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BRITISH  
COLUMBIA

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Principles of Mathematics 12

AUGUST 2001

Course Code = MA

### Student Instructions

1. Place the stickers with your Personal Education Number (PEN) in the allotted spaces above. **Under no circumstance is your name or identification, other than your Personal Education Number, to appear on this booklet.**
2. Ensure that in addition to this examination booklet, you have an **Examination Response Form**. Follow the directions on the front of the Response Form.
3. **Disqualification** from the examination will result if you bring books, paper, notes or unauthorized electronic devices into the examination room.
4. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by **END OF EXAMINATION**.
5. At the end of the examination, place your Response Form inside the front cover of this booklet and return the booklet and your Response Form to the supervisor.

Question 1:

1.  .

(4)

Question 2:

2.  .

(4)

Question 3:

3.  .

(4)

Question 4:

4.  .

(5)

Question 5:

5.  .

(4)

Question 6:

6.  .

(4)

Question 7:

7.  .

(4)

Question 8:

8.  .

(5)

**PRINCIPLES OF  
MATHEMATICS 12**

**AUGUST 2001**

COURSE CODE = MA

## GENERAL INSTRUCTIONS

1. Aside from an approved calculator, electronic devices, including dictionaries and pagers, are **not** permitted in the examination room.
2. All multiple-choice answers must be entered on the Response Form using an **HB pencil**. Multiple-choice answers entered in this examination booklet will **not** be marked.
3. For each of the written-response questions, write your answer in the space provided in this booklet.

You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.

4. Ensure that you use language and content appropriate to the purpose and audience of this examination. Failure to comply may result in your paper being awarded a zero.
5. This examination is designed to be completed in **two hours**. *Students may, however, take up to 30 minutes of additional time to finish.*

## PRINCIPLES OF MATHEMATICS 12 PROVINCIAL EXAMINATION

- |   | Value                   | Suggested Time     |
|---|-------------------------|--------------------|
| 1. This examination consists of <b>two</b> parts: |                         |                    |
| PART A: 44 multiple-choice questions              | 66                      | 75                 |
| PART B: 8 written-response questions              | 34                      | 45                 |
|   | <b>Total: 100 marks</b> | <b>120 minutes</b> |
2. The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae**, **Rough Work for Graphing**, and **Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.
3. **A graphing calculator is essential for the Principles of Mathematics 12 Provincial Examination.** The calculator must be a hand-held device designed primarily for mathematical computations involving logarithmic and trigonometric functions as well as for graphing functions. Computers, calculators with a QWERTY keyboard or symbolic manipulation abilities, and electronic writing pads will not be allowed. Students must not bring any external devices (peripherals) to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, CD-ROMs, libraries or external keyboards. Students may have more than one calculator available during the examination, of which one may be a scientific calculator. Calculators may not be shared and must not have the ability to either transmit or receive electronic signals. In addition to an approved calculator, students will be allowed to use rulers, compasses, and protractors during the examination.
- Calculators must not have any information programmed into the memory which would not be acceptable in paper form. Specifically, calculators must not have any built-in notes, definitions, or libraries. There is no requirement to clear memories at the beginning of the examination but the use of calculators with built-in notes is equivalent to the use of notes in paper form. Any student deemed to have cheated on a provincial examination will receive a “0” on that examination and will be permanently disqualified from the Provincial Examination Scholarship Program.
4. If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.
5. When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

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**PART A: MULTIPLE CHOICE**

**Value: 66 marks**

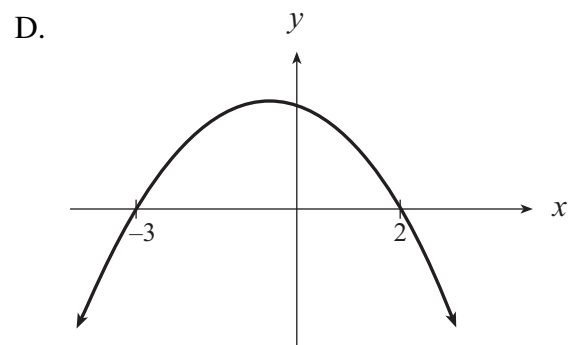
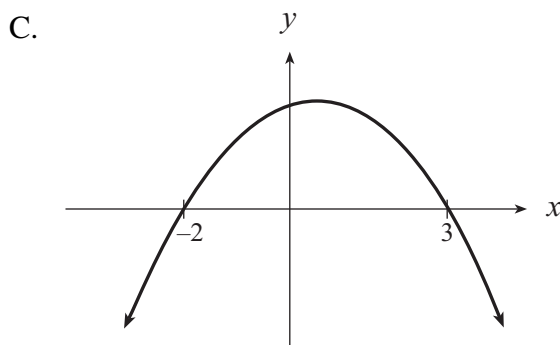
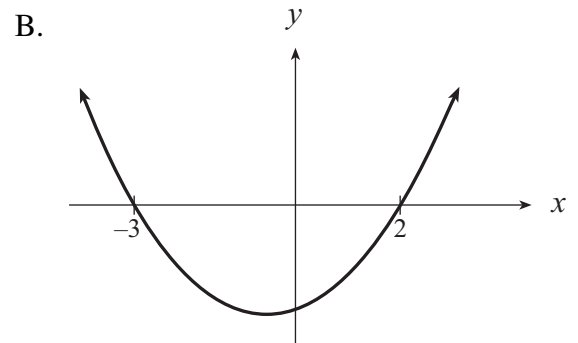
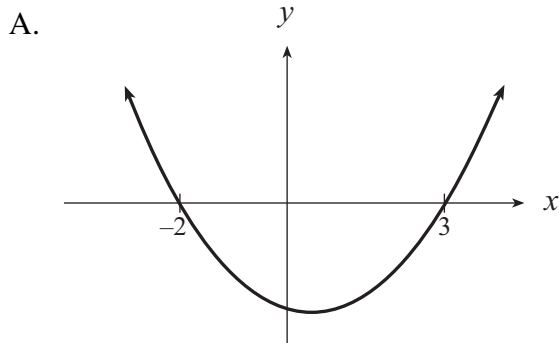
**Suggested Time: 75 minutes**

**INSTRUCTIONS:** For each question, select the **best** answer and record your choice on the Response Form provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. According to the Rational Root Theorem, list all possible rational roots of  $3x^3 + 2x + 7 = 0$ .

