

JANUARY 1996

## PROVINCIAL EXAMINATION

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MINISTRY OF EDUCATION

# MATHEMATICS 12

### GENERAL INSTRUCTIONS

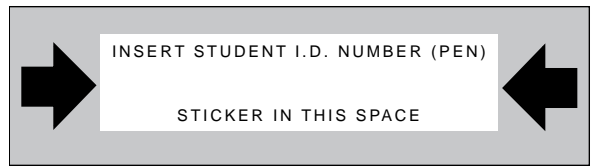
1. Insert the stickers with your Student I.D. Number (PEN) in the allotted spaces above.  
**Under no circumstance is your name or identification, other than your Student I.D. Number, to appear on this paper.**
2. Take the separate Answer Sheet and follow the directions on its front page.
3. Be sure you have an HB pencil and an eraser for completing your Answer Sheet.  
Follow the directions on the Answer Sheet when answering multiple-choice questions.
4. For each of the written-response questions, write your answer in the space provided.
5. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

**END OF EXAMINATION** .

6. At the end of the examination, place your Answer Sheet inside the front cover of this booklet and return the booklet and your Answer Sheet to the supervisor.

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**FOR OFFICE USE ONLY**



**MATHEMATICS 12 JANUARY 1996 PROVINCIAL**

**Course Code = MA**

**Examination Type = P**

1.  $\frac{\quad}{(2)}$

2.  $\frac{\quad}{(3)}$

3.  $\frac{\quad}{(3)}$

4.  $\frac{\quad}{(2)}$

5.  $\frac{\quad}{(1)}$

6.  $\frac{\quad}{(3)}$

7.  $\frac{\quad}{(2)}$

8.  $\frac{\quad}{(4)}$

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## MATHEMATICS 12 PROVINCIAL EXAMINATION

	Value	Suggested Time
1. This examination consists of <b>two</b> parts:		
PART A: 50 multiple-choice questions	50	75
PART B: 7 written-response questions	20	45
2 questions worth <b>two</b> marks each, 4 questions worth <b>three</b> marks each, and 1 question worth <b>four</b> marks.		
	<b>Total: 70 marks</b>	<b>120 minutes</b>
2. The last <b>three</b> pages inside the back cover contain <b>A Summary of Basic Identities and Formulae</b> , <b>Rough Work for Graphing</b> , and <b>Rough Work for Multiple-Choice</b> . These pages may be detached for convenient reference prior to writing this examination.		
3. You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.		
4. An approved scientific calculator is essential for the examination. The calculator must be a hand-held device designed <b>only</b> for mathematical computations such as logarithmic and trigonometric functions. It <b>can be</b> programmable, but <b>must not</b> contain any graphing capabilities. You <b>must not</b> bring into the examination room any devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or keyboards.		
5. You are permitted to use rulers, compasses, and protractors.		
6. You have <b>two hours</b> to complete this examination.		

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**PART A: MULTIPLE-CHOICE**

**Value: 50 marks**

**Suggested Time: 75 minutes**

**INSTRUCTIONS:** For each question, select the **best** answer and record your choice on the Answer Sheet provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. Identify the conic which is described by  $9x^2 - 9y^2 - 18y - 45 = 0$ .

- A. circle
- B. ellipse
- C. parabola
- D. hyperbola

2. Determine the distance between the points  $(7, 3)$  and  $(2, -1)$ .

- A. 5
- B. 9
- C.  $\sqrt{41}$
- D.  $\sqrt{85}$

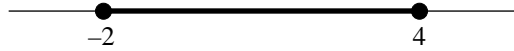
3. What is the maximum possible number of solutions for the following system of equations?  
( $m$  and  $b$  are real numbers.)

$$x^2 + y^2 = 16$$

$$y = mx + b$$

- A. 1
- B. 2
- C. 3
- D. 4

4. Which absolute value inequality describes the solution shown?

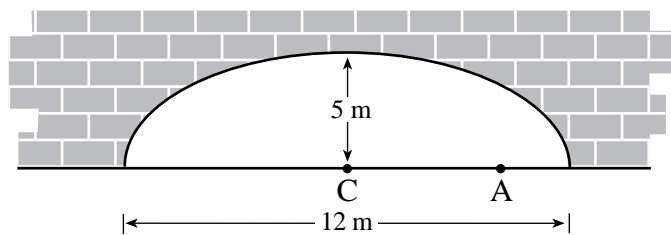


- A.  $|x + 1| \leq 3$
- B.  $|x + 1| < 3$
- C.  $|x - 1| \leq 3$
- D.  $|x - 1| < 3$

5. Determine the vertex for the parabola given by the equation  $y = 2x^2 + 12x + 13$ .

- A.  $(-3, -5)$
- B.  $(-3, 4)$
- C.  $(-3, 5)$
- D.  $(-3, 22)$

6. A tunnel is semi-elliptical in shape, with a maximum height of 5 m and a maximum width of 12 m. Determine the height of the tunnel at point A which is 4 m from the centre C.



- A. 1.67 m
- B. 3.33 m
- C. 3.73 m
- D. 4.71 m



7. A point P moves such that it is always the same distance from the point (4, 4) as it is from the line  $x = 2$ . Find an equation of the locus.

A.  $(x - 4)^2 + (y - 4)^2 = (x - 2)^2$

B.  $(x + 4)^2 + (y + 4)^2 = (x + 2)^2$

C.  $(x - 4)^2 + (y - 4)^2 = (y - 2)^2$

D.  $(x + 4)^2 + (y + 4)^2 = (y + 2)^2$

8. Determine the equations of the asymptotes of the hyperbola defined by  $\frac{x^2}{16} - \frac{(y+2)^2}{25} = 1$ .

A.  $y = \pm \frac{4}{5}x + 2$

B.  $y = \pm \frac{4}{5}x - 2$

C.  $y = \pm \frac{5}{4}x + 2$

D.  $y = \pm \frac{5}{4}x - 2$

9. Determine the radius  $r$  so that the following system has exactly two different real solutions.

$$xy = 4$$

$$x^2 + y^2 = r^2$$

A. 2

B. 4

C.  $2\sqrt{2}$

D.  $4\sqrt{2}$

10. All the points on a line are equidistant from the points  $P(x_1, y_1)$  and  $Q(x_2, y_2)$ . Determine the slope of this line.

