

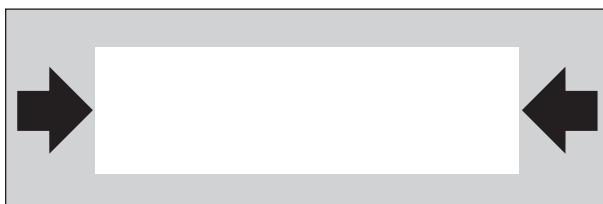
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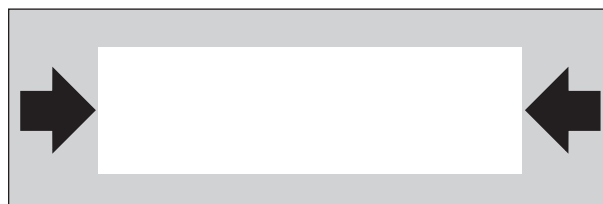
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## Principles of Mathematics 12

**JANUARY 2001**

**Course Code = MA**

### Student Instructions

1. Place the stickers with your Personal Education Number (PEN) in the allotted spaces above. **Under no circumstance is your name or identification, other than your Personal Education Number, to appear on this booklet.**
2. Ensure that in addition to this examination booklet, you have an **Examination Response Form**. Follow the directions on the front of the Response Form.
3. **Disqualification** from the examination will result if you bring books, paper, notes or unauthorized electronic devices into the examination room.
4. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by **END OF EXAMINATION**.
5. At the end of the examination, place your Response Form inside the front cover of this booklet and return the booklet and your Response Form to the supervisor.

Question 1:

1.  .

(4)

Question 2:

2.  .

(5)

Question 3:

3.  .

(4)

Question 4:

4.  .

(4)

Question 5:

5.  .

(4)

Question 6:

6.  .

(4)

Question 7:

7.  .

(4)

Question 8:

8.  .

(5)



**PRINCIPLES OF  
MATHEMATICS 12**

**JANUARY 2001**

COURSE CODE = MA

## GENERAL INSTRUCTIONS

1. Aside from an approved calculator, electronic devices, including dictionaries and pagers, are **not** permitted in the examination room.
2. All multiple-choice answers must be entered on the Response Form using an **HB pencil**. Multiple-choice answers entered in this examination booklet will **not** be marked.
3. For each of the written-response questions, write your answer in the space provided in this booklet.

You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.

4. Ensure that you use language and content appropriate to the purpose and audience of this examination. Failure to comply may result in your paper being awarded a zero.
5. This examination is designed to be completed in **two hours**. *Students may, however, take up to 30 minutes of additional time to finish.*

## PRINCIPLES OF MATHEMATICS 12 PROVINCIAL EXAMINATION

- |   | Value                   | Suggested Time     |
|---|-------------------------|--------------------|
| 1. This examination consists of <b>two</b> parts: |                         |                    |
| PART A: 44 multiple-choice questions              | 66                      | 75                 |
| PART B: 8 written-response questions              | 34                      | 45                 |
|   | <b>Total: 100 marks</b> | <b>120 minutes</b> |
2. The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae, Rough Work for Graphing, and Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.
3. **A graphing calculator is essential for the Principles of Mathematics 12 Provincial Examination.** The calculator must be a hand-held device designed primarily for mathematical computations involving logarithmic and trigonometric functions as well as for graphing functions. Computers, calculators with a QWERTY keyboard or symbolic manipulation abilities, and electronic writing pads will not be allowed. Students must not bring any external devices (peripherals) to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, CD-ROMs, libraries or external keyboards. Students may have more than one calculator available during the examination, of which one may be a scientific calculator. Calculators may not be shared and must not have the ability to either transmit or receive electronic signals. In addition to an approved calculator, students will be allowed to use rulers, compasses, and protractors during the examination.
- Calculators must not have any information programmed into the memory which would not be acceptable in paper form. Specifically, calculators must not have any built-in notes, definitions, or libraries. There is no requirement to clear memories at the beginning of the examination but the use of calculators with built-in notes is equivalent to the use of notes in paper form. Any student deemed to have cheated on a provincial examination will receive a “0” on that examination and will be permanently disqualified from the Provincial Examination Scholarship Program.
4. If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.
5. When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

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**PART A: MULTIPLE CHOICE**

**Value: 66 marks**

**Suggested Time: 75 minutes**

**INSTRUCTIONS:** For each question, select the **best** answer and record your choice on the Response Form provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. Which expression represents the remainder when the polynomial  $P(x)$  is divided by  $x - 9$  ?
  - A.  $P(9)$
  - B.  $P(-9)$
  - C.  $P(0)$
  - D.  $P(x - 9)$
  
2. According to the Rational Root Theorem, which of the following is a possible root of the equation  $5x^3 + mx^2 + nx + 20 = 0$ , where  $m$  and  $n$  are integers?
  - A.  $\frac{1}{10}$
  - B.  $\frac{1}{5}$
  - C.  $\frac{1}{4}$
  - D.  $\frac{1}{2}$
  
3. Determine the quotient when  $x^4 - 8x^2 + 2x - 7$  is divided by  $x + 3$ .
  - A.  $x^2 - 5x - 13$
  - B.  $x^2 - 11x + 35$
  - C.  $x^3 - 3x^2 + x - 1$
  - D.  $x^3 + 3x^2 + x + 5$
  
4. Determine the value of  $k$  if  $x - 2$  is a factor of the polynomial  $x^3 - 4x^2 + kx + 6$ .
  - A.  $-9$
  - B.  $-1$
  - C.  $1$
  - D.  $9$

5. Solve  $(x+a)^2(x+b)(x+c) < 0$ , where  $a, b, c$  are real number constants and  $0 < a < b < c$ .

- A.  $b < x < c$
- B.  $-b < x < -c$
- C.  $-c < x < -b$
- D.  $-b < x < -a, x < -c$

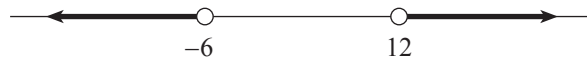
6. Which conic is represented by  $2x^2 + 3y^2 + 4x - 12y - 10 = 0$  ?