- A.  $\pm 1, \pm 3$
- B.  $\pm 1, \pm 7$
- C.  $\pm 1, \pm 3, \pm \frac{1}{7}, \pm \frac{3}{7}$
- D.  $\pm 1, \pm 7, \pm \frac{1}{3}, \pm \frac{7}{3}$

2. Which sketch best represents the graph of  $y = (2 - x)(x + 3)$  ?



**OVER**

3. Given that  $(x - 2)$  is a factor of  $p(x)$ , what must be true for the graph of  $y = p(x)$  ?

- A.  $y$ -intercept of  $-2$
- B.  $y$ -intercept of  $2$
- C.  $x$ -intercept of  $-2$
- D.  $x$ -intercept of  $2$

4. Given the following table of values for the polynomial function,  $f(x)$ , between which two consecutive  $x$ -values must a zero be located?

$x$	$y$
$-1$	$8$
$0$	$3$
$1$	$1$
$2$	$-4$
$3$	$-7$

- A. between  $-1$  and  $0$
- B. between  $0$  and  $1$
- C. between  $1$  and  $2$
- D. between  $2$  and  $3$

5. Solve:  $x^3 - 14x^2 > 17x - 380$

- A.  $-4.95 < x < 5.87$  or  $x > 13.08$
- B.  $x < -4.95$  or  $5.87 < x < 13.08$
- C.  $x > 14.62$
- D.  $x > -8.82$

6. If the polynomial  $p(x) = ax^2 + bx - 6$  is divided by  $(x - 1)$ , the remainder is  $-9$ . When  $p(x)$  is divided by  $(x + 2)$ , the remainder is  $12$ . Find the value of  $b$ .

- A.  $-5$
- B.  $-2$
- C.  $2$
- D.  $5$



7. Which conic is described by the equation  $4x^2 + 4x - 3y^2 + 6y - 53 = 0$  ?

- A. circle
- B. ellipse
- C. parabola
- D. hyperbola

8. Determine the distance between  $(2, 4)$  and  $(-3, 7)$ .

- A.  $\sqrt{10}$
- B.  $\sqrt{34}$
- C.  $2\sqrt{30}$
- D.  $\sqrt{122}$

9. A point  $P(x, y)$  moves such that it is always 9 units from the point  $A(3, 5)$ . Determine an equation of this locus.

- A.  $(x + 3)^2 + (y + 5)^2 = 9$
- B.  $(x - 3)^2 + (y - 5)^2 = 9$
- C.  $(x + 3)^2 + (y + 5)^2 = 81$
- D.  $(x - 3)^2 + (y - 5)^2 = 81$

10. Change  $6x^2 + 5y^2 - 36x - 6 = 0$  to standard form.

- A.  $(x - 3)^2 + \frac{y^2}{6} = 1$
- B.  $\frac{(x - 3)^2}{5} + \frac{y^2}{3} = 1$
- C.  $\frac{(x - 3)^2}{10} + \frac{y^2}{12} = 1$
- D.  $\frac{(x - 3)^2}{55} + \frac{y^2}{66} = 1$

**OVER**

11. Determine the slopes of the asymptotes of the hyperbola  $\frac{x^2}{25} - \frac{(y-1)^2}{36} = 1$ .

- A.  $\pm \frac{5}{6}$
- B.  $\pm \frac{6}{5}$
- C.  $\pm \frac{25}{36}$
- D.  $\pm \frac{36}{25}$

12. Solve the following system for  $x$  only:

$$2x^2 - 3y^2 = 6$$

$$x^2 + y^2 = 3$$

- A. 0
- B.  $\pm \frac{\sqrt{10}}{2}$
- C.  $\pm \sqrt{3}$
- D.  $\pm \sqrt{10}$

13. Give an absolute value inequality that represents the solution for  $x^2 - 2x - 15 > 0$ .

- A.  $|x + 4| > 1$
- B.  $|x - 4| > 1$
- C.  $|x + 1| > 4$
- D.  $|x - 1| > 4$

14. Determine all values of  $k$  such that the following system has two different real solutions.

$$x = (y - 2)^2 + 3$$

$$\frac{(x - 1)^2}{k} + \frac{(y - 2)^2}{5} = 1$$

- A.  $k > 2$   
B.  $k > 4$   
C.  $k > 8$   
D.  $k > 16$
15. Evaluate:  $\log_3 42$

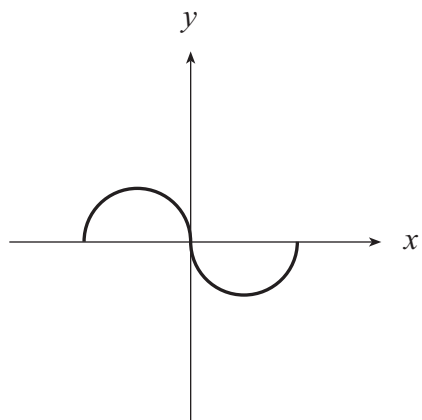
- A. 1.15  
B. 3.40  
C. 4.87  
D. 20.04