A.  $-\frac{x_2 - x_1}{y_2 - y_1}$

B.  $-\frac{y_2 - y_1}{x_2 - x_1}$

C.  $\frac{x_2 - x_1}{y_2 - y_1}$

D.  $\frac{y_2 - y_1}{x_2 - x_1}$

11. Determine the period of the function  $y = -3 \sin 2x$ .

A.  $-3$

B.  $3$

C.  $\pi$

D.  $2\pi$

12. In which quadrant does the terminal arm of a position angle lie if the angle measures 5 radians?

A. I

B. II

C. III

D. IV

13. Evaluate:  $\cot \frac{2\pi}{5}$

A. 0.203

B. 0.325

C. 1.021

D. 3.078

14. Simplify:  $\cos^4 \theta - \sin^4 \theta$

- A.  $-1$
- B.  $-\cos 2\theta$
- C.  $\cos 2\theta$
- D.  $\cos 4\theta$

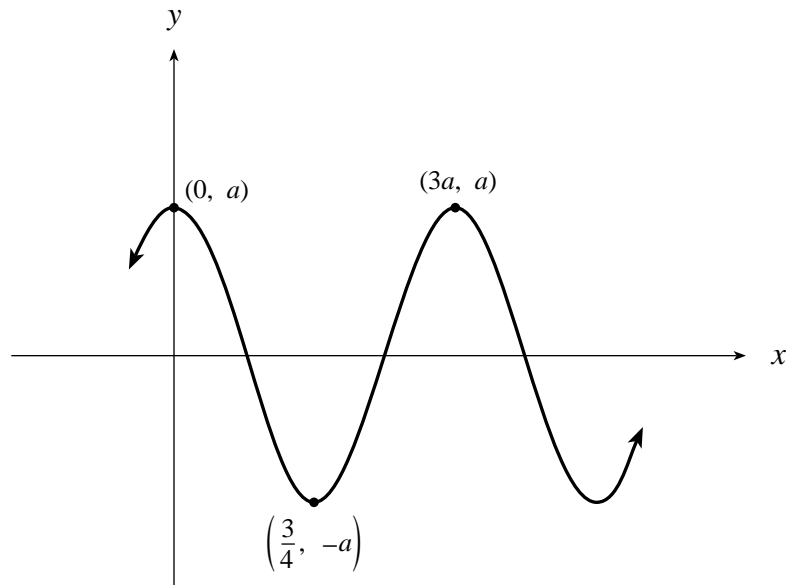
15. Given  $\sin \theta = \frac{4}{5}$  and  $\sec \theta < 0$ , determine the ratio for  $\cot \theta$ .

- A.  $-\frac{4}{3}$
- B.  $-\frac{3}{4}$
- C.  $\frac{3}{4}$
- D.  $\frac{4}{3}$

16. Simplify:  $\cos(630^\circ - \theta)$

- A.  $-\cos \theta$
- B.  $-\sin \theta$
- C.  $\cos \theta$
- D.  $\sin \theta$

17. Given the graph of  $y = a \cos kx$  as shown, determine  $a$ .



- A.  $\frac{1}{2}$
- B. 1
- C.  $\frac{3}{2}$
- D. 3

18. Determine a single geometric mean between  $\sec x - 1$  and  $\sec x + 1$ .

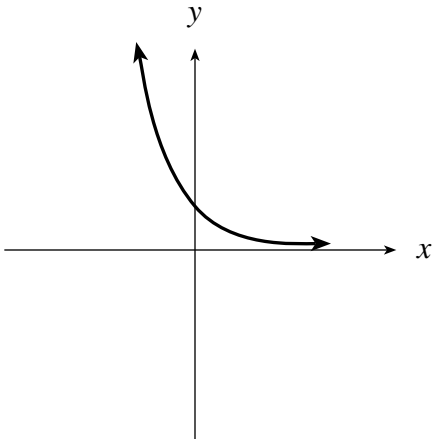
- A.  $-1$
- B. 1
- C.  $\cos x$
- D.  $\tan x$

19. Write  $a^b = c$  in logarithmic form.

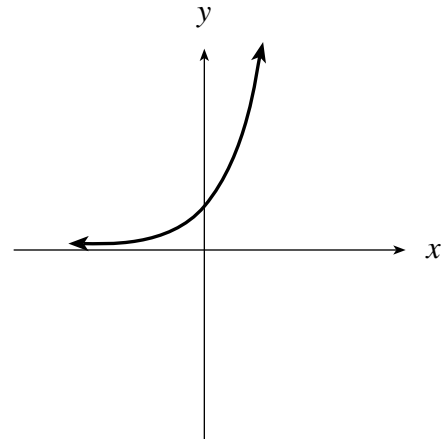
- A.  $\log_a c = b$
- B.  $\log_a b = c$
- C.  $\log_c a = b$
- D.  $\log_b a = c$

20. Determine which graph **best** represents  $y = \left(\frac{1}{2}\right)^x$ .

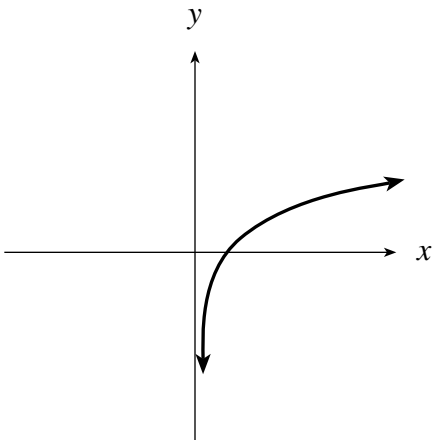
A.