- A. circle
- B. ellipse
- C. parabola
- D. hyperbola

7. Determine the midpoint of the line segment joining the points  $(-8, 4)$  and  $(4, 6)$ .

- A.  $(-2, 5)$
- B.  $(2, -5)$
- C.  $(6, 1)$
- D.  $(-6, -1)$

8. Which absolute value inequality describes the solution shown?



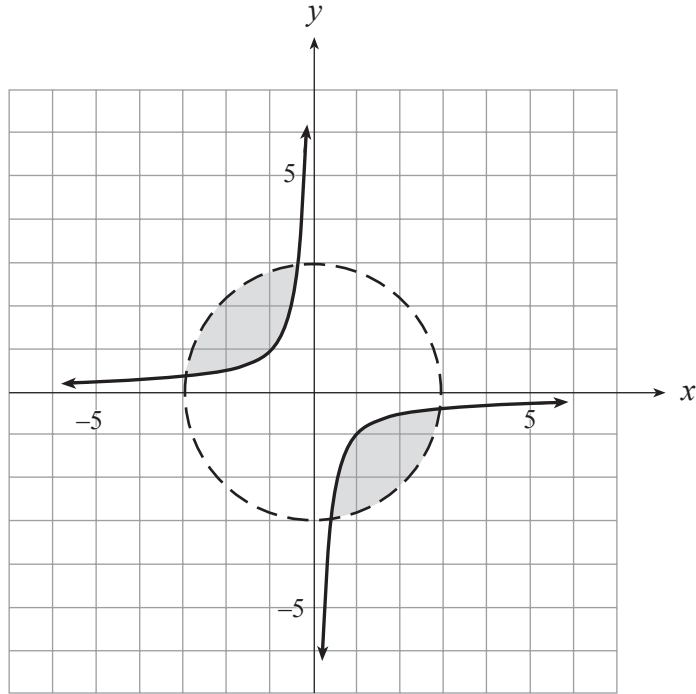
- A.  $|x - 3| < 9$
- B.  $|x - 3| > 9$
- C.  $|x - 9| < 3$
- D.  $|x - 9| > 3$

9. Determine the vertex of the parabola  $x = 4(y - 3)^2 + 2$ .

- A.  $(2, 3)$
- B.  $(2, -3)$
- C.  $(3, -2)$
- D.  $(3, 2)$



10. The shaded region below is the solution of which of the following systems?



A.  $x^2 + y^2 < 9$   
 $xy \leq -1$

B.  $x^2 + y^2 < 9$   
 $xy \geq -1$

C.  $x^2 + y^2 < 9$   
 $xy \leq 1$

D.  $x^2 + y^2 < 9$   
 $xy \geq 1$

11. Solve the following system for  $y$  only.

$$x^2 - y^2 = 4$$

$$x^2 + y = 6$$

- A. 1
- B.  $\pm 1$
- C. -2, 1
- D. -1, 2

**OVER**

12. The point  $P(x, y)$  moves such that it is equidistant from the point  $(-3, 2)$  and the line  $x = 4$ . Which equation represents this locus?

A.  $\sqrt{(x+3)^2 + (y-2)^2} = \sqrt{(x-0)^2 + (y-y)^2}$

B.  $\sqrt{(x+3)^2 + (y-2)^2} = \sqrt{(x-x)^2 + (y-4)^2}$

C.  $\sqrt{(x+3)^2 + (y-2)^2} = \sqrt{(x-4)^2 + (y-0)^2}$

D.  $\sqrt{(x+3)^2 + (y-2)^2} = \sqrt{(x-4)^2 + (y-y)^2}$

13. Which system could be used to solve the following problem?

A rectangle has length  $\ell$ , width  $w$  and the length of a diagonal is 20 cm. If the perimeter of the rectangle is 60 cm, determine the length and the width of the rectangle.

A.  $\ell^2 + w^2 = 400$   
 $\ell + w = 60$

B.  $\ell w = 400$   
 $\ell + w = 60$

C.  $\ell w = 400$   
 $2\ell + 2w = 60$

D.  $\ell^2 + w^2 = 400$   
 $2\ell + 2w = 60$

14. The vertices of a hyperbola are  $(-4, 3)$  and  $(4, 3)$ , and the slopes of its asymptotes are  $\pm \frac{1}{2}$ . Determine an equation of the hyperbola.

