

Insert Personal Education Number (PEN) here.

Insert **only** pre-printed PEN label here.

STUDENT INSTRUCTIONS

1. Insert the stickers with your Personal Education Number (PEN) in the allotted spaces above. **Under no circumstance is your name or identification, other than your Personal Education Number, to appear on this booklet.**
2. Ensure that in addition to this examination booklet, you have an **Examination Response Form**. Follow the directions on the front of the Response Form.
3. **Disqualification** from the examination will result if you bring books, paper, notes or unauthorized electronic devices into the examination room.
4. When instructed to open this booklet, **check the numbering of the pages** to ensure that they are numbered in sequence from page one to the last page, which is identified by

END OF EXAMINATION.

5. At the end of the examination, place your Response Form inside the front cover of this booklet and return the booklet and your Response Form to the supervisor.

PRINCIPLES OF MATHEMATICS 12

AUGUST 2000

COURSE CODE = MA

Insert **only** hand-printed PEN here.

Ministry use only.

Question 1:

1. .
(3)

Question 2:

2. .
(3)

Question 3:

3. .
(3)

Question 4:

4. .
(3)

Question 5:

5. .
(3)

Question 6:

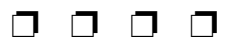
6. .
(3)

Question 7:

7. .
(3)

Question 8:

8. .
(4)



**PRINCIPLES OF
MATHEMATICS 12**

AUGUST 2000

COURSE CODE = MA

GENERAL INSTRUCTIONS

1. Aside from an approved calculator, electronic devices, including dictionaries and pagers, are **not** permitted in the examination room.
2. All multiple-choice answers must be entered on the Response Form using an **HB pencil**. Multiple-choice answers entered in this examination booklet will **not** be marked.
3. For each of the written-response questions, write your answer in the space provided in this booklet.

You will not be provided with any additional paper since rough-work space for the written-response questions has been incorporated into the space allowed for answering each question. You may not need all of the space provided to answer each question.

4. Ensure that you use language and content appropriate to the purpose and audience of this examination. Failure to comply may result in your paper being awarded a zero.
5. This examination is designed to be completed in **two hours**. *Students may, however, take up to 30 minutes of additional time to finish.*

PRINCIPLES OF MATHEMATICS 12 PROVINCIAL EXAMINATION

- | | Value | Suggested Time |
|---|------------------------|--------------------|
| 1. This examination consists of two parts: | | |
| PART A: 45 multiple-choice questions | 45 | 75 |
| PART B: 8 written-response questions | 25 | 45 |
| | Total: 70 marks | 120 minutes |
2. The last **three** pages inside the back cover contain **A Summary of Basic Identities and Formulae, Rough Work for Graphing, and Rough Work for Multiple-Choice**. These pages may be detached for convenient reference prior to writing this examination.
3. **A graphing calculator is essential for the Principles of Mathematics 12 Provincial Examination.** The calculator must be a hand-held device designed primarily for mathematical computations involving logarithmic and trigonometric functions as well as for graphing functions. Computers, calculators with a QWERTY keyboard, and electronic writing pads will not be allowed. Students must not bring any external devices to support calculators such as manuals, printed or electronic cards, printers, memory expansion chips or cards, or external keyboards. Students may have more than one calculator available during the examination of which one may be a scientific calculator. Calculators may not be shared and must not have the ability to either transmit or receive electronic signals. In addition to an approved calculator, students will be allowed to use rulers, compasses, and protractors during the examination.
4. If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.
5. When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

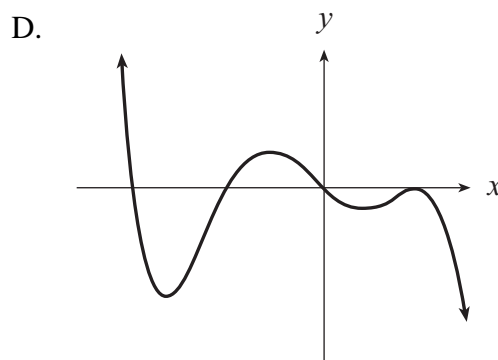
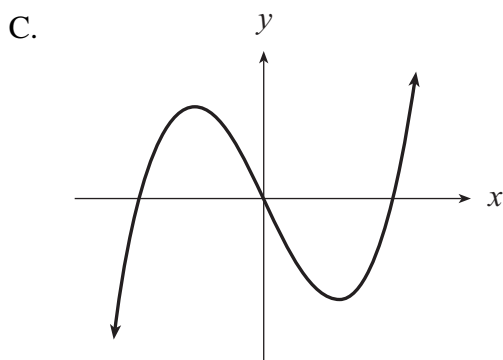
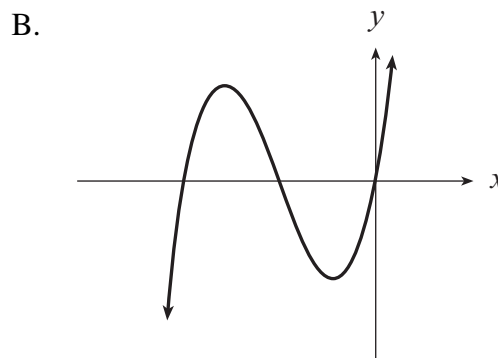
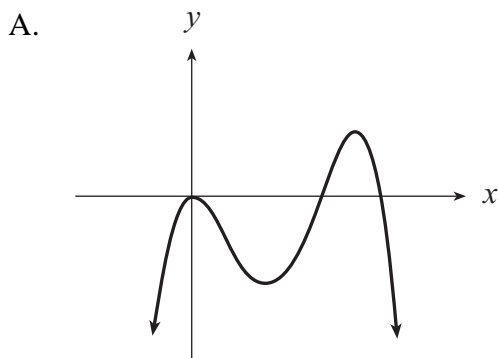
PART A: MULTIPLE CHOICE

Value: 45 marks

Suggested Time: 75 minutes

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the Response Form provided. Using an HB pencil, completely fill in the circle that has the letter corresponding to your answer.