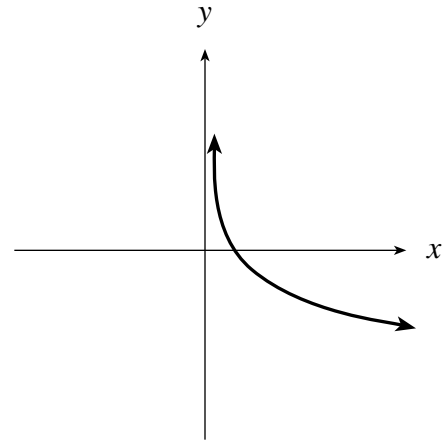
B.



C.



D.



21. Solve for  $x$ :  $\log(4x + 3) - \log x = \log 10$

- A.  $\frac{1}{2}$
- B.  $\frac{1}{3}$
- C.  $\frac{7}{3}$
- D.  $\frac{13}{3}$

22. Solve for  $x$ :  $\log_2 8 - \log_3 3 = x$

A. 2

B. 3

C. 5

D.  $\frac{8}{3}$

23. Write as a single logarithm:  $2 \log a + 3 \log a^2$

A.  $\log a^7$

B.  $\log a^8$

C.  $\log a^{10}$

D.  $\log a^{12}$

24. If  $f(x) = \log_3(x - 5) + 1$ , determine an equation of an asymptote of  $f^{-1}(x)$ , the inverse of  $f(x)$ .

A.  $y = 5$

B.  $x = 5$

C.  $y = 1$

D.  $x = 1$

25. Determine all values of  $x$  such that the expression  $\frac{1}{\log(2x - 5)}$  is defined.

A.  $x \neq 3$

B.  $x > \frac{5}{2}$

C.  $0 < x < \frac{5}{2}$

D.  $x > \frac{5}{2}, x \neq 3$

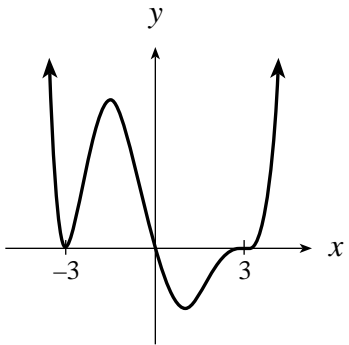
26. According to the Rational Root Theorem, determine all possible rational roots of  $3x^3 - 8x^2 + 16x - 4 = 0$ .
- A.  $\pm 1, \pm 3$
  - B.  $\pm 1, \pm 2, \pm 4$
  - C.  $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}$
  - D.  $\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 4, \pm \frac{4}{3}$
27. If  $p(x)$  is a polynomial function where  $p(-2) = 5$ , then which of the following **could not** be a zero of this function?
- A.  $-5$
  - B.  $-2$
  - C.  $2$
  - D.  $5$
28. Find the remainder when  $x^3 - 2x^2 + 5$  is divided by  $x^2 + x - 1$ .
- A.  $4$
  - B.  $2x + 2$
  - C.  $2x + 4$
  - D.  $4x + 2$
29. Solve for  $x$ :  $x^3 - 2x^2 - 5x + 6 = 0$
- A.  $-1, -2, 3$
  - B.  $-1, 2, 3$
  - C.  $1, -2, 3$
  - D.  $1, 2, -3$

30. Determine the value of  $k$  such that  $x + 2$  is a factor of the polynomial  $2x^3 + 5x^2 + kx - 12$ .

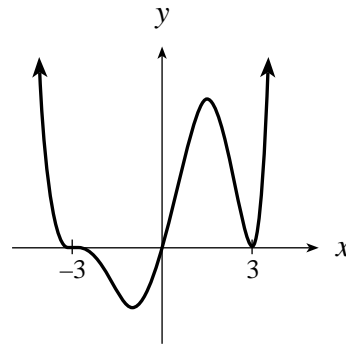
- A. -12
- B. -4
- C. 4
- D. 12

31. Which graph **best** represents  $y = -x(x + 3)^2(x - 3)^3$  ?

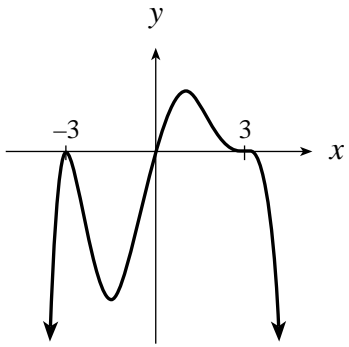
A.



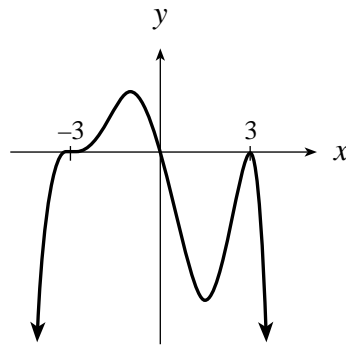
B.



C.

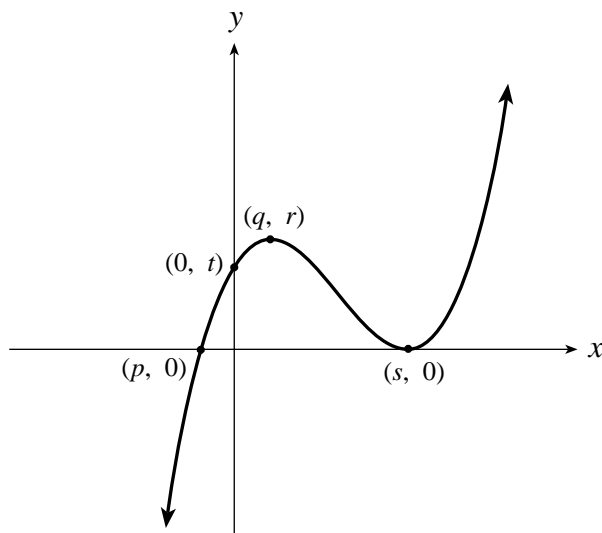


D.