16. Express as a single logarithm:  $\log_3 A - \log_3 B^2$

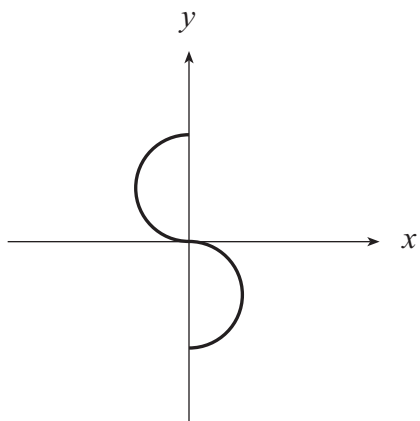
- A.  $\log_3 \frac{A}{B^2}$   
B.  $\log_3(A - B^2)$   
C.  $\frac{\log_3 A}{\log_3 B^2}$   
D.  $\log_3 \frac{A}{2B}$

**OVER**

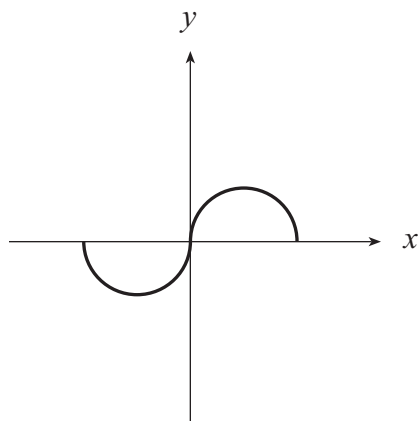
17. Which graph best represents the inverse relation of the graph shown below?



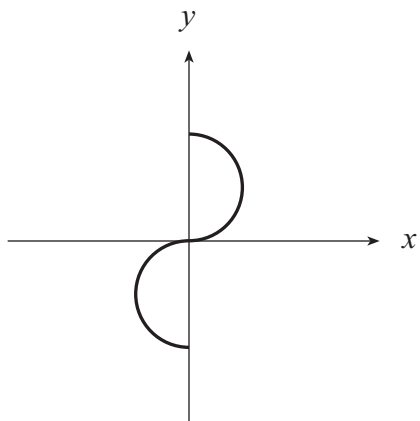
A.



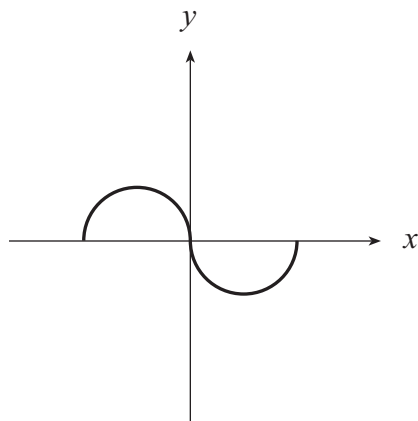
B.



C.



D.



18. Solve:  $2\log_3(x - 4) - \log_3 x = 2$

- A. 1
- B. 10
- C. 16
- D. no solution

19. What amount of money would grow to \$1 200 in five years if it is invested at 6% compounded annually?

- A. \$ 896.71
- B. \$ 950.51
- C. \$1 164.64
- D. \$1 605.87

20. Solve for  $x$ :  $a^{x+3} = b^x$

- A.  $\frac{\log b}{\log a} - 3$
- B.  $\frac{a^3}{\log b - \log a}$
- C.  $\frac{3\log a}{\log b - 1}$
- D.  $\frac{3\log a}{\log b - \log a}$

21. Determine the number of solutions of the system, where  $0 \leq x < 2\pi$ .

$$y = -7 \cos 2x$$

$$y = 1 + 2 \log(x - 3)$$

- A. 2
- B. 3
- C. 4
- D. 5

**OVER**

22. Determine the number of terms in the series:  $\sum_{k=21}^{63} 3(2)^{k-1}$

- A. 21
- B. 42
- C. 43
- D. 63

23. The recursive formula for a sequence is:

$$t_1 = 3$$

$$t_n = t_{n-1} + 4^n, \quad n > 1$$

Determine  $t_2$ .

- A. 7
- B. 11
- C. 19
- D. 49

24. Find the 50<sup>th</sup> term of the arithmetic sequence  $-6, -2, 2, \dots$

- A. 186
- B. 190
- C. 194
- D. 198

25. The first term of a geometric sequence is 3. If the sum of the second and third terms is 36, determine  $r$ .

- A. -10
- B. 10
- C. -3, 4
- D. -4, 3

26. Bob worked for a company for 18 years. In the first year, he earned \$23 000. He received an annual raise of \$2 000 each year. Determine the total amount of money that Bob earned during his time with the company.
- A. \$ 57 000
  - B. \$720 000
  - C. \$738 000
  - D. \$779 000
27. If the sum of  $t_6$  and  $t_8$  in an arithmetic sequence is  $m$ , determine  $t_7$ .
- A.  $\frac{m}{2}$
  - B.  $\frac{m}{7}$
  - C.  $\frac{m}{14}$
  - D.  $\pm\sqrt{m}$
28. An infinite geometric series has a finite sum and a common ratio  $r = \frac{1}{x-1}$ . Which of the following could not be a value for  $x$  ?
- A. -0.6
  - B. -0.5
  - C. 2
  - D. 3
29. Convert  $\frac{2\pi}{7}$  radians to degrees.
- A.  $0.02^\circ$
  - B.  $0.90^\circ$
  - C.  $51.43^\circ$
  - D.  $102.86^\circ$