1. Which graph could represent a polynomial function of degree 4 ?



2. According to the Rational Root Theorem, which of the following equations has possible rational roots of ± 1 , ± 2 , $\pm \frac{1}{4}$, $\pm \frac{1}{2}$?

- A. $2x^3 + 5x^2 - 4 = 0$
- B. $4x^3 - 5x^2 - 2 = 0$
- C. $x^3 - 4x^2 + 2x - 4 = 0$
- D. $4x^3 + 3x^2 - x + 1 = 0$

3. Solve: $x^3 + x^2 - 4x - 4 = 0$

- A. -2, -1, 2
- B. -2, 1, 2
- C. -4, -2, 1
- D. -1, 2, 4

4. Determine the range of the function $f(x) = x^4 - 3x^3 - 8$.

- A. $y \geq -18.81$
- B. $y \geq -16.54$
- C. $y \geq -8$
- D. all real numbers

5. Solve the inequality: $(x + 1)(2 - x)(x - 2) \geq 0$



6. Determine the midpoint of the line segment joining the points A(-5, 7) and B(5, -3).

- A. (0, 2)
- B. (2, 0)
- C. (-5, 5)
- D. (5, -5)

7. Which relation below describes a circle?

- A. $4x^2 + 3y^2 + 4x + 4y = 0$
- B. $4x^2 + 3y^2 + 3x + 3y = 0$
- C. $3x^2 + 4y^2 + 3x + 4y = 0$
- D. $4x^2 + 4y^2 + 3x + 3y = 0$

8. Which absolute value inequality describes the solution shown?



- A. $|x - 4| < 3$
- B. $|x - 4| > 3$
- C. $|x + 4| < 3$
- D. $|x + 4| > 3$

9. What is the length of the transverse axis of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$?

- A. 3
- B. 4
- C. 6
- D. 8

10. Change $3y^2 + 6y + x + 1 = 0$ to standard form.

- A. $x = -3(y+1)^2$
- B. $x = 3(y+1)^2 - 4$
- C. $x = -(y+3)^2 + 8$
- D. $x = -3(y+1)^2 + 2$

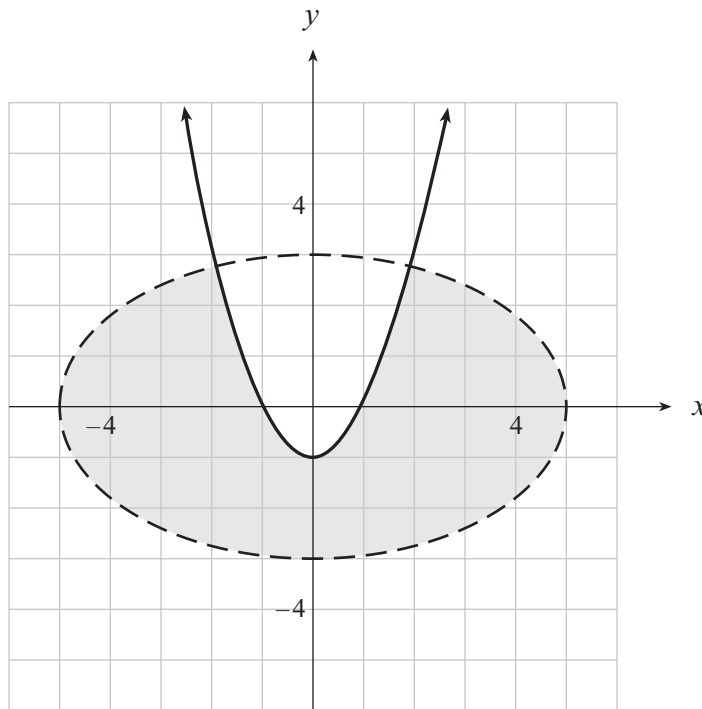
11. Solve the following system:

$$x = -y^2$$

$$x^2 + y^2 = 30$$

- A. $(-5, \sqrt{5})$, $(-5, -\sqrt{5})$
- B. $(5, \sqrt{5})$, $(5, -\sqrt{5})$
- C. $(-6, \sqrt{6})$, $(-6, -\sqrt{6})$
- D. $(6, \sqrt{6})$, $(6, -\sqrt{6})$

12. Which system has the shaded region shown below as its solution?



- A. $y \leq x^2 - 1$
 $\frac{x^2}{25} + \frac{y^2}{9} < 1$
- B. $y \leq x^2 - 1$
 $\frac{x^2}{25} + \frac{y^2}{9} > 1$
- C. $y \geq x^2 - 1$
 $\frac{x^2}{25} + \frac{y^2}{9} < 1$
- D. $y \geq x^2 - 1$
 $\frac{x^2}{25} + \frac{y^2}{9} > 1$

13. An ellipse with vertices $(2, 3)$ and $(2, -5)$ has a minor axis of length 6. Determine an equation of this ellipse.

A. $\frac{(x-2)^2}{3} + \frac{(y-1)^2}{4} = 1$

B. $\frac{(x-2)^2}{9} + \frac{(y-1)^2}{16} = 1$

C. $\frac{(x-2)^2}{3} + \frac{(y+1)^2}{4} = 1$

D. $\frac{(x-2)^2}{9} + \frac{(y+1)^2}{16} = 1$

14. Determine the coordinates of the point on the circle defined by $x^2 + y^2 = 5$ that is closest to the point $P(1, 3)$.

A. $(1, 2)$

B. $(0.52, 2.17)$

C. $(0.71, 2.12)$

D. $(2.12, 0.71)$

15. A point $P(x, y)$ moves such that it is always twice as far from the line $y = -2$ as it is from the point $R(0, 1)$. Determine an equation of this locus.