32. Given that  $p(x)$  and  $f(x)$  are polynomial functions such that  $p(x) = x f(x) + c$ , determine  $c$  if the graph of  $p(x)$  is shown.



- A.  $p$   
B.  $t$   
C.  $r$   
D.  $s$
33. Which of the following sequences is arithmetic?
- A.  $c, c^2, c^3, c^4$   
B.  $c, c^2, c^4, c^8$   
C.  $c, 2c, 3c, 4c$   
D.  $c, c+2, c+4, c+8$
34. Determine the sum of the infinite geometric series  $3 - \frac{3}{4} + \frac{3}{16} \dots$

- A.  $\frac{12}{5}$   
B.  $\frac{39}{16}$   
C. 4  
D. 12

35. Determine the second term of the sequence defined by

$$t_1 = -5$$

$$t_n = 1 - 2t_{n-1}, \quad n > 1$$

- A. -11
- B. -9
- C. 9
- D. 11

36. Evaluate:  $\sum_{k=1}^{10} 3(-2)^{k-1}$

- A. -3 069
- B. -1 023
- C. 1 025
- D. 3 075

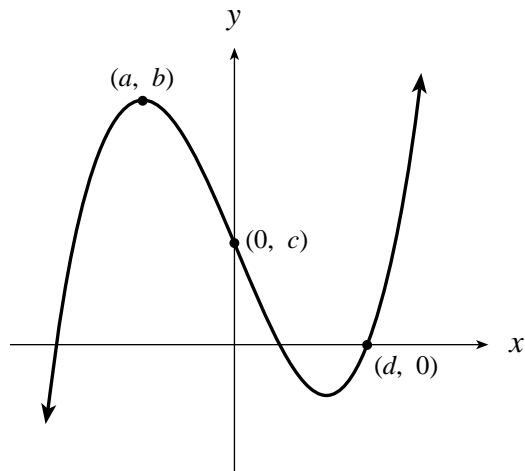
37. In a geometric sequence,  $t_{10} = 129$  and the common ratio is  $\frac{5}{3}$ . Determine the value of the first term. (Accurate to 2 decimal places.)

- A. 0.78
- B. 1.09
- C. 1.30
- D. 1.88

38. For any series, which of the following is equivalent to  $S_{n-1} + S_n$  ?

- A.  $2S_{n-1} + t_n$
- B.  $2S_{n-1} - t_n$
- C.  $2S_n - 1$
- D.  $2S_n + t_n$

39. Determine the local maximum value for the function whose graph is shown.



- A.  $a$
- B.  $b$
- C.  $c$
- D.  $d$

40. Find  $f'(x)$  if  $f(x) = 2x^3$ .

- A.  $5x$
- B.  $6x$
- C.  $5x^2$
- D.  $6x^2$

41. Evaluate:  $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 + x - 12}$

- A.  $\frac{3}{4}$
- B.  $\frac{6}{7}$
- C.  $1$
- D. limit does not exist

42. Evaluate:  $\lim_{n \rightarrow \infty} \frac{3n-1}{4-2n}$

A.  $-\frac{3}{2}$

B.  $-\frac{1}{4}$

C.  $\frac{3}{2}$

D. limit does not exist

43. Given the function  $f(x) = x^2 - 3x$ , determine the slope of the **secant line** intersecting the graph of  $f$  at the points where  $x = 2$  and  $x = 3$ .

A.  $-2$

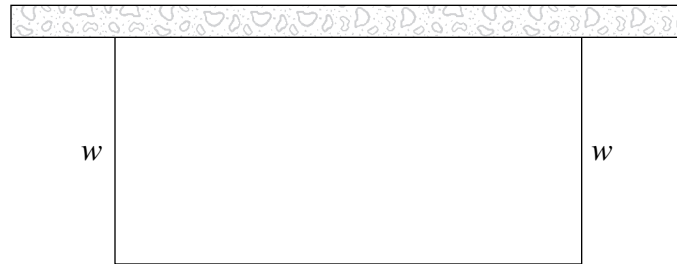
B.  $-\frac{1}{2}$

C.  $\frac{1}{2}$

D.  $2$

44. Choose the **most** appropriate function to solve the following problem:

You want to construct a rectangular enclosure of area  $A$  having one side along an existing wall and the other three sides fenced. If you have 50 m of fencing, what is the largest possible area your enclosure can have?



- A.  $A = (25 + w)w$
- B.  $A = (25 - w)w$
- C.  $A = (50 + 2w)w$
- D.  $A = (50 - 2w)w$

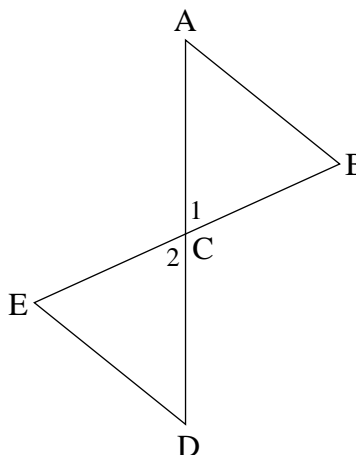
45. For what values of  $x$  is the function  $y = a^2x^3 - 3x$  increasing, where  $a$  is a positive constant?

- A.  $-\frac{1}{a} < x < \frac{1}{a}$
- B.  $-1 < x < 1$
- C.  $x < -\frac{1}{a}$  or  $x > \frac{1}{a}$
- D.  $x < -1$  or  $x > 1$

Use the following diagram and proof to answer question 46.