30. Determine the amplitude of  $y = -2 \sin 3(x - 5)$ .

- A. -2
- B. 2
- C. 3
- D. 5

31. Determine the phase shift of  $y = -4 \sin(2x - 6)$ .

- A. 2 to the right
- B. 3 to the right
- C. 4 to the right
- D. 6 to the right

32. Solve for  $x$ :  $\cot x = -2$ ,  $0 \leq x < 2\pi$

- A. 1.11, 4.25
- B. 2.60, 5.74
- C. 2.68, 5.82
- D. 3.61, 5.82

33. Simplify:  $\cos 7x \cos 5x - \sin 7x \sin 5x$

- A.  $\cos 2x$
- B.  $\sin 2x$
- C.  $\cos 12x$
- D.  $\sin 12x$

34. Simplify:  $\frac{\cos x}{\cos x - \cos^3 x}$

- A.  $\csc^2 x$
- B.  $\sec^2 x$
- C.  $\tan^2 x$
- D.  $-\sec^3 x$



35. Simplify:  $\frac{3 - 6 \sin^2 x}{2 \sin x \cos x}$

A.  $3 \cot 2x$

B.  $3 \tan 2x$

C.  $-\frac{3}{2} \tan x$

D.  $\frac{3}{2} \cot x$

36. Which equation represents the same graph as  $y = 3 \cos\left(x - \frac{\pi}{4}\right)$  ?

A.  $y = 3 \sin\left(x - \frac{\pi}{4}\right)$

B.  $y = 3 \sin\left(x + \frac{3\pi}{4}\right)$

C.  $y = -3 \sin\left(x + \frac{\pi}{4}\right)$

D.  $y = -3 \sin\left(x - \frac{3\pi}{4}\right)$

37. Evaluate:  $\sum_{k=1}^5 (k + \cos k\pi)$

A. 5

B. 12

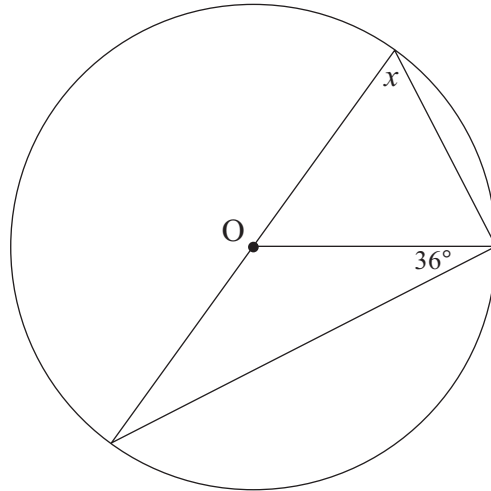
C. 14

D. 15

**OVER**

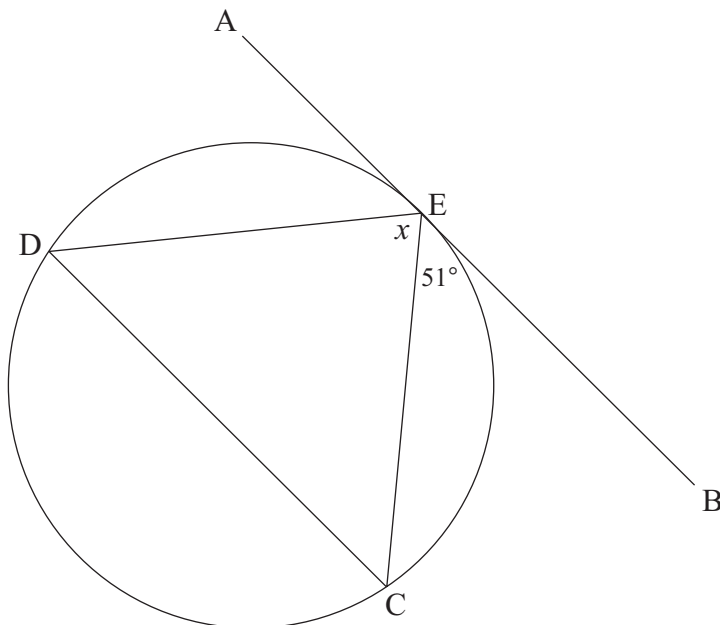
For questions 38 to 41, diagrams are not drawn to scale.

38. In the diagram below, O is the centre of the circle. Determine the measure of  $\angle x$ .



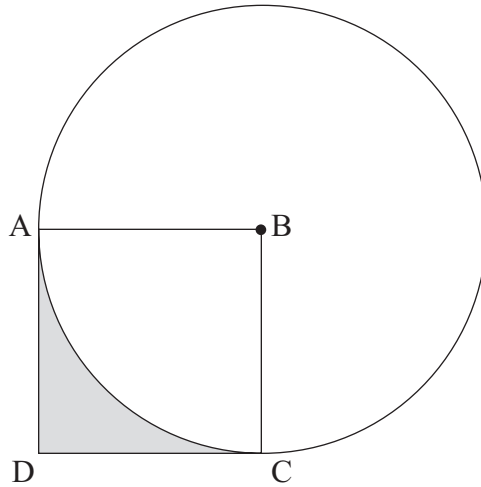
- A.  $36^\circ$
- B.  $45^\circ$
- C.  $54^\circ$
- D.  $72^\circ$

39. In the diagram below,  $AB \parallel DC$  and AB is tangent to the circle at E. Determine the measure of  $\angle x$ .

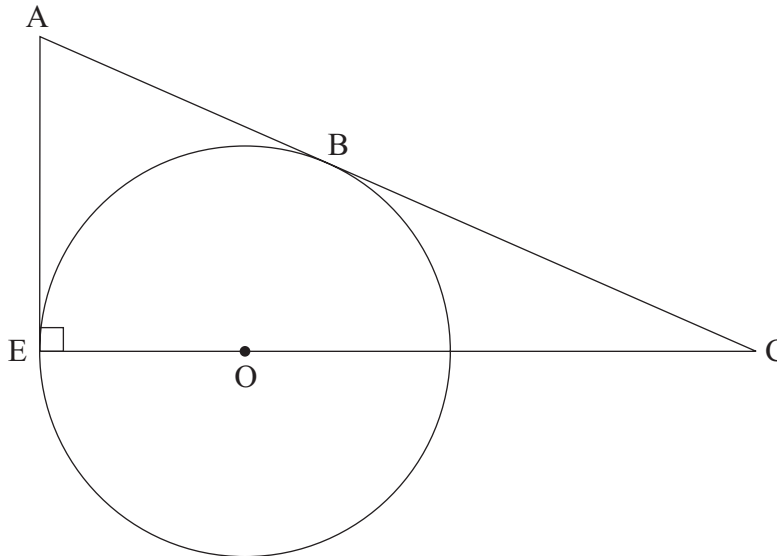


- A.  $78^\circ$
- B.  $83^\circ$
- C.  $90^\circ$
- D.  $102^\circ$

40. In the diagram below, the circle has a centre B and an area of  $40 \text{ cm}^2$ . If ABCD is a square, determine the shaded area.