A. $\sqrt{(x-0)^2 + (y-1)^2} = 2\sqrt{(x-x)^2 + (y+2)^2}$

B. $\sqrt{(x-0)^2 + (y-1)^2} = 2\sqrt{(x-0)^2 + (y+2)^2}$

C. $2\sqrt{(x-0)^2 + (y-1)^2} = \sqrt{(x-0)^2 + (y+2)^2}$

D. $2\sqrt{(x-0)^2 + (y-1)^2} = \sqrt{(x-x)^2 + (y+2)^2}$

16. Which of the following must be true given that the following system has exactly four different real solutions? ($A > 0$, $B > 0$)

$$\frac{(x-2)^2}{25} + \frac{(y-1)^2}{16} = 1$$

$$\frac{(x-2)^2}{A} - \frac{(y-1)^2}{B} = -1$$

- A. $A > 25$
B. $B > 16$
C. $0 < A < 25$
D. $0 < B < 16$
17. Change $y = \log x$ to exponential form.
- A. $x = 10^y$
B. $x = 10y$
C. $x = y^{10}$
D. $x = 10^{-y}$
18. Express as a single logarithm: $\log a - 3 \log b + 2 \log c$

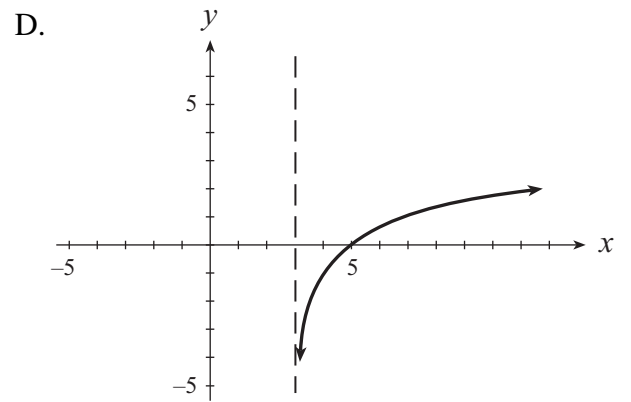
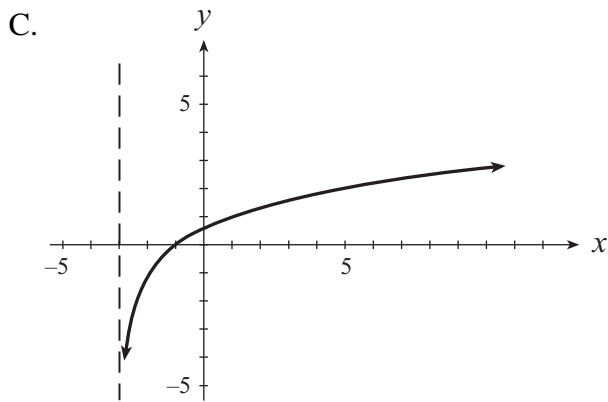
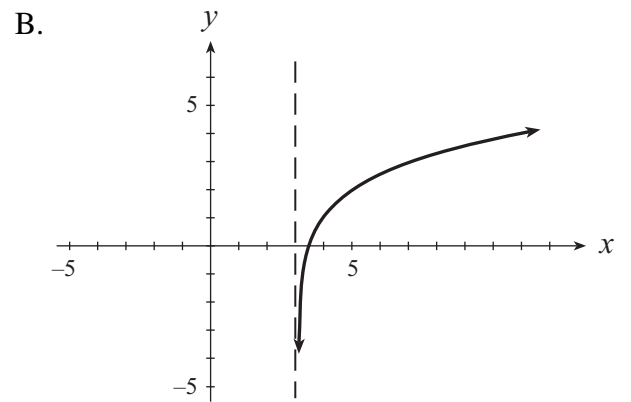
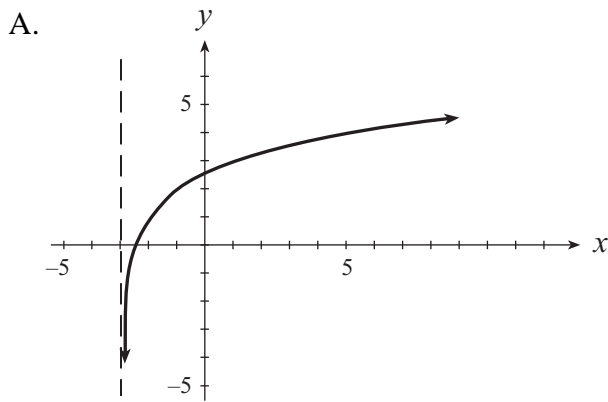
A. $\log \frac{ac^2}{b^3}$

B. $\log \frac{a}{b^3 c^2}$

C. $\log \frac{ab^3}{c^2}$

D. $\log(a - 3b + 2c)$

19. Which graph best represents the function $y = \log_2(x - 3) - 1$?



20. Determine the equation of the asymptote of the graph of $y = \left(\frac{1}{2}\right)^{x+3} - 2$.

- A. $x = -3$
- B. $x = 3$
- C. $y = -2$
- D. $y = 2$

21. Solve for x : $A^x - B = 0$

A. $x = \log A - \log B$

B. $x = \log B - \log A$

C. $x = \frac{\log A}{\log B}$

D. $x = \frac{\log B}{\log A}$

22. A colony of termites has a population of 12 000. The population is decreasing at a rate of 4% per annum. Select an expression for the population after 26 years.

A. $12\,000(0.04)^{26}$

B. $12\,000(0.60)^{26}$

C. $12\,000(0.96)^{26}$

D. $12\,000(1.04)^{26}$

23. The point (m, n) is on the graph of $f(x) = a^x$. Which of the following must be a point on the graph of $g(x) = -\log_a x$ ($a > 0$ and $a \neq 1$)?