Given:  $AB \parallel ED$   
 $BE$  bisects  $AD$

Prove:  $\triangle ABC \cong \triangle DEC$



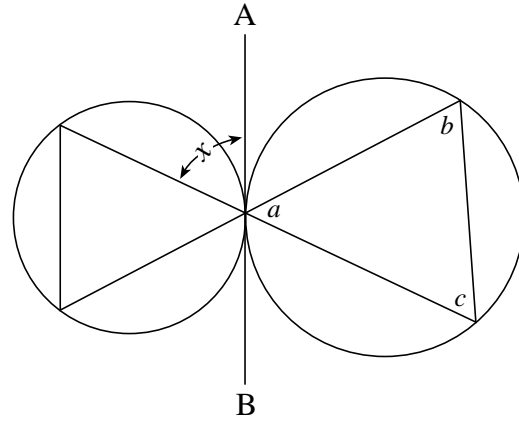
Proof	
Statement	Reason
$AB \parallel ED$	given
(a) $\angle A = \angle D$	alternate interior angles are =
$BE$ bisects $AD$	given
(b) $BC = EC$	definition of bisector
(c) $AC = CD$	definition of bisector
(d) $\angle 1 = \angle 2$	vertically opposite angles are =
$\triangle ABC \cong \triangle DEC$	ASA

46. Which line is **incorrect** in the given proof?

- A. a
- B. b
- C. c
- D. d

Use the following diagram to answer question 47.

Given: Line AB is tangent to both circles at the same point as shown.



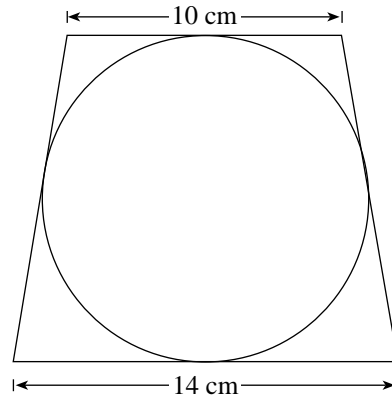
47. Determine the value for  $\angle x$ .

- A.  $a$
- B.  $b$
- C.  $c$
- D.  $a + b$

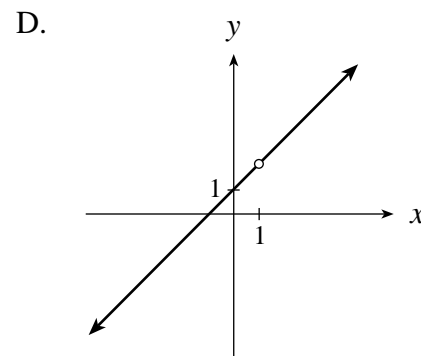
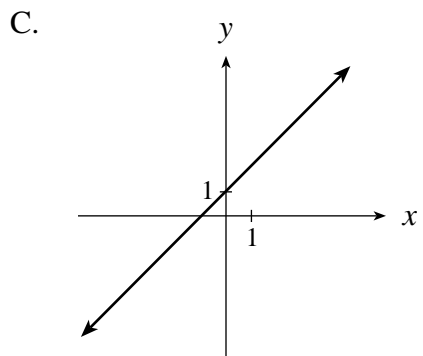
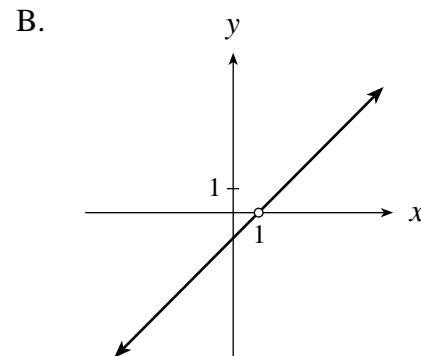
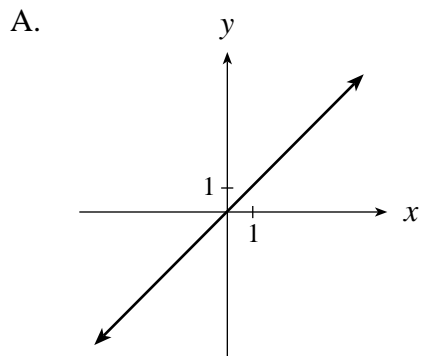
48. What is the probability of obtaining a sum of 16 if three fair six-sided dice are rolled?

- A.  $\frac{2}{216}$
- B.  $\frac{3}{216}$
- C.  $\frac{6}{216}$
- D.  $\frac{12}{216}$

49. A circle is inscribed in an isosceles trapezoid having bases 10 cm and 14 cm. Determine the diameter of the circle. (Accurate to 2 decimal places.)



- A. 11.26 cm  
 B. 11.83 cm  
 C. 12.00 cm  
 D. 12.17 cm
50. Which of the following **best** represents the graph of  $y = \frac{x^2 - 1}{x - 1}$  ?



**This is the end of the multiple-choice section.  
 Answer the remaining questions directly in this examination booklet.**



**PART B: WRITTEN-RESPONSE**

**Value: 20 marks**

**Suggested Time: 45 minutes**

**INSTRUCTIONS:** Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

**Full marks will NOT be given for the final answer only.**

1. Determine the sum of the arithmetic series  $7 + 2 + (-3) + (-8) + \dots + (-213)$ .

**(2 marks)**

ANSWER:

Score for  
Question 1:

1.           
(2)

