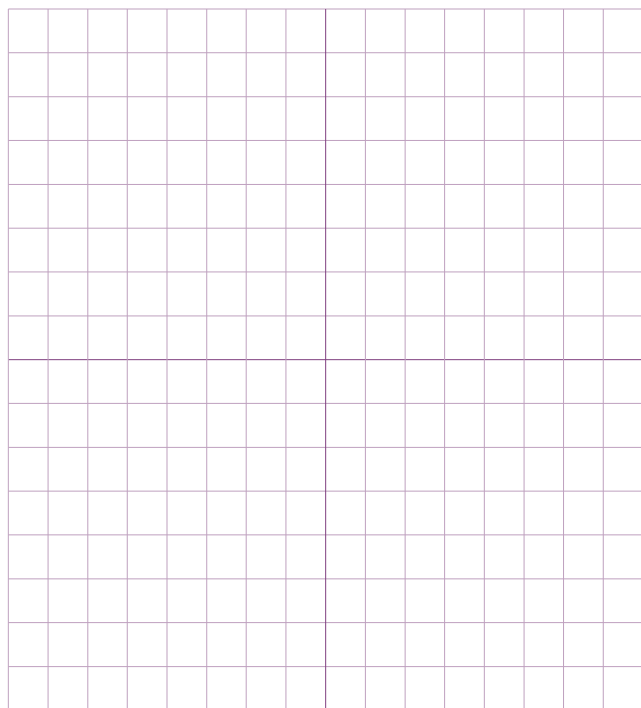
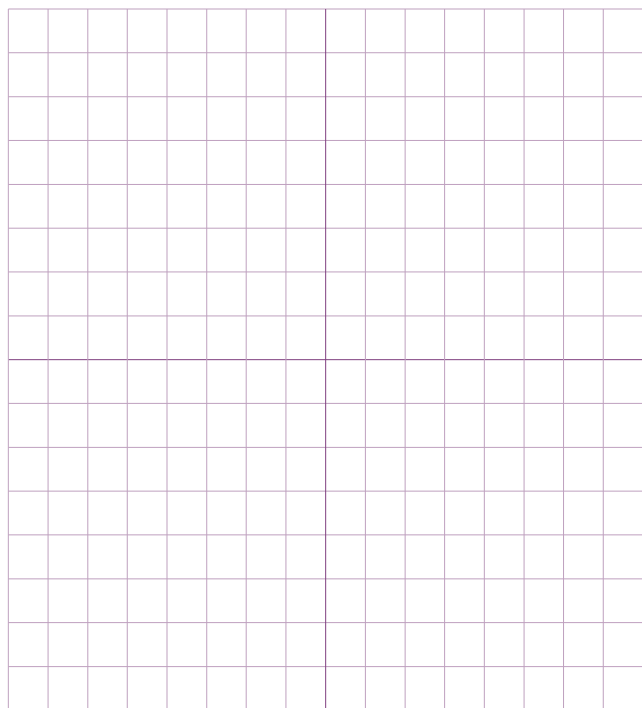
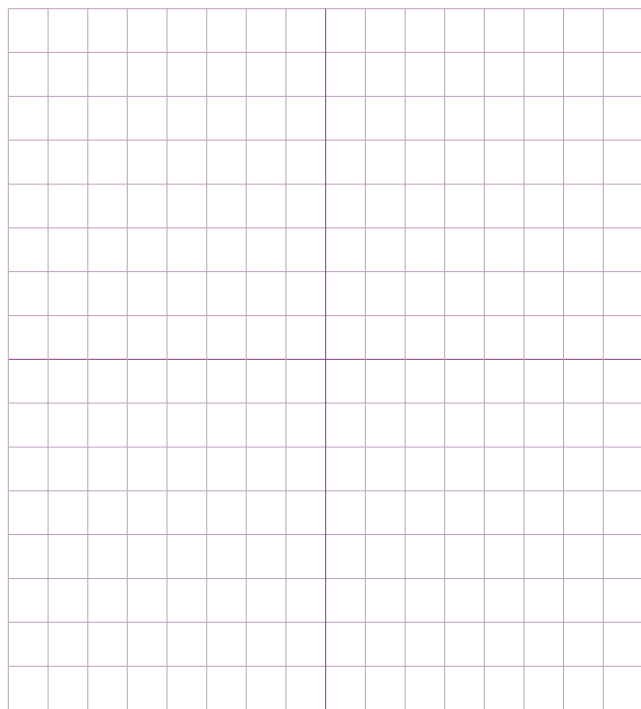
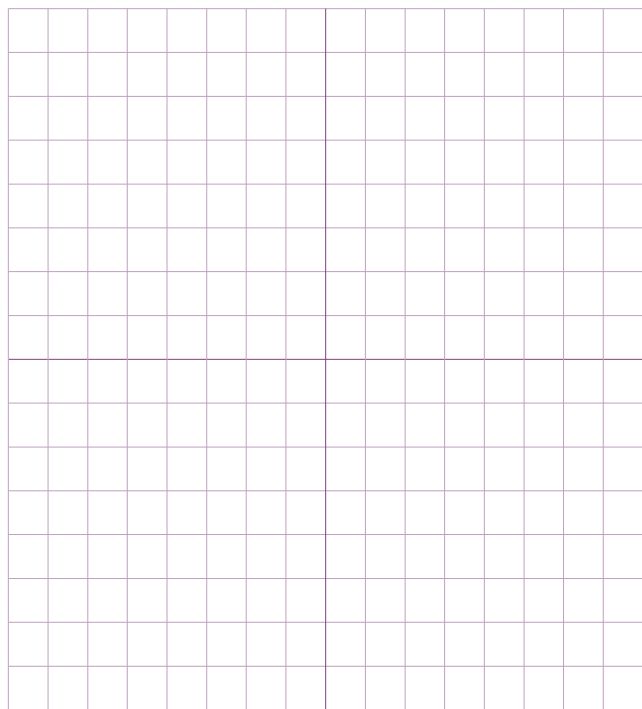