A. (n, m)

B. $(n, -m)$

C. $(m, -n)$

D. $(-m, n)$

24. Determine the next term in the geometric sequence $-4, 1, \dots$

A. -6

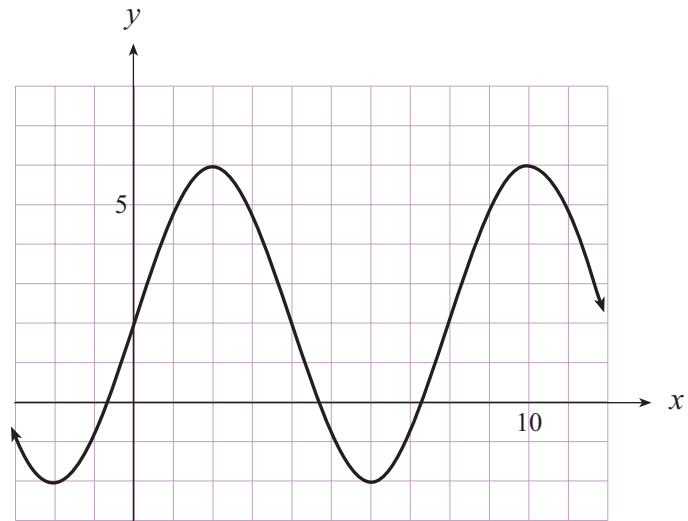
B. $-\frac{1}{4}$

C. $\frac{1}{4}$

D. 6

25. Determine the number of terms in the series $\sum_{k=3}^{19} 2k^3$.
- A. 16
 - B. 17
 - C. 18
 - D. 19
26. Find the sum of the infinite geometric series $200 - 120 + 72 - \dots$
- A. 125
 - B. 152
 - C. 500
 - D. no finite sum
27. Given that $t_n = 3t_{n-1} + 5$, $n > 1$ and $t_3 = 74$, find t_1 .
- A. 6
 - B. $\frac{59}{9}$
 - C. $\frac{28}{3}$
 - D. 23
28. When k arithmetic means are inserted between 4 and 67, the sum of these means is 781. Determine k , the number of means.
- A. 20
 - B. 21
 - C. 22
 - D. 24
29. Convert $\frac{11\pi}{6}$ radians to degrees.
- A. 300°
 - B. 330°
 - C. 344°
 - D. 367°

30. Determine the amplitude of the trigonometric function graphed below.

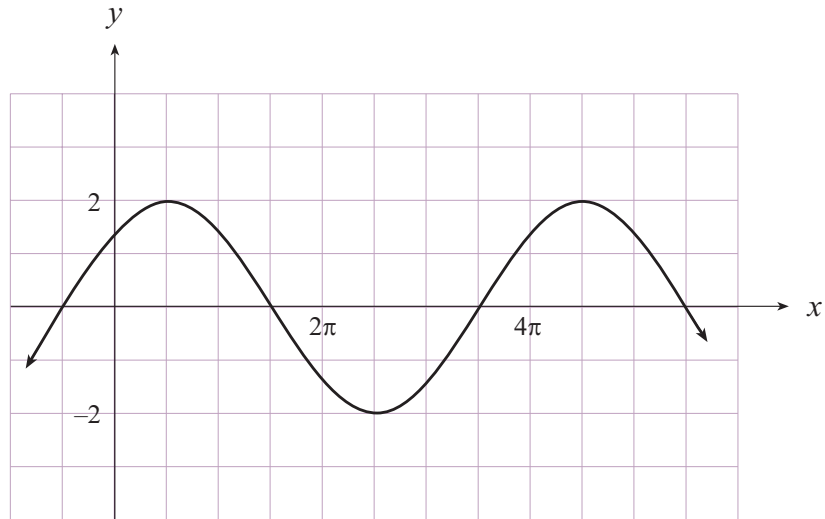


- A. 2
- B. 4
- C. 6
- D. 8

31. Determine the range of the trigonometric function $y = -5 \cos(x + 2)$.

- A. $-7 \leq y \leq 3$
- B. $-5 \leq y \leq 5$
- C. $-3 \leq y \leq 7$
- D. $3 \leq y \leq 7$

32. Which cosine function represents the following graph?



- A. $y = 2 \cos \frac{1}{2} \left(x - \frac{\pi}{2} \right)$
- B. $y = 2 \cos \frac{1}{2} \left(x + \frac{\pi}{2} \right)$
- C. $y = 2 \cos 2 \left(x - \frac{\pi}{2} \right)$
- D. $y = 2 \cos 2 \left(x + \frac{\pi}{2} \right)$

33. Evaluate: $\sec \frac{3\pi}{2}$

- A. -0.5
- B. 0
- C. 0.98
- D. undefined

34. If $\cot \theta = -\frac{12}{5}$, $\frac{\pi}{2} < \theta < \pi$, determine the value of $\sin(\theta + \pi)$.

- A. $-\frac{5}{12}$
- B. $-\frac{5}{13}$
- C. $\frac{5}{13}$
- D. $\frac{5}{12}$

35. Determine the largest solution of $\sin x = \log x$.

- A. 2.70
- B. 7.33
- C. 8.26
- D. 14.11

36. Solve: $\sin 3\theta - \sin 2\theta - \sin \theta = -2$, $0 \leq \theta < 2\pi$

- A. 1.17, 1.57
- B. 4.71, 5.11
- C. 1.73, 3.31, 4.44, 5.26
- D. 1.23, 1.80, 3.47, 4.41, 5.18, 5.95

37. If $(\sin \theta + \cos \theta)^2 = a$, find the value of $\sin 2\theta$.

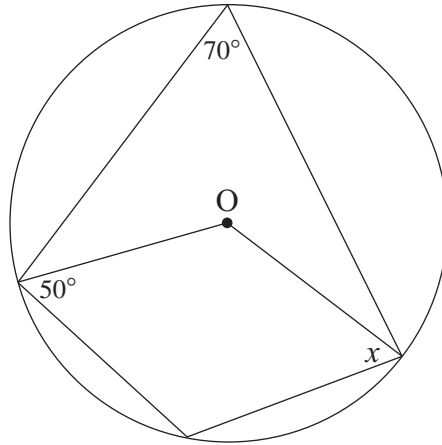
- A. $2a - 2$
- B. $2a + 2$
- C. $a - 1$
- D. $a + 1$