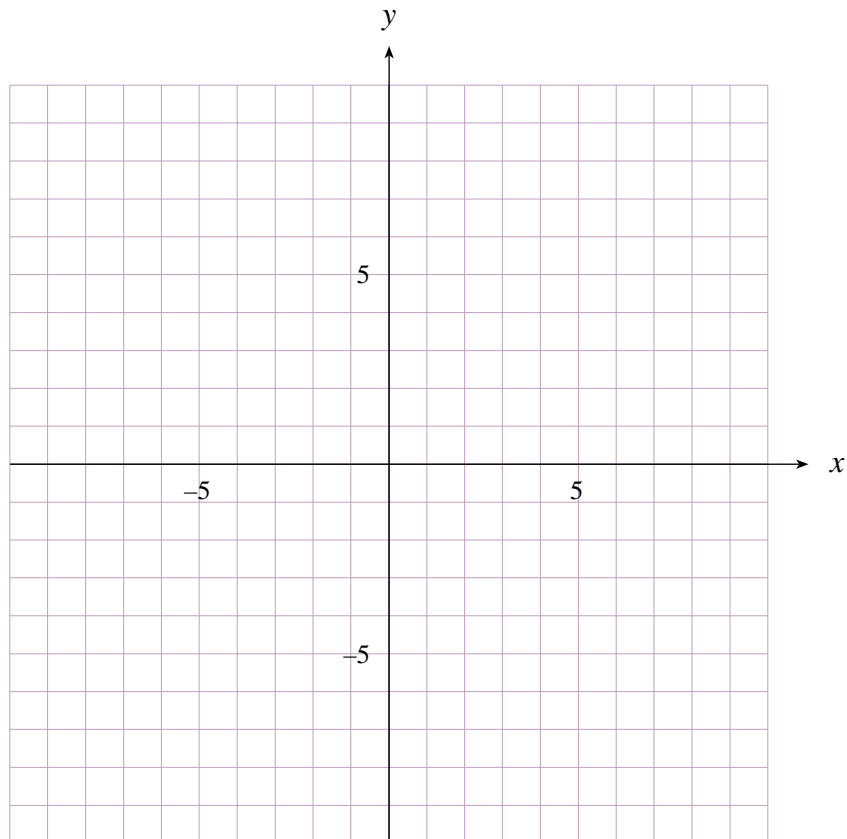
**OVER**

2. Graph the solution of the following system of inequalities on the grid provided.

**(3 marks)**

$$x < -(y-2)^2 + 4$$

$$\frac{(x-3)^2}{9} + \frac{(y-2)^2}{4} \leq 1$$



Score for  
Question 2:

2.           
(3)

**OVER**

3. Solve:  $2 \tan^2 x + 3 \tan x - 2 = 0$ ,  $0 \leq x < 2\pi$ . (Accurate to at least 2 decimal places.) (3 marks)

ANSWER:

Score for  
Question 3:

3.           
(3)

**OVER**

4. A particle moves along the  $x$ -axis so that its position at time  $t$  is  $x(t) = 2t^3 - 5t^2 - 4t + 3$ .  
( $x$  in cm and  $t$  in seconds.)

a) At what time(s) is the particle stationary?

**(2 marks)**

ANSWER:	Score for Question 4a:  4. <u>        </u> (2)
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b) At what time(s) is the particle moving to the left?

**(1 mark)**

ANSWER:

Score for  
Question 4b:

5.           
(1)

**OVER**

5. Suppose you invest \$10 at 10.2% per annum compounded annually. How many years would it take for your investment to grow to \$15 000? (Accurate to at least 1 decimal place.) **(3 marks)**

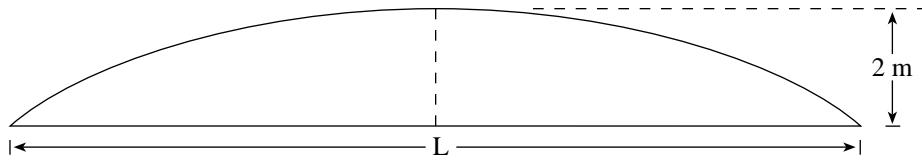
ANSWER:

Score for  
Question 5:

6.           
(3)

**OVER**

6. A circular arch of a footbridge rises 2 m at the centre. If the horizontal length  $L$  of the footbridge is the same as the radius of the circle, calculate the value of  $L$ . (Accurate to at least 2 decimal places.) **(2 marks)**



ANSWER:

Score for  
Question 6:

7.           
(2)

**OVER**

7. Complete the proof.

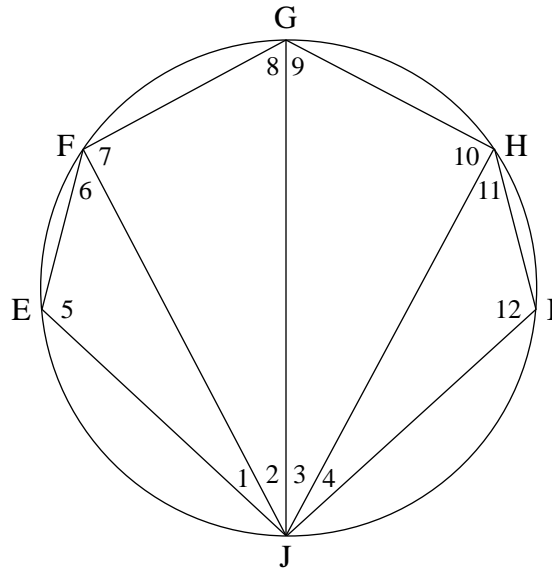
(4 marks)

Given: Circle with diameter GJ

$EF = IH$

$\angle 6 = \angle 11$

Prove:  $\angle 2 = \angle 3$



Proof	
Statement	Reason

Score for  
Question 7:

8. \_\_\_\_\_  
(4)

**END OF EXAMINATION**

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## A SUMMARY OF BASIC IDENTITIES AND FORMULAE

### Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

### Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

### Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

### Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

### Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_n = \frac{a - \ell r}{1-r}$$