A.  $\frac{x^2}{4} - \frac{(y-3)^2}{2} = 1$

B.  $\frac{x^2}{4} - \frac{(y-3)^2}{16} = 1$

C.  $\frac{x^2}{16} - \frac{(y-3)^2}{4} = 1$

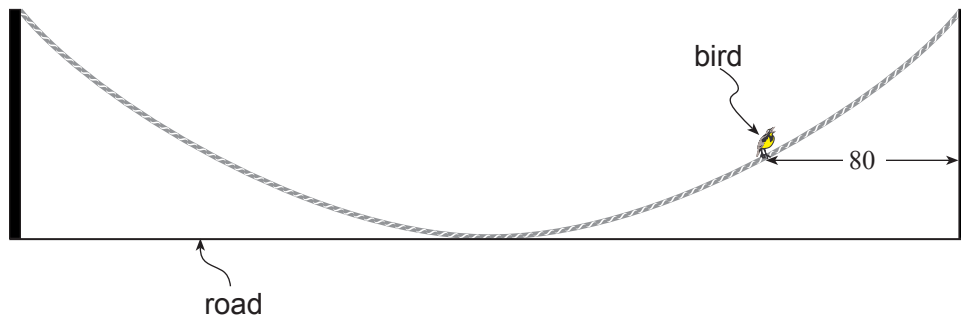
D.  $\frac{x^2}{16} - \frac{(y-3)^2}{8} = 1$

15. Determine all values of  $k$ ,  $k > 0$ , such that the following system has exactly 2 different real solutions.

$$x^2 + (y - 8)^2 = 25$$

$$y = x^2 + k$$

- A.  $0 < k < 3$   
B.  $0 < k < 13$   
C.  $4 \leq k \leq 12$   
D.  $3 < k < 13$
16. A suspension bridge has a cable in the shape of a parabola, as shown in the diagram below. The road passes through the vertex. The supporting towers are 60 metres high and 400 metres apart. A bird sits on the cable at a horizontal distance of 80 metres from a supporting tower. Determine the height of the bird above the road.



- A. 9.6 m  
B. 21.6 m  
C. 24.0 m  
D. 36.0 m

17. Solve for  $x$ :  $\log_3 x = \log_3 5 + \log_3 7$

- A. 12
- B. 35
- C.  $\log_3 12$
- D.  $\log_3 35$

18. Express  $\frac{1}{2} \log a - \log b$  as a single logarithm.

- A.  $\log \frac{a}{2b}$
- B.  $\log \frac{ab}{2}$
- C.  $\log b\sqrt{a}$
- D.  $\log \frac{\sqrt{a}}{b}$

19. Give the domain of the function  $y = \log_4(x + 7) - 3$ .

- A.  $x > -7$
- B.  $x > -3$
- C.  $x > 3$
- D.  $x > 7$

20. If  $f(x) = 3 \log x$ , determine  $f^{-1}(x)$ , the inverse of  $f(x)$ .

- A.  $f^{-1}(x) = \frac{1}{3 \log x}$
- B.  $f^{-1}(x) = \frac{1}{3} \log \frac{1}{x}$
- C.  $f^{-1}(x) = 10^{\frac{x}{3}}$
- D.  $f^{-1}(x) = 10^{x-3}$

21. A culture has 300 bacteria. The number of bacteria doubles every 4 hours. How long will it take for the number of bacteria to reach 72 000 ?
- A. 31.63 hours
  - B. 33.27 hours
  - C. 36.88 hours
  - D. 60 hours

22. Determine the number of solutions for the following system:

$$y = -4 \log_{12} x$$

$$y = 4 \sin x$$

- A. 2
  - B. 3
  - C. 4
  - D. 5
23. Simplify:  $2^{\log_8 x^{27}}$

- A.  $3x$
- B.  $9x$
- C.  $x^3$
- D.  $x^9$

24. Determine the 200<sup>th</sup> term of the arithmetic sequence  $-12, -2, 8, \dots$

- A. 1 580
- B. 1 604
- C. 1 978
- D. 2 002

25. Determine the sum of the infinite geometric series:  $100 - 60 + 36 - \dots$

- A. 62.5
- B. 76
- C. 160
- D. 250

26. Determine two positive geometric means between 16 and 800.

- A. 42.55, 113.14
- B. 58.94, 217.15
- C. 212, 408
- D. 277.33, 538.67

27. Which expression represents the sum of the series  $\sum_{k=1}^n 2(3)^k$  ?

- A.  $-2(1-3^n)$
- B.  $-3(1-3^n)$
- C.  $-6(1-3^n)$
- D.  $-2(1-3^{n-1})$

28. If the sum of  $n$  terms of a series is given by  $S_n = n^2 + 4n$ , find a formula for  $t_n$ .

- A.  $t_n = 3n + 2$
- B.  $t_n = 2n + 3$
- C.  $t_n = 2n - 3$
- D.  $t_n = 2n + 5$

29. Convert  $210^\circ$  to radians.

- A.  $\frac{7\pi}{12}$
- B.  $\frac{6\pi}{7}$
- C.  $\frac{7\pi}{6}$
- D.  $\frac{12\pi}{7}$