38. Determine an expression equivalent to $\frac{\sec^2 \theta}{\sec^2 \theta - 2}$.

- A. $-\csc 2\theta$
- B. $-\sec 2\theta$
- C. $\csc 2\theta$
- D. $\sec 2\theta$

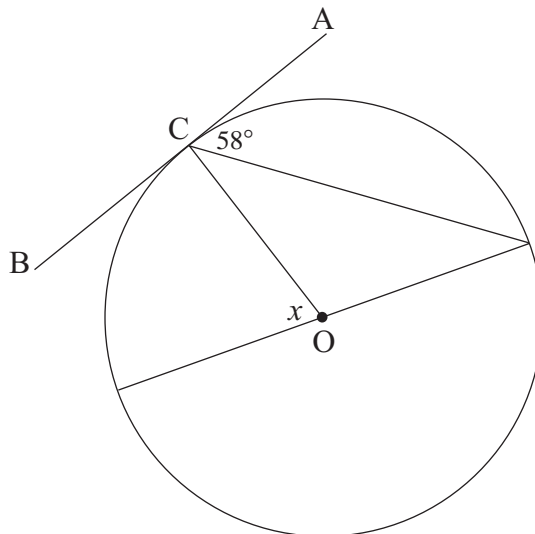
For questions 39 to 42, diagrams are not drawn to scale.

39. In the circle with centre O , determine the measure of $\angle x$.



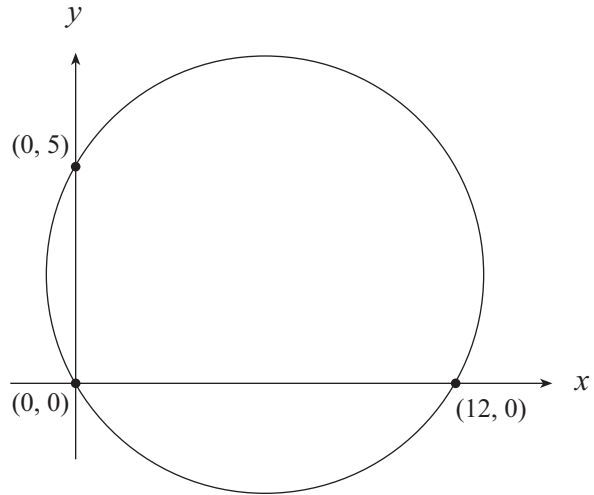
- A. 50°
- B. 60°
- C. 70°
- D. 130°

40. In the circle with centre O , AB is tangent at C . Determine the measure of $\angle x$.



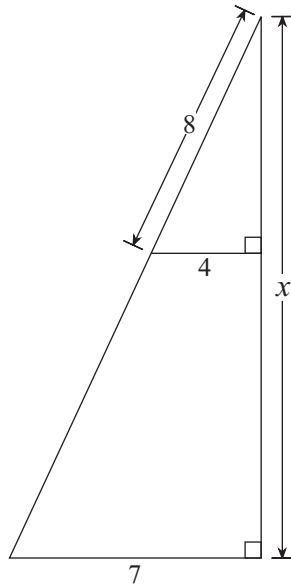
- A. 32°
- B. 54°
- C. 58°
- D. 64°

41. Determine the radius of the circle that passes through the points $(0, 5)$, $(12, 0)$ and $(0, 0)$.



- A. 5.5
- B. 6.5
- C. 7.5
- D. 8.5

42. In the diagram below, determine the length, x .

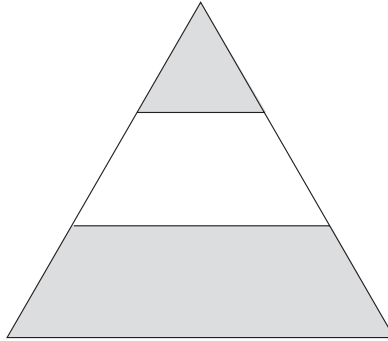


- A. 12
- B. 14
- C. $7\sqrt{3}$
- D. $7\sqrt{5}$

43. Find the product of all the positive divisors of 200.

- A. $(2^4)(5^2)$
- B. $(2^{15})(5^{10})$
- C. $(2^{18})(5^{12})$
- D. $(2^{36})(5^{24})$

44. What fraction of the following figure is shaded? All bands are of equal width.



- A. $\frac{7}{12}$
- B. $\frac{5}{8}$
- C. $\frac{2}{3}$
- D. $\frac{3}{4}$

45. Which equation is equivalent to $y = \sin^{-1} x$, where $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$?

- A. $y = \frac{1}{\sin x}$
- B. $x = \frac{1}{\sin y}$
- C. $y = \sin \frac{1}{x}$
- D. $x = \sin y$

**This is the end of the multiple-choice section.
Answer the remaining questions directly in this examination booklet.**

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PART B: WRITTEN RESPONSE

Value: 25 marks

Suggested Time: 45 minutes

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

Full marks will NOT be given for the final answer only.

1. Determine the cubic polynomial function with zeros -1 , 2 , and ± 3 that passes through $(3, -10)$.
(Answer may be left in factored form.) **(3 marks)**

ANSWER:



2. The sum of the first three terms of an arithmetic series is -12 and the 8th term is 32.
Determine the 4th term of this series.