$$S = \frac{a}{1-r}$$

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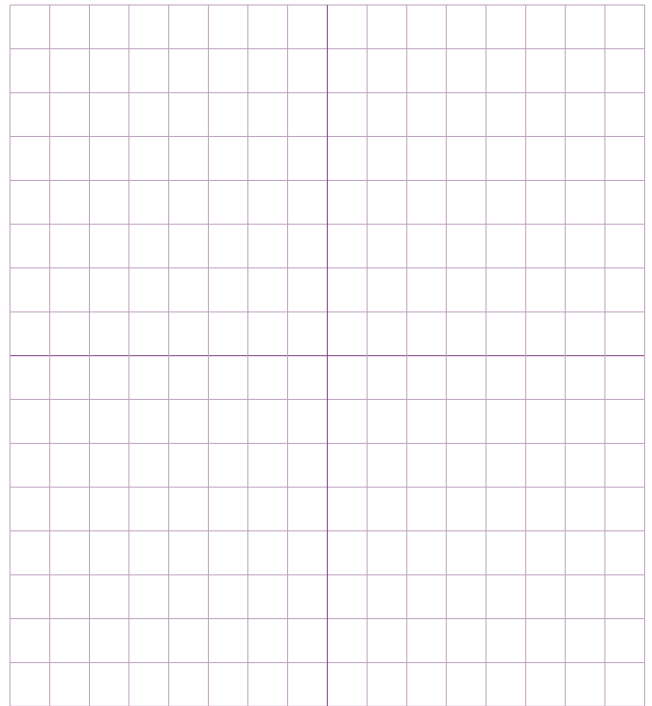
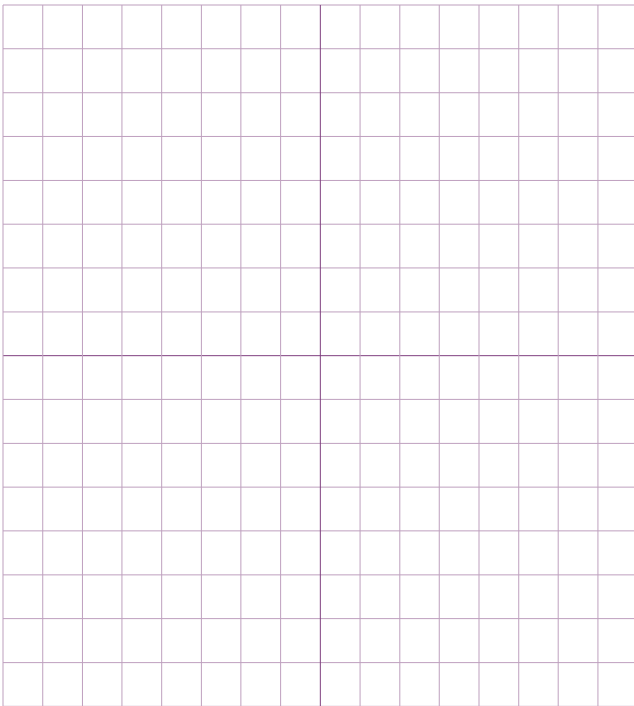