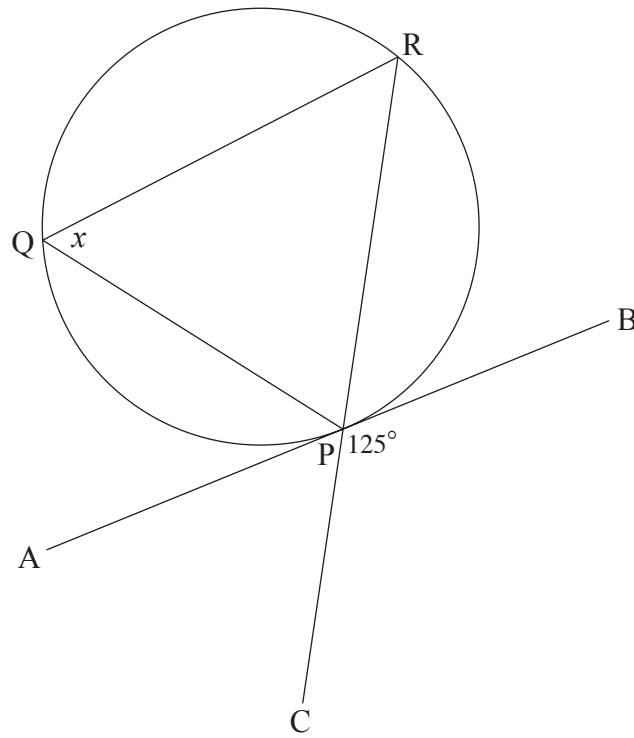
30. Determine the phase shift of the function  $y = 4 \cos 2\left(x - \frac{\pi}{4}\right) + 5$ .
- A.  $\frac{\pi}{4}$  to the right
  - B.  $\frac{\pi}{2}$  to the right
  - C.  $\frac{\pi}{4}$  to the left
  - D.  $\frac{\pi}{2}$  to the left
31. Solve:  $\tan x = 3.2$  ,  $0 \leq x < 2\pi$
- A. 0.06, 3.20
  - B. 1.27, 1.87
  - C. 1.27, 4.41
  - D. 1.87, 5.02
32. If the point  $(-7, -24)$  is on the terminal arm of angle  $\theta$  in standard position, determine the value of  $\csc \theta$ .
- A.  $-\frac{25}{7}$
  - B.  $-\frac{25}{24}$
  - C.  $\frac{7}{25}$
  - D.  $\frac{24}{25}$
33. Which expression is equivalent to  $\frac{\cot \theta \sin \theta}{\sec \theta}$  ?
- A.  $\sin^2 \theta$
  - B.  $\cos^2 \theta$
  - C.  $\sec^2 \theta$
  - D.  $\csc^2 \theta$

34. Solve:  $3 \tan\left(\frac{1}{2}x - 2\right) = 4 \sin 2x$ ,  $0 \leq x < 2\pi$
- A. 1.48, 5.28, 5.94  
B. 1.64, 3.56, 3.84  
C. 2.20, 2.90, 4.60  
D. 3.28, 4.90, 5.74
35. Which of the following is equivalent to  $\cos(2\theta + \pi)$  ?
- A.  $2 \sin \theta \cos \theta$   
B.  $-2 \sin \theta \cos \theta$   
C.  $1 - 2 \sin^2 \theta$   
D.  $2 \sin^2 \theta - 1$
36. Determine the amplitude of the function  $y = k \sin \theta \cos \theta$ , where  $k$  is a positive constant.
- A.  $\frac{k}{2}$   
B.  $k$   
C.  $2k$   
D.  $4k$
37. A sine function has a maximum point at  $(4, 32)$  and the nearest minimum point to the right is  $(16, 18)$ . Determine an equation for this function.
- A.  $y = 7 \sin \frac{\pi}{6}(x - 4) + 25$   
B.  $y = 7 \sin \frac{\pi}{6}(x + 4) + 25$   
C.  $y = 7 \sin \frac{\pi}{12}(x - 2) + 25$   
D.  $y = 7 \sin \frac{\pi}{12}(x + 2) + 25$



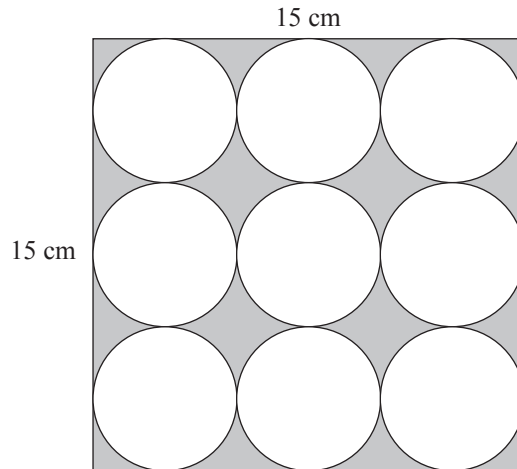
For questions 38 to 41, diagrams are not drawn to scale.

38. In the diagram below, AB is tangent to the circle at P. If  $\angle BPC = 125^\circ$ , determine the measure of  $\angle x$ .



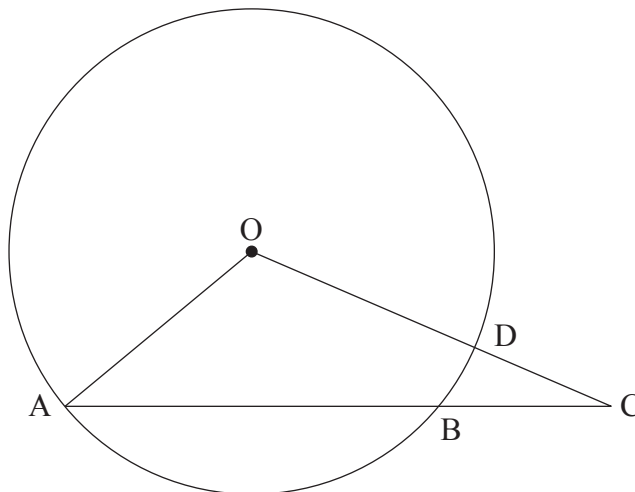
- A.  $45^\circ$
- B.  $55^\circ$
- C.  $65^\circ$
- D.  $75^\circ$