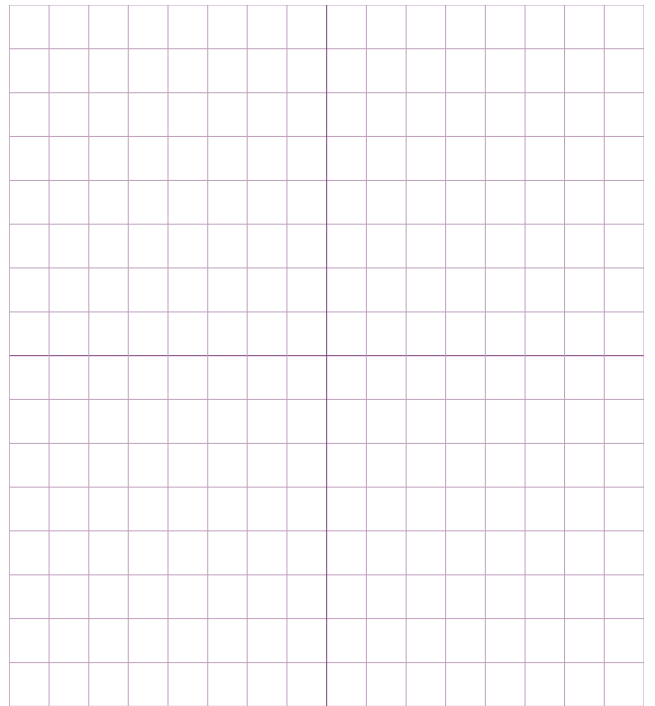
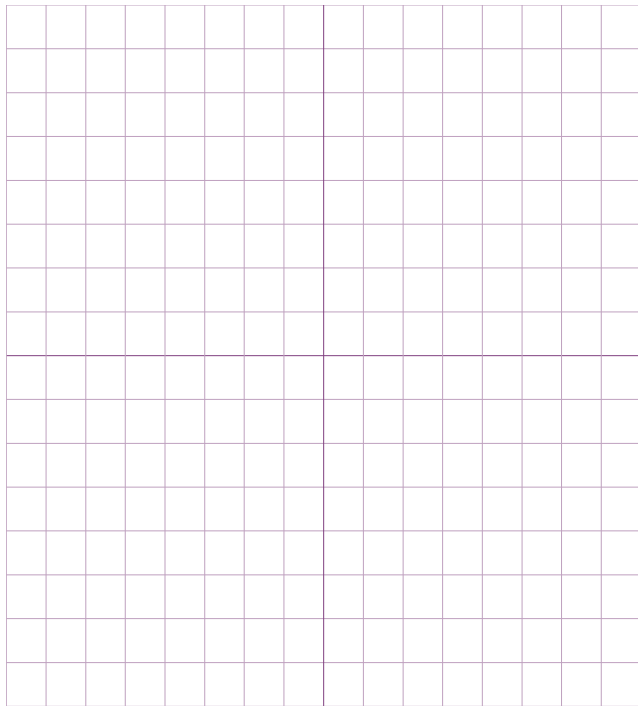
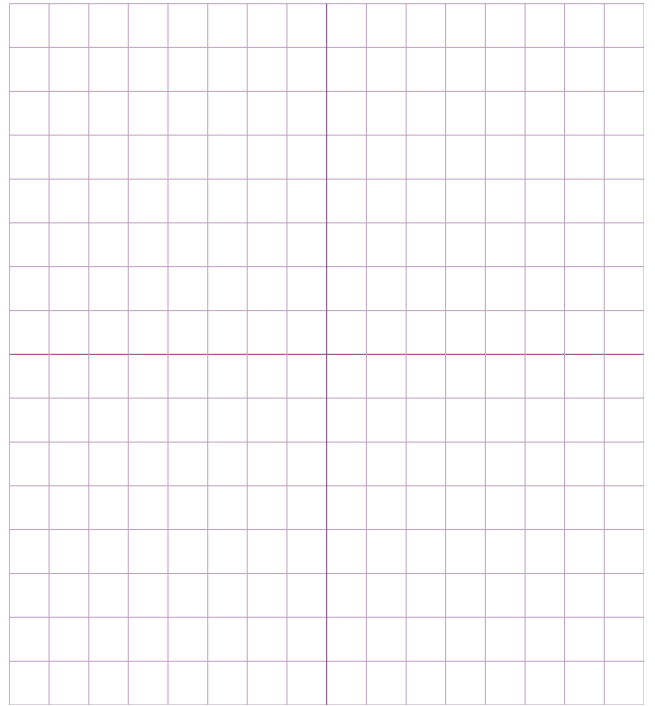
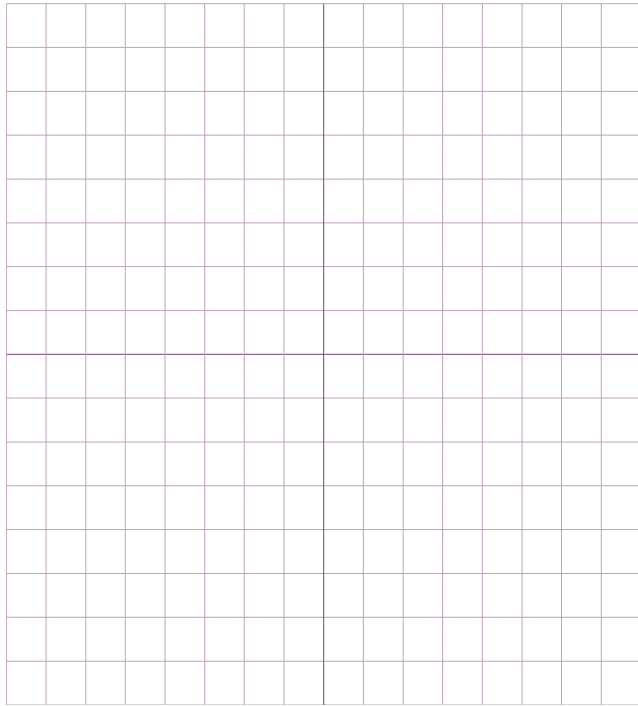
$$S = \frac{a}{1-r}$$