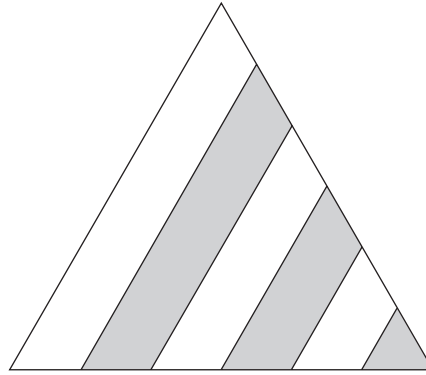
- A.  $2.73 \text{ cm}^2$   
 B.  $2.83 \text{ cm}^2$   
 C.  $3.57 \text{ cm}^2$   
 D.  $4.27 \text{ cm}^2$
41. In the diagram below, O is the centre of the circle with radius 11 cm and AC is tangent to the circle at B. If  $OC = 29 \text{ cm}$  and  $AE \perp EC$ , determine the length of AC.



- A. 37.01 cm  
 B. 37.40 cm  
 C. 42.78 cm  
 D. 43.23 cm

**OVER**

42. In the diagram below, what fraction of the entire figure is shaded? All bands are of equal width.



- A.  $\frac{5}{12}$   
B.  $\frac{1}{2}$   
C.  $\frac{7}{12}$   
D.  $\frac{5}{7}$

43. Solve:  $|x + 3| = 2|x - 3|$

- A.  $x = 9$   
B.  $x = 1, 9$   
C.  $x = 0, 1, 9$   
D.  $x = 1, 4, 9, 12$

44.  $\sin^{-1}(x)$  is defined to be the number between  $-\frac{\pi}{2}$  and  $\frac{\pi}{2}$  (inclusive) whose sine is  $x$ .

Determine an algebraic expression for  $\cos(\sin^{-1}(x))$ ,  $0 < x < 1$ .

- A.  $\sqrt{1 - x^2}$   
B.  $\sqrt{1 + x^2}$   
C.  $\sqrt{x^2 - 1}$   
D.  $\frac{x}{\sqrt{1 - x^2}}$

**This is the end of the multiple-choice section.  
Answer the remaining questions directly in this examination booklet.**

## PART B: WRITTEN RESPONSE

Value: 34 marks

Suggested Time: 45 minutes

**INSTRUCTIONS:** Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

**Full marks will NOT be given for the final answer only.**

1. If 2 is a root of the polynomial equation  $6x^3 + kx^2 + x + 2 = 0$ , determine the other roots.

**(4 marks)**

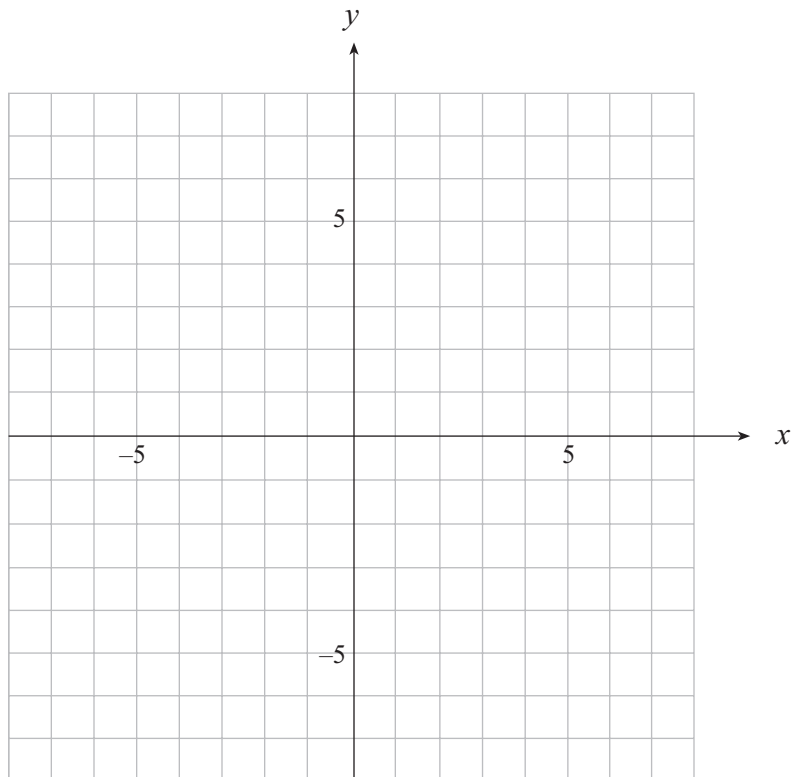
ANSWER:

2. Graph the solution of the following system of inequalities on the grid provided.

**(4 marks)**

$$x^2 - (y - 1)^2 \geq 4$$

$$y > (x - 3)^2 + 1$$







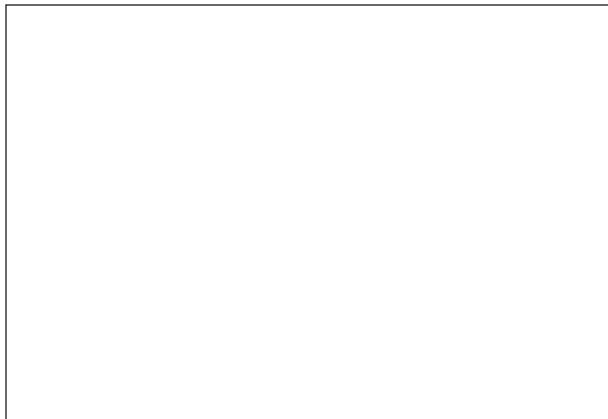
3. Solve the following system using a graphing calculator.