(3 marks)

ANSWER:



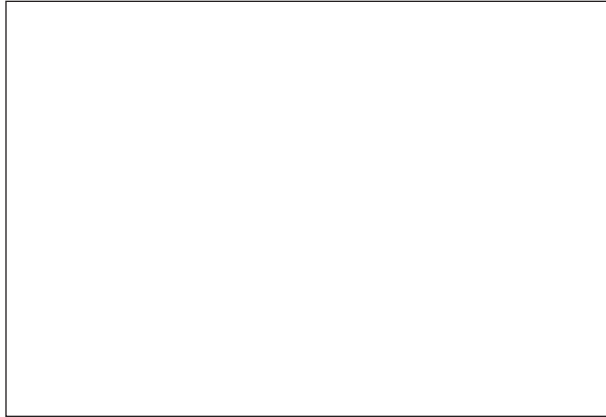
3. Solve the following system using a graphing calculator.

(3 marks)

$$y^2 - x^2 = 16$$

$$y = 4 \log(x + 3)$$

Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the function(s) and all intersection points are visible.



$Y_1 =$

$Y_2 =$

$Y_3 =$

$Y_4 =$

[,] [,]

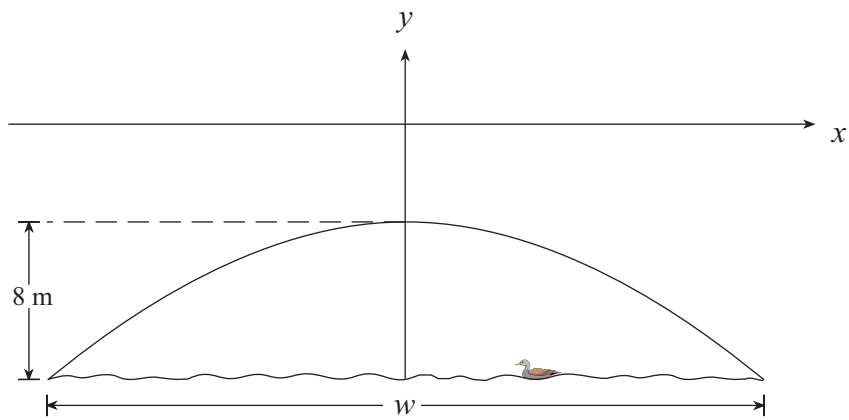
x x
min max

y y
min max

ANSWER:



4. A bridge over a river is supported by an arch in the shape of a rectangular hyperbola as shown in the diagram. The equation of the arch is $y^2 - x^2 = 25$. If the maximum height of the arch above the water is 8 m, determine the width, w , of the river. **(3 marks)**



ANSWER:



5. Prove the identity:

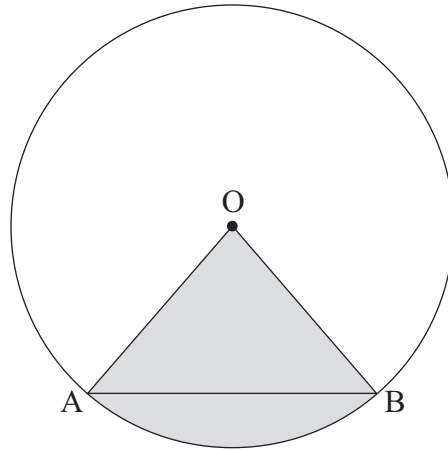
(3 marks)

$$\frac{\cos 2\theta}{\sin \theta} = \frac{\cot^2 \theta - 1}{\csc \theta}$$

LEFT SIDE

RIGHT SIDE

6. A circle with centre O has a radius of 5 cm. If chord $AB = 8$ cm, find the area of the shaded region. **(3 marks)**



ANSWER:

7. Solve the following using a graphing calculator.

(3 marks)

$$|x + 4| = 1.2|x - 4|$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Indicate appropriate window dimensions that will provide enough of the graph so that all intersection points or all zeros are visible.



$Y_1 =$

$Y_2 =$

$Y_3 =$

$Y_4 =$

[,] [,]

x
min x
max

y
min y
max

ANSWER:

Students should choose one or the other method of proof.

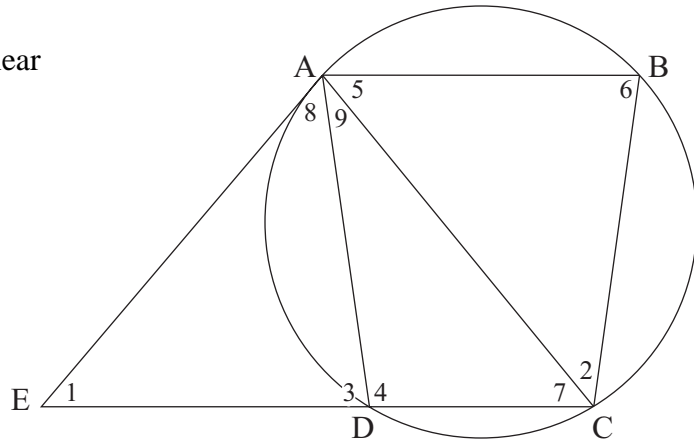
8. Complete the proof.

(4 marks)

Diagram clarification: E, D, C are collinear

Given: EA is tangent to the circle at A
 $AB \parallel EC$

Prove: $\angle 1 = \angle 2$



Paragraph proof method:

Two-column proof method:

STATEMENT	REASON

END OF EXAMINATION

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A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Reciprocal and Quotient Identities

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Addition Identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Double-Angle Identities