39. Nine identical circles are drawn in a square of side 15 cm, as shown in the diagram. Determine the shaded area.



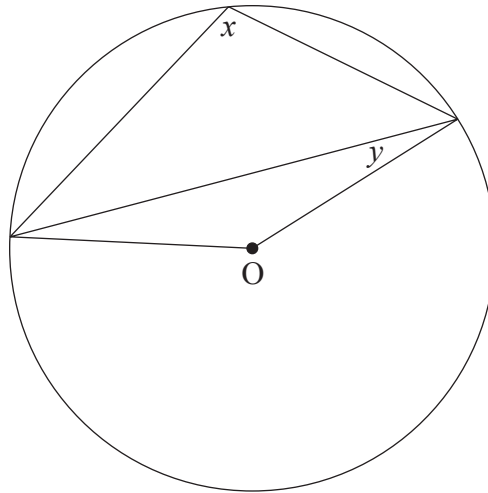
- A.  $48.29 \text{ cm}^2$   
 B.  $59.71 \text{ cm}^2$   
 C.  $83.63 \text{ cm}^2$   
 D.  $112.50 \text{ cm}^2$
40. In the diagram below, the circle with centre O has a radius of 12. If  $AC = 30$  and  $AB = 20$ , determine the length of segment OC.

**Note:** A, B, C are collinear  
 O, D, C are collinear



- A. 20  
 B. 21.07  
 C. 21.42  
 D. 22

41. In the diagram below, the circle has centre O. Determine an equation expressing the measure of  $\angle y$  in terms of  $\angle x$ .



- A.  $y = \frac{1}{2}x$
- B.  $y = 2x - 180^\circ$
- C.  $y = 90^\circ - x$
- D.  $y = x - 90^\circ$
42. Male bees have a mother but no father. Female bees have both a mother and a father. How many ancestors does a female bee have in the 5<sup>th</sup> generation back?
- A. 8
- B. 12
- C. 13
- D. 15

43. Determine the exact value of  $\tan\left(\sin^{-1}\left(\frac{2}{x}\right)\right)$ ,  $x > 2$ .

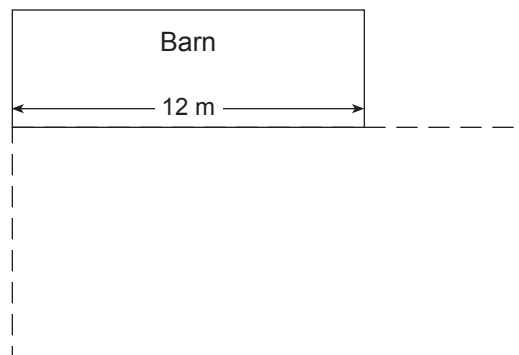
A.  $\frac{2}{x-2}$

B.  $\frac{x}{\sqrt{4-x^2}}$

C.  $\frac{2}{\sqrt{4-x^2}}$

D.  $\frac{2}{\sqrt{x^2-4}}$

44. A farmer has 80 metres of fencing to enclose a rectangular pen for his ducks. He uses his barn to form part of one side of the pen, as shown in the diagram. If the length of the barn is 12 metres, determine the maximum area of the pen.



A.  $408 \text{ m}^2$

B.  $480 \text{ m}^2$

C.  $529 \text{ m}^2$

D.  $576 \text{ m}^2$

**This is the end of the multiple-choice section.  
Answer the remaining questions directly in this examination booklet.**

## PART B: WRITTEN RESPONSE

Value: 34 marks

Suggested Time: 45 minutes

**INSTRUCTIONS:** Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

**Full marks will NOT be given for the final answer only.**

1. Solve the following equation using a graphing calculator.

(4 marks)

$$x^3 - 15x^2 = -10x - 30$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window.



$Y_1 =$

$Y_2 =$

$Y_3 =$

$Y_4 =$

[       ,       ]       [       ,       ]

$x$         $x$   
min       max

$y$         $y$   
min       max

ANSWER:

2. Change the following equation to standard form.

**(5 marks)**

$$4x^2 - 32x - 9y^2 - 36y - 116 = 0$$



ANSWER:



3. Find the sum of the arithmetic series:  $20 + 26 + 32 + \dots + 734$

**(4 marks)**

ANSWER:

4. Prove the identity:

(4 marks)

$$\frac{\cot \theta - 1}{1 - \tan \theta} = \frac{\csc \theta}{\sec \theta}$$

LEFT SIDE

RIGHT SIDE



5. Solve the following system using a graphing calculator. Express all solutions as ordered pairs.

(4 marks)

$$y = 5 \log(x + 4) + 2$$

$$y = 2^{x-1} - 5$$

Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the function(s) and all intersection points are visible.



Y<sub>1</sub> =

Y<sub>2</sub> =

Y<sub>3</sub> =

Y<sub>4</sub> =

[            ,            ]

[            ,            ]