(4 marks)

$$y = 5 \log_3(x + 5)$$

$$y = x - 1$$

Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the function(s) and all intersection points are visible.



$Y_1 =$

$Y_2 =$

$Y_3 =$

$Y_4 =$

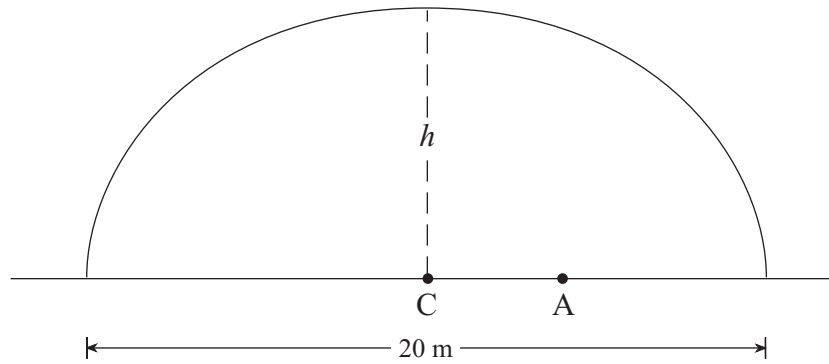
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$x$   
min             $x$   
max

$y$   
min             $y$   
max

ANSWER:

4. An arch is semi-elliptical in shape with a maximum width of 20 m, as shown in the diagram. Point A is 4 m from the centre, C. If the height of the arch at point A is 7 m, determine the maximum height,  $h$ , of the arch. **(5 marks)**



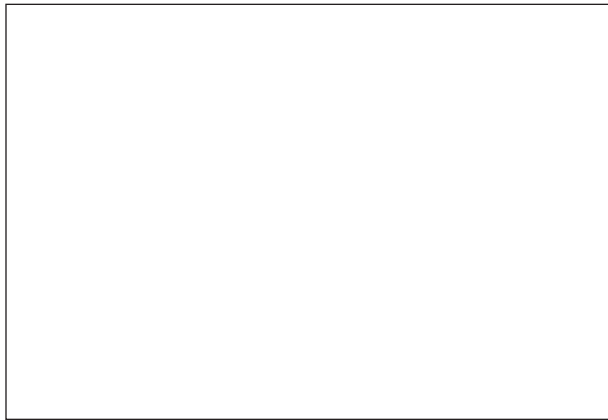
ANSWER:

5. Solve the following equation using a graphing calculator:

(4 marks)

$$5 \cos(x - 1) = \cot x, \quad 0 \leq x < 2\pi$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that all intersection points or zeros are visible within the viewing window.



$Y_1 =$

$Y_2 =$

$Y_3 =$

$Y_4 =$

[            ,            ]

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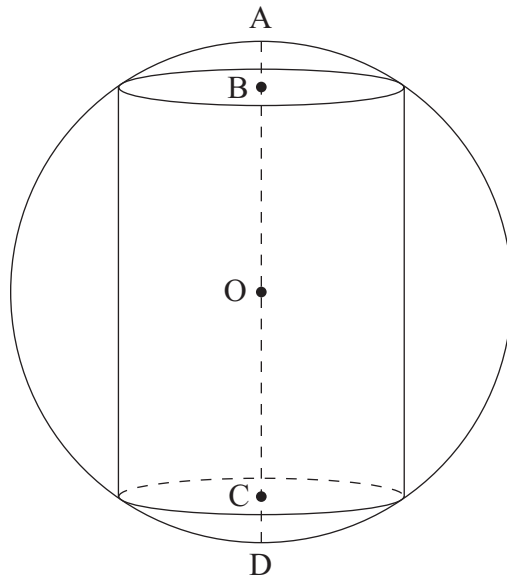
$x$   
min         $x$   
max

$y$   
min         $y$   
max

ANSWER:

6. A sphere with centre  $O$  has a volume of  $36\pi \text{ cm}^3$ . A cylinder is inscribed in the sphere so that  $AB = CD = 1 \text{ cm}$ , as shown in the diagram. If the diameter  $AD$  passes through the centre of the cylinder, determine the volume of the cylinder. **(4 marks)**

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3 \quad V_{\text{cylinder}} = \pi r^2 h$$





ANSWER:

7. State the restrictions on  $x$  and  $y$  in the following equation, then express  $y$  as a polynomial function of  $x$ . **(4 marks)**

$$\frac{1}{\log_y 7} = \log_7 4 - \log_7 \left( \frac{1}{x^2} \right)$$

ANSWER:

**Students must choose one or the other method of proof.**

8. Complete the proof.

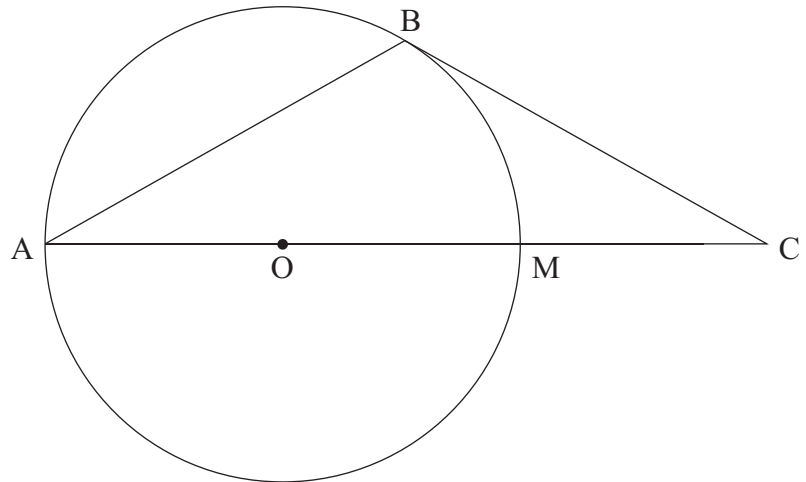
**(5 marks)**

Diagram clarification:  $O$  is the centre of the circle  
 $A, O, M, C$  are collinear

Given:  $AB = BC$   
 $BC$  is tangent to the circle

Prove:  $AM = OC$

**Note:** Students are encouraged to number angles.



**Paragraph proof method:**

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**Two-column proof method:**

STATEMENT	REASON

**END OF EXAMINATION**

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## A SUMMARY OF BASIC IDENTITIES AND FORMULAE

### Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

### Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

### Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

### Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

### Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n - 1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_n = \frac{a - \ell r}{1 - r}$$