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$t_n = ar^{n-1}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$S_n = \frac{n}{2}(a + \ell)$$

$$S_n = \frac{a - \ell r}{1-r}$$

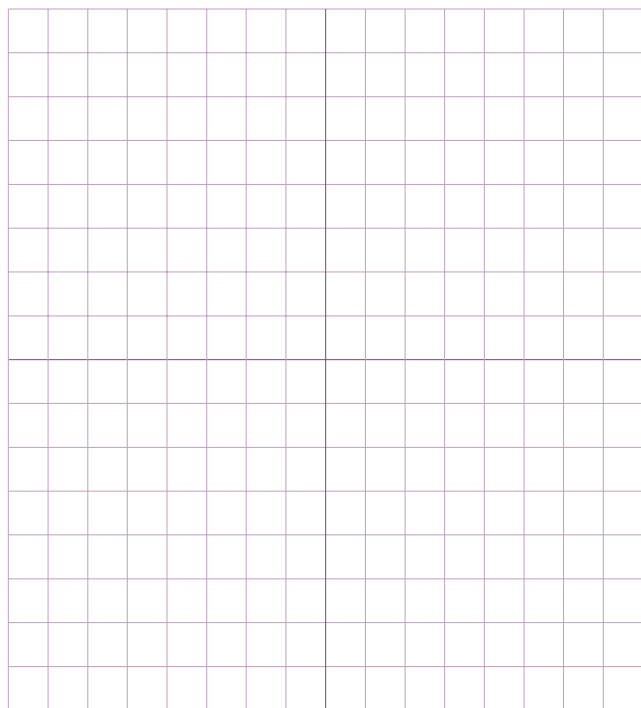
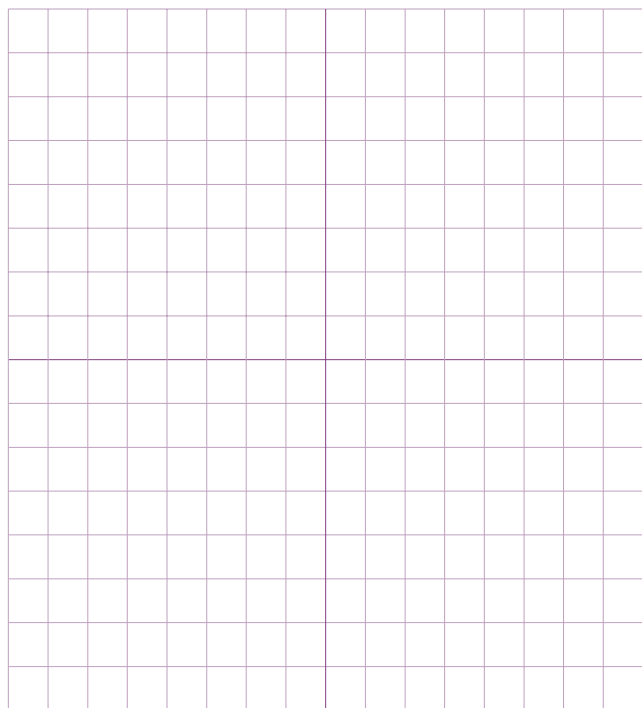
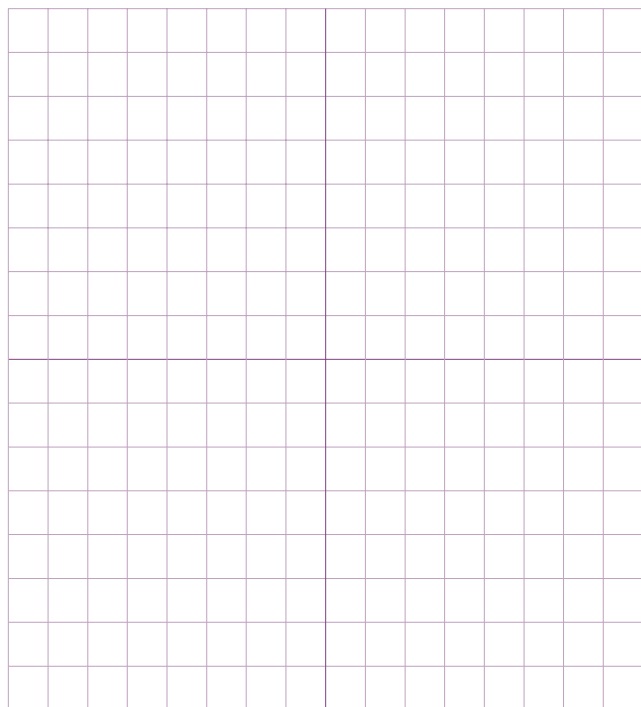
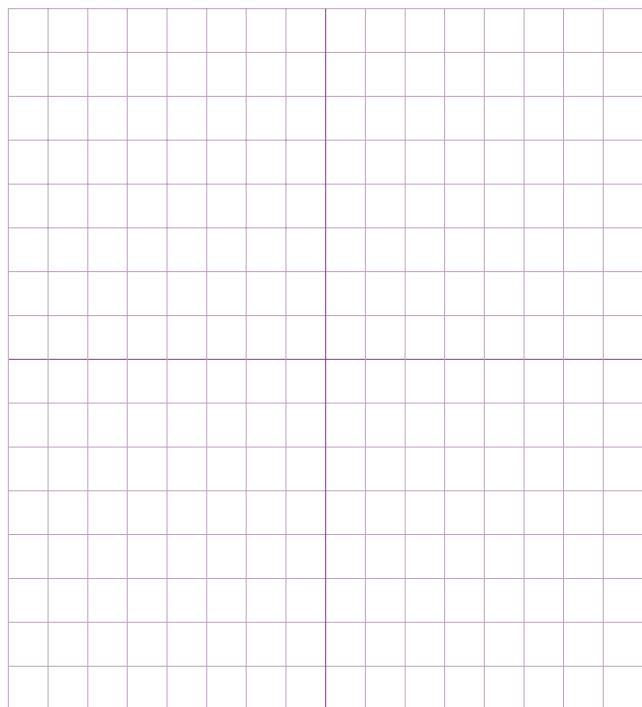
$$S = \frac{a}{1-r}$$