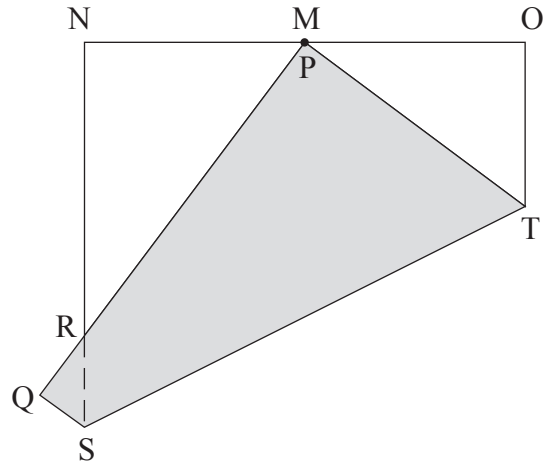
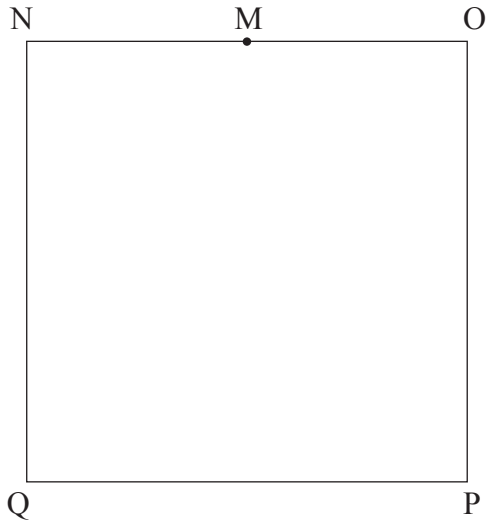
$x$   
min       $x$   
max

$y$   
min       $y$   
max

ANSWER:



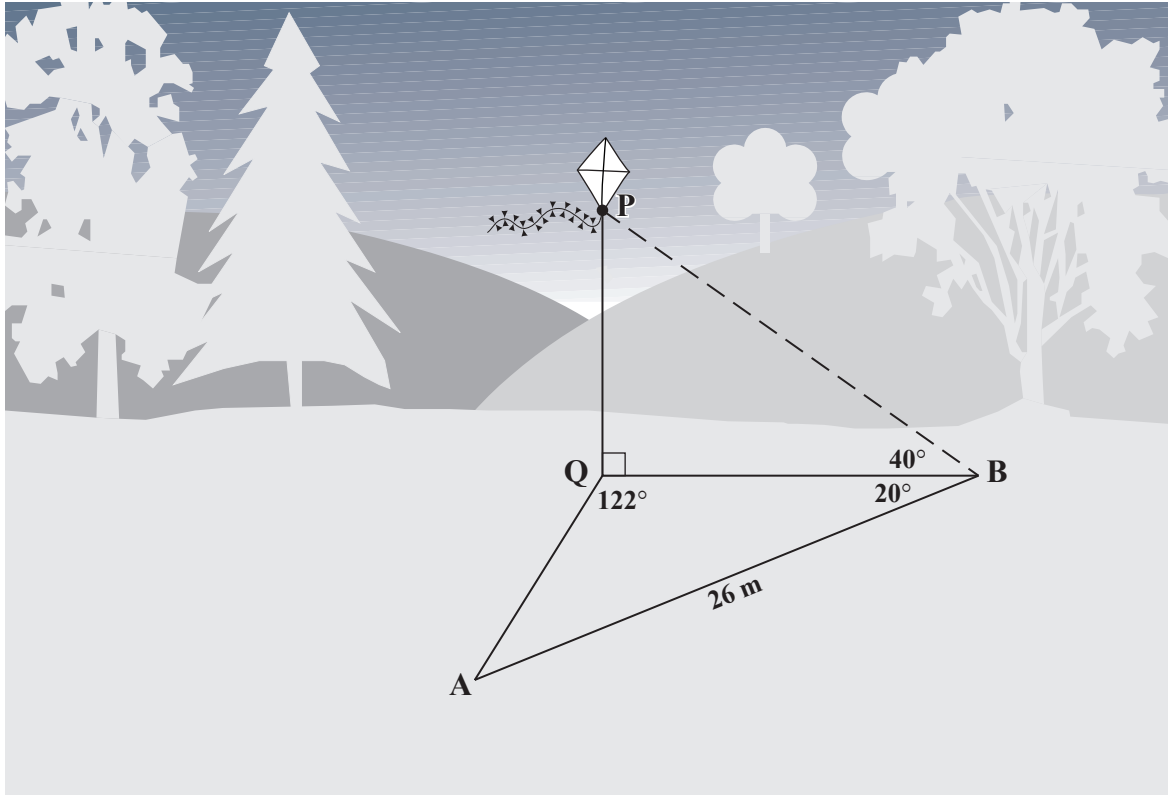
6. A square piece of paper  $24\text{ cm} \times 24\text{ cm}$  is folded in such a way that the lower right hand corner at P just touches the midpoint M of the top side, as shown in the diagram. In  $\triangle MNR$ , determine the length of side NR. **(4 marks)**





ANSWER:

7. A kite is flying at a point P vertically above a point Q which is in the same horizontal plane as two observers standing at A and B. The distance between the observers is 26 metres.  $\angle QBA = 20^\circ$  and  $\angle AQB = 122^\circ$ . The angle of elevation of the kite from the observer at B is  $40^\circ$ . How high is the kite flying above point Q? **(4 marks)**



ANSWER:



Students must choose one or the other method of proof.