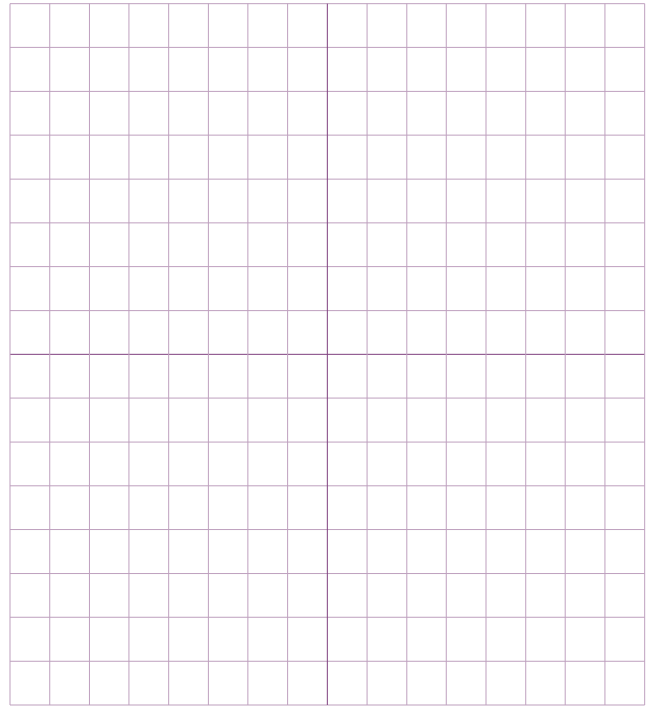
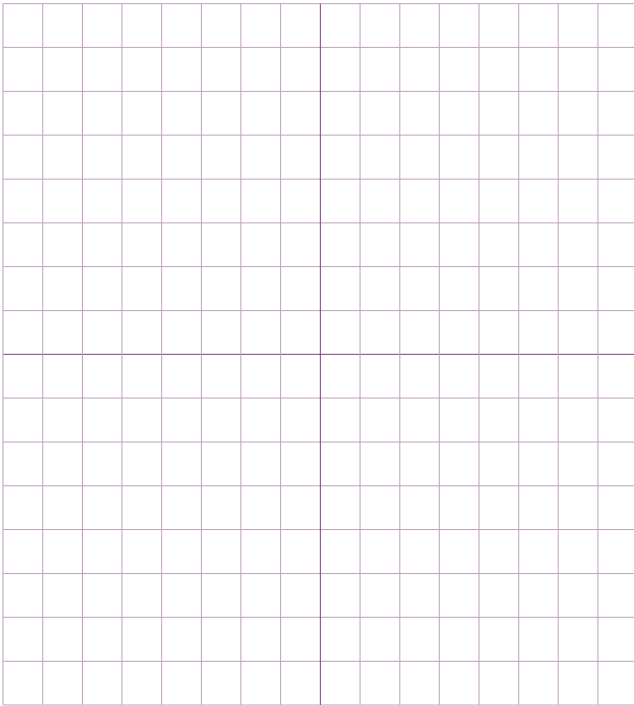
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