$$S = \frac{a}{1 - r}$$

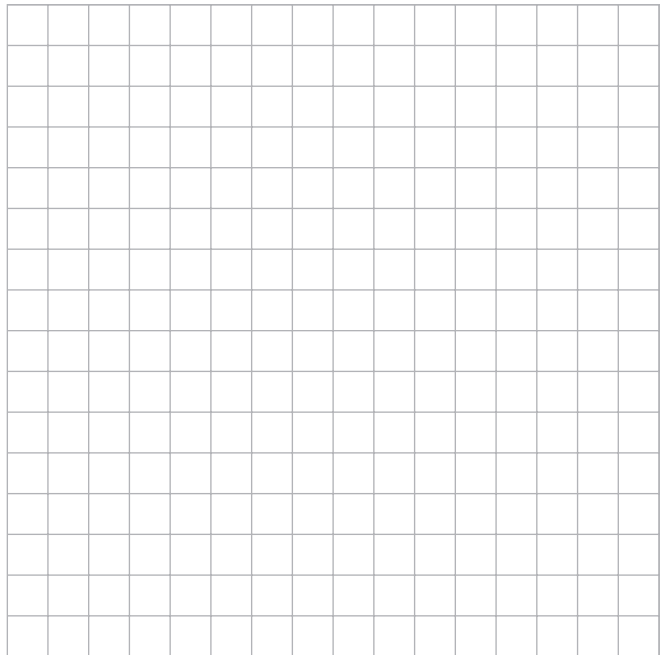
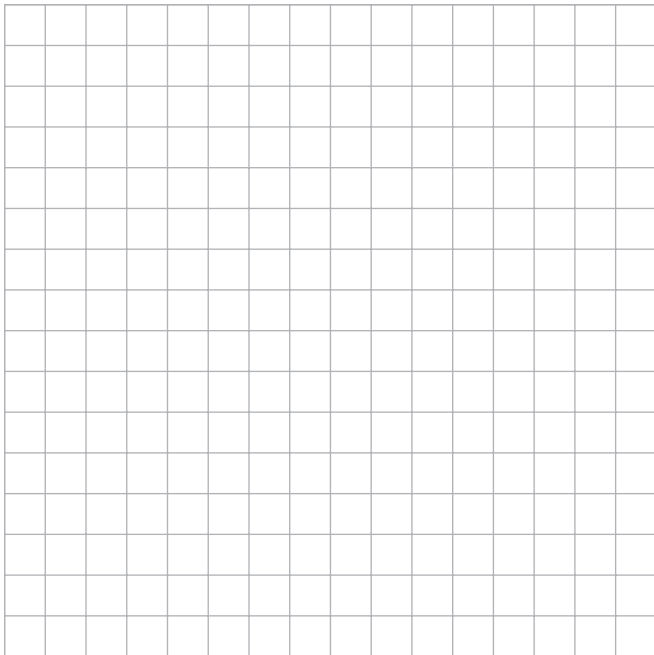
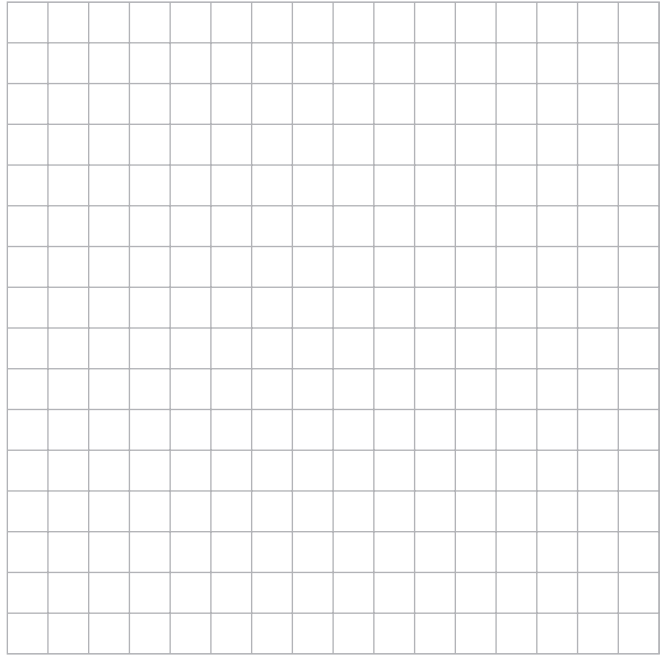
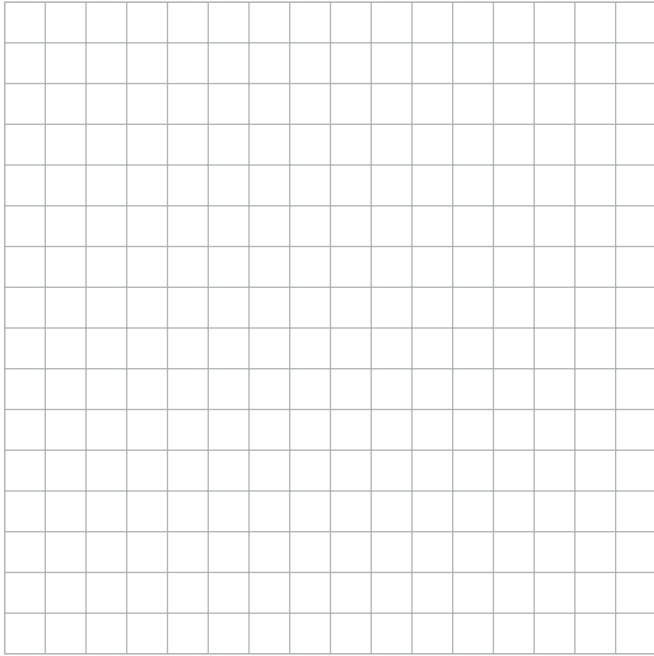
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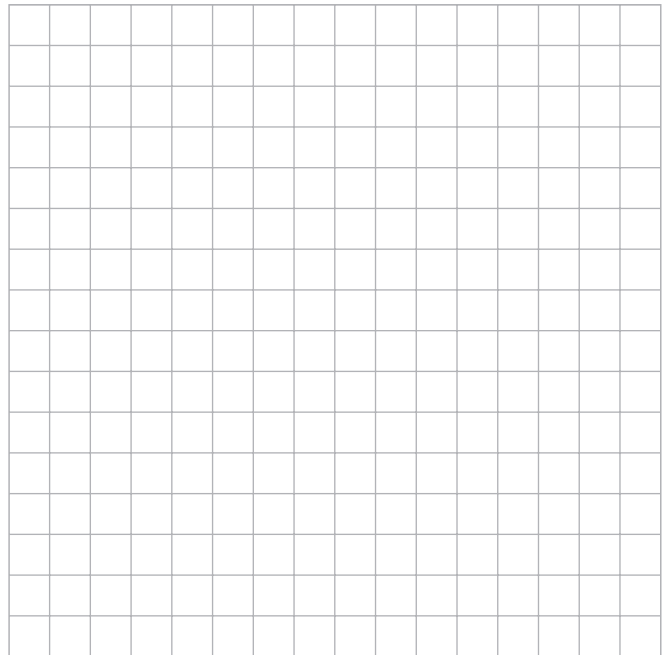
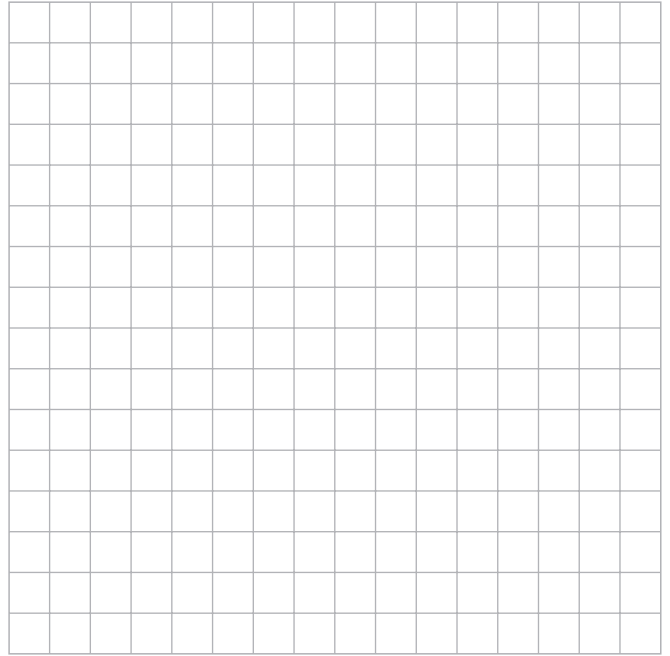
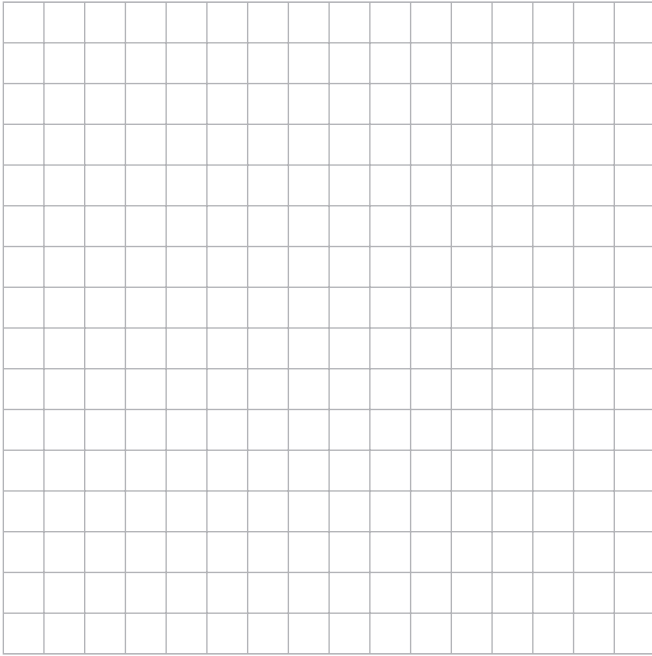
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## ROUGH WORK FOR MULTIPLE-CHOICE