8. Complete the proof.

(5 marks)

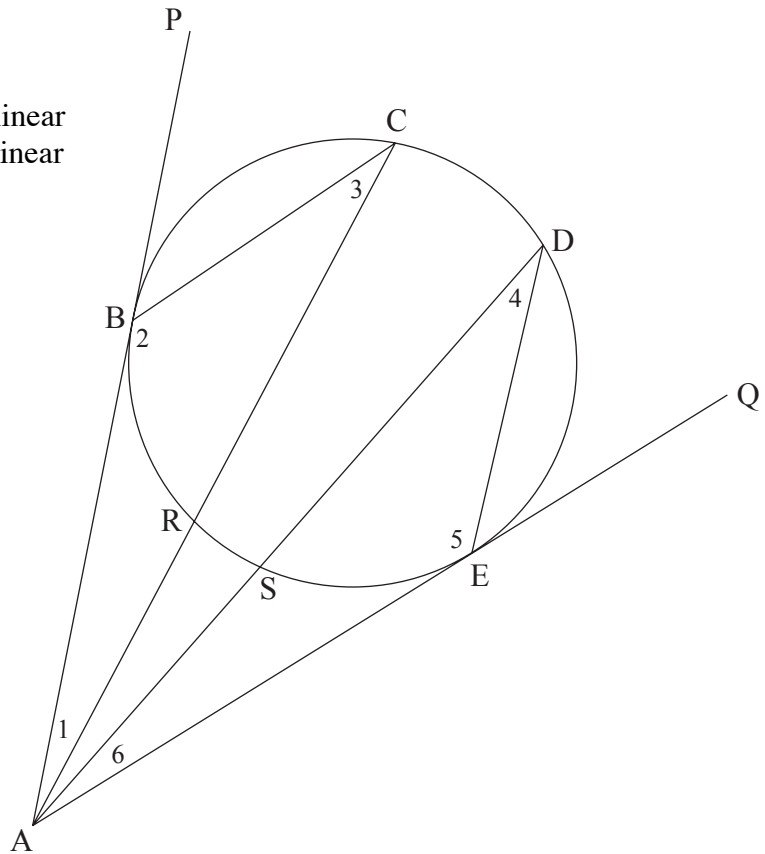
Diagram clarification: A, R, C are collinear  
A, S, D are collinear

Given: AP and AQ are tangents

$$\widehat{BR} = \widehat{SE}$$

$$\angle 2 = \angle 5$$

Prove:  $BC = ED$



Paragraph proof method:

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**Two-column proof method:**

STATEMENT	REASON

**END OF EXAMINATION**

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## A SUMMARY OF BASIC IDENTITIES AND FORMULAE

### Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

### Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

### Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

### Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

### Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_n = \frac{a - \ell r}{1 - r}$$