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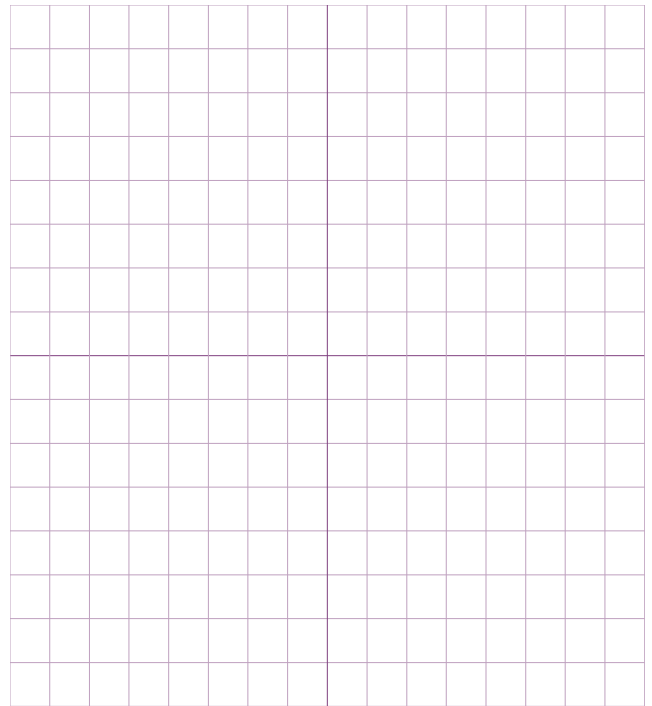
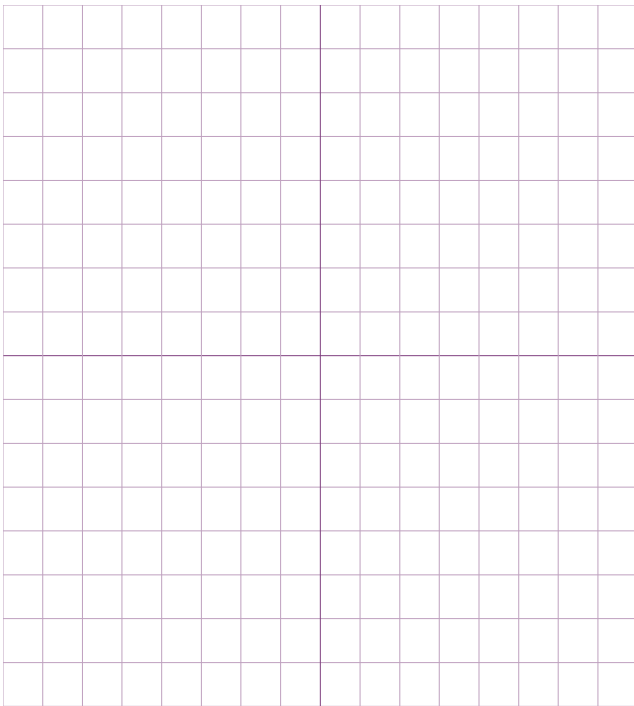
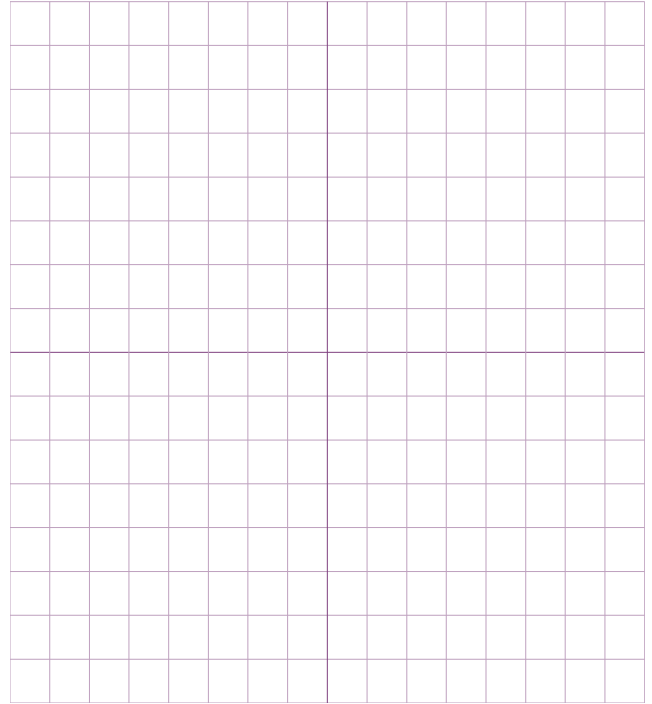
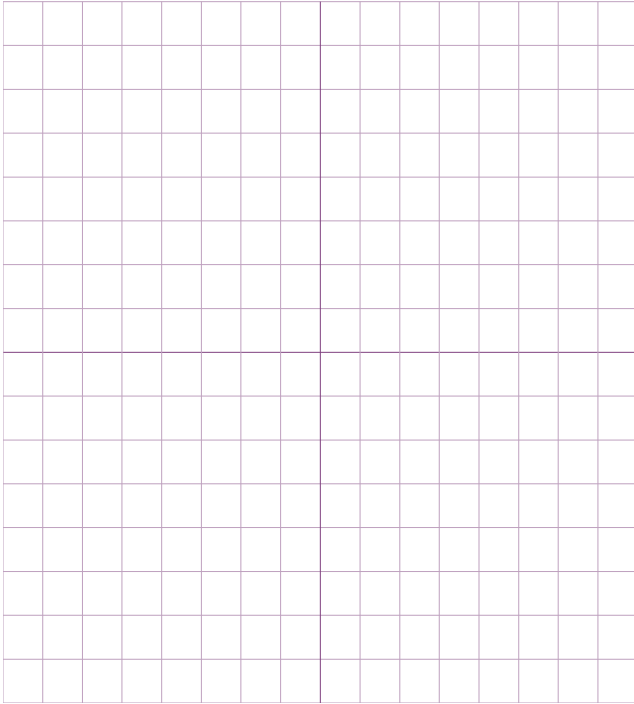
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## ROUGH WORK FOR MULTIPLE-CHOICE