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ROUGH WORK FOR GRAPHING

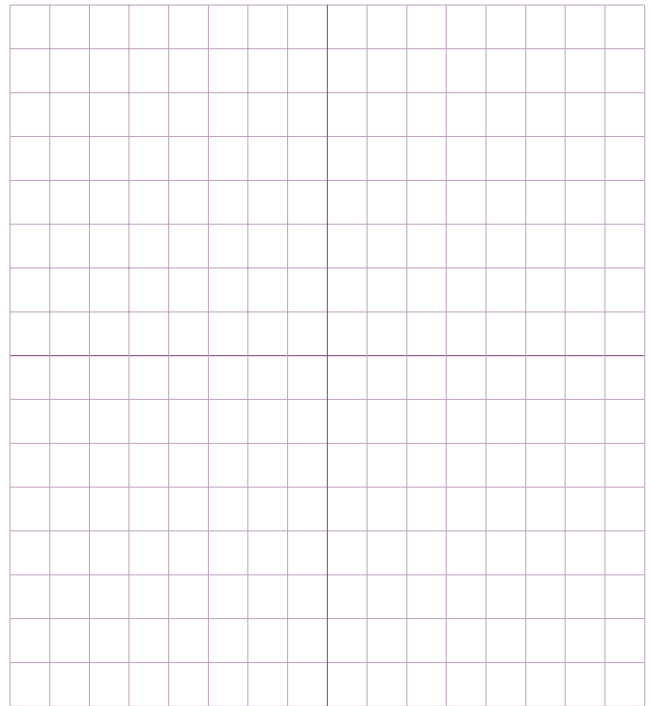
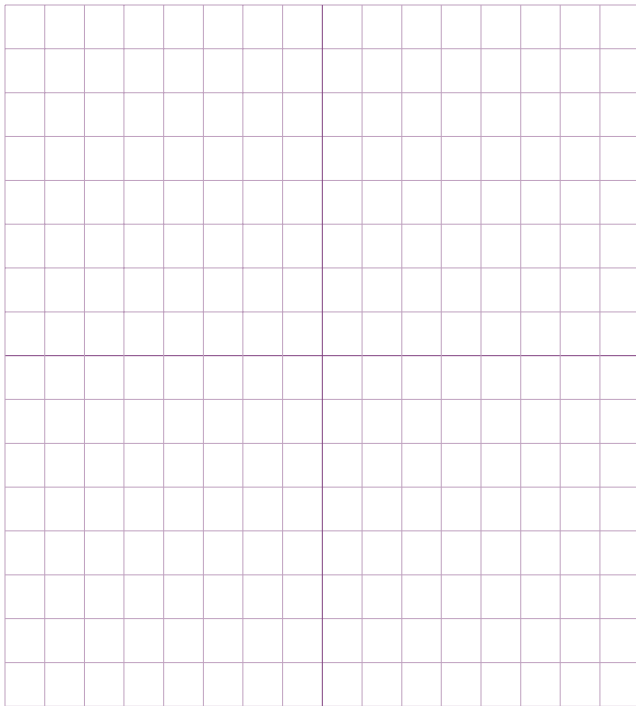
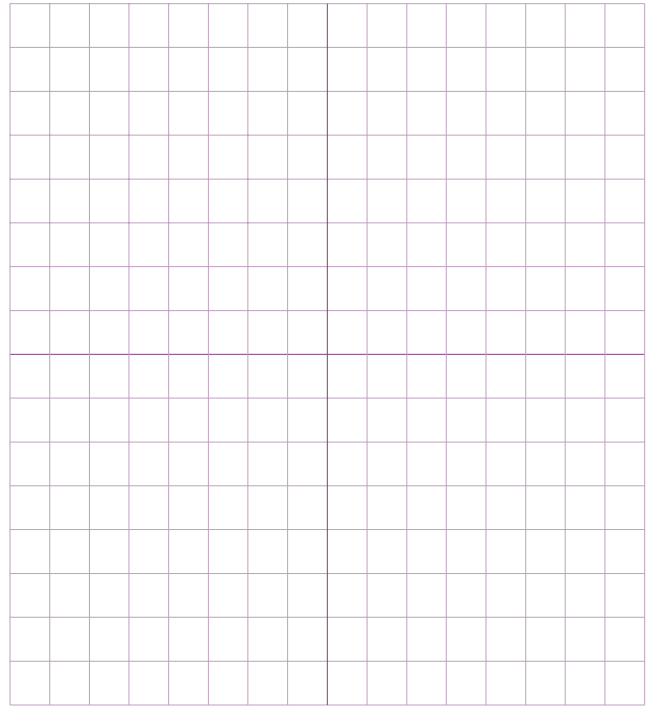
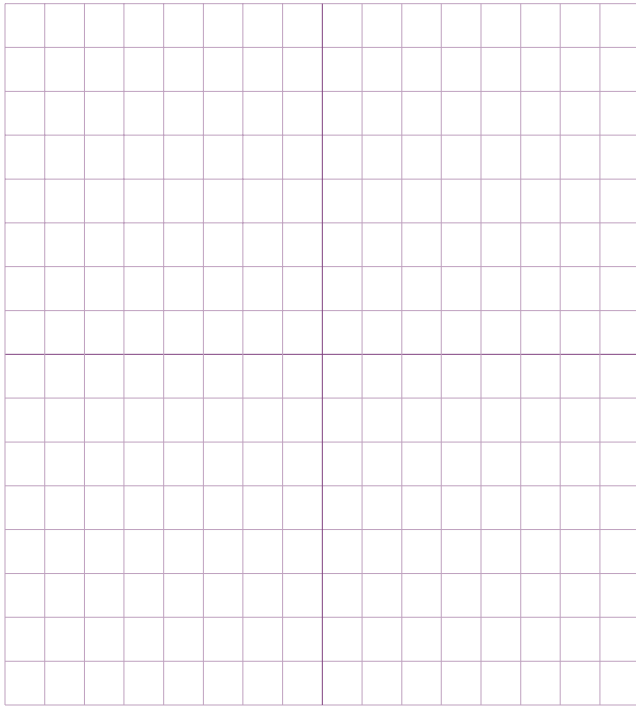
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ROUGH WORK FOR GRAPHING

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ROUGH WORK FOR MULTIPLE-CHOICE

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ROUGH WORK FOR MULTIPLE-CHOICE