$$S = \frac{a}{1 - r}$$

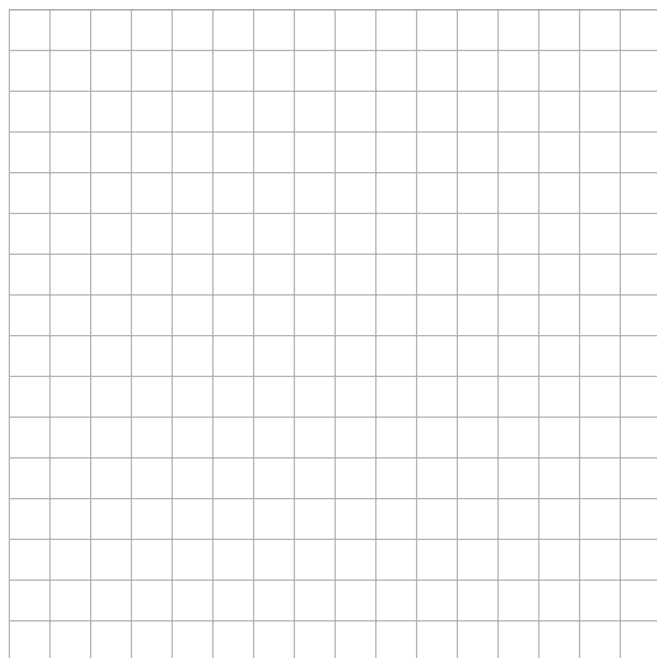
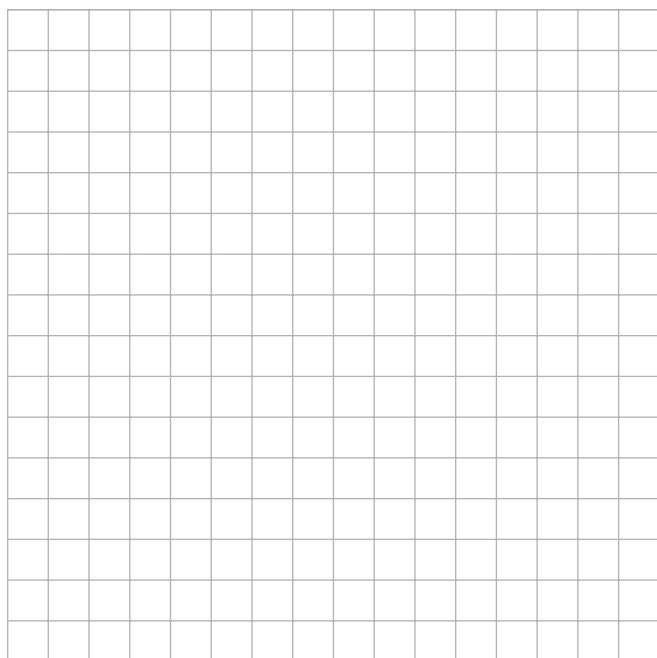
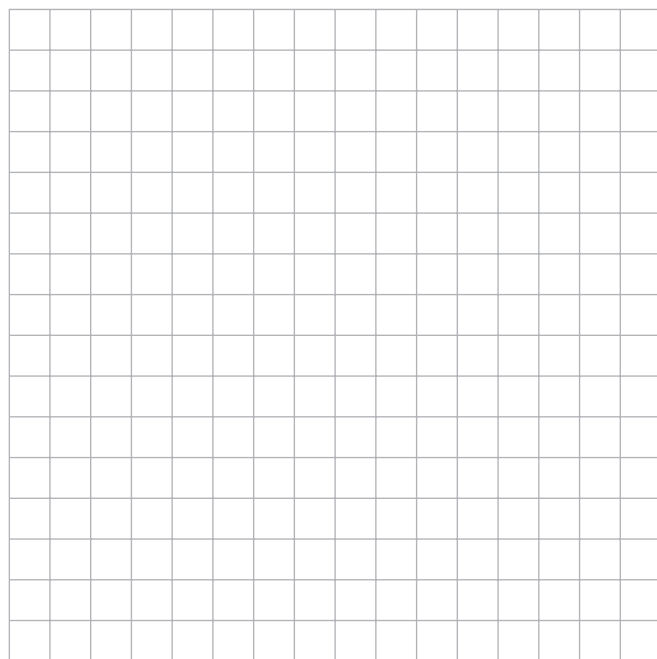
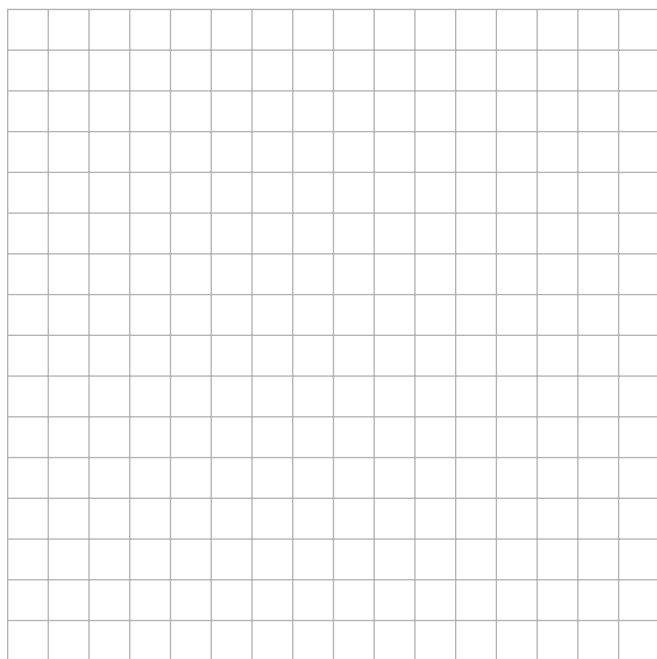
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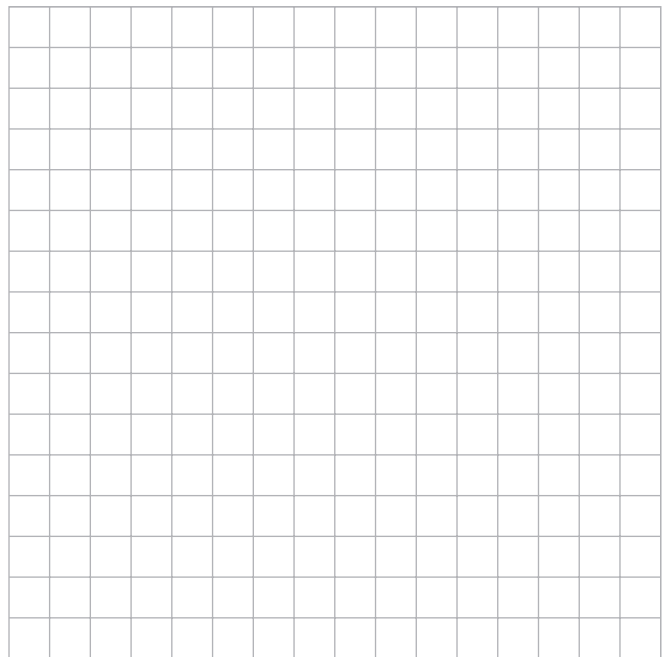
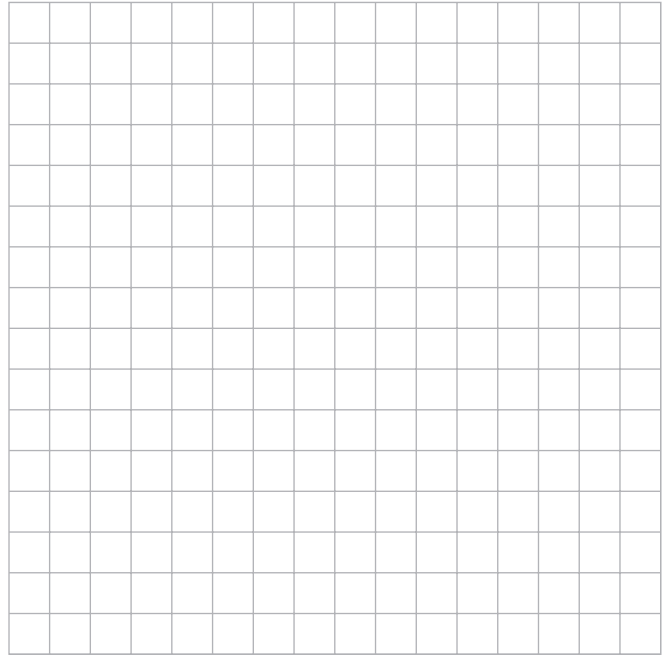
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**ROUGH WORK FOR GRAPHING**

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**ROUGH WORK FOR MULTIPLE-CHOICE**

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## ROUGH WORK FOR MULTIPLE-CHOICE