Applications of Mathematics 12

June 1999 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers	Sub-Organizers
1. Problem Solving	A Problem Set
2. Number	B Matrices
	C Financial Decision-Making
3. Patterns and Relations	D Fractals
	E Linear Programming
	F Non-Linear Functions
4. Shape and Space	G Periodic Functions
	H Geometry Applications
5. Statistics and Probability	I Data Analysis
	J Applications of Probability

Part A: Multiple Choice

Q	K	C	CO	PLO	Q	K	C	CO	PLO
1.	D	K	2	B2	24.	D	H	4	G2
2.	C	K	2	B2	25.	C	K	4	H1
3.	D	U	2	B2	26.	C	U	4	H1
4.	D	U	2	B2	27.	C	U	4	H2
5.	A	U	2	B3	28.	C	U	4	H1
6.	C	U	2	B2	29.	B	H	4	H1
7.	B	H	2	B3	30.	C	H	4	H1
8.	B	U	2	C1	31.	A	H	4	H1
9.	C	U	2	C1	32.	B	K	5	I1
10.	D	U	2	C2	33.	B	U	5	I2
11.	A	U	2	C2	34.	D	H	5	I4
12.	A	U	2	C2	35.	A	K	5	J4
13.	B	U	2	C1	36.	A	U	5	J4
14.	B	K	3	D2	37.	D	U	5	J1
15.	D	H	3	D1	38.	A	U	5	J1
16.	A	K	3	E2	39.	D	U	5	J3
17.	C	H	3	E3	40.	B	H	5	J1
18.	B	K	3	F1	41.	C	H	5	J1
19.	D	U	3	F2	42.	B	U	1	A1
20.	A	H	3	F3	43.	B	U	1	A2
21.	A	U	3	G3	44.	D	U	1	A1
22.	B	U	4	G4	45.	A	U	1	A2
23.	A	U	4	G4					

Multiple Choice = 45 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1a.	1	U	2	5	I2
1b.	2	U	1	5	I2
2a.	3	U	2	3	E4
2b.	4	U	1	3	E4
2c.	5	U	1	3	E4
3a.	6	U	2	2	B3
3b.	7	H	1	2	B3
4.	8	U	3	5	J5
5a.	9	U	1	3	F3
5b.	10	U	2	3	F3
6a.	11	U	2	3	D3
6b.	12	U	1	3	D3
7.	13	U	3	4	H1, 2
8.	14	H	3	1	A1

Written Response = 25 marks

Multiple Choice = 45 (45 questions)
Written Response = 25 (8 questions)
EXAMINATION TOTAL = 70 marks

LEGEND:
Q = Question Number **K** = Keyed Response **C** = Cognitive Level
B = Score Box Number **S** = Score **CO** = Curriculum Organizer
PLO = Prescribed Learning Outcome

PART B: WRITTEN RESPONSE

Value: 25 marks **Suggested Time: 45 minutes**

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

Full marks will NOT be given for the final answer only.

1. A fast-food restaurant chain collects the following data to determine how the number of restaurants within a 1.5 km radius affects the daily sales.

Number of restaurants in 1.5 km radius	Daily sales ($)
1	3 600
2	3 100
3	2 700
4	2 500
5	2 300

a) If x represents the number of restaurants and y the daily sales, determine the equation of the least squares line of best fit. **(2 marks)**

Solution

Using linear regression:

$y = ax + b$ $\quad a = -320$ $\leftarrow$ **1 mark**

$b = 3\,800$ $\leftarrow$ **1 mark**

$\therefore y = -320x + 3\,800$

b) Predict the daily sales if there are nine restaurants in the 1.5 km radius. **(1 mark)**

Solution

$y = -320(9) + 3\,800$

$y = 920$

$\therefore$ \$920 could be the expected daily sales

← **1 mark**

2. A man can invest at most $18 000 in two different mutual funds, A and B. Mutual fund A returns 8% annually, and mutual fund B returns 12% annually, but at a higher risk than A. To be safe, he wants to invest at least three times as much in fund A as in fund B. Let x represent the amount in fund A and y the amount in fund B.

a) Write the constraints and the objective function that would be used to determine the maximum return on his investment. **(2 marks)**

Solution

Constraints:

$x + y \le 18\,000$ ← $\frac{1}{2}$ **mark**

$x \ge 3y$ ← $\frac{1}{2}$ **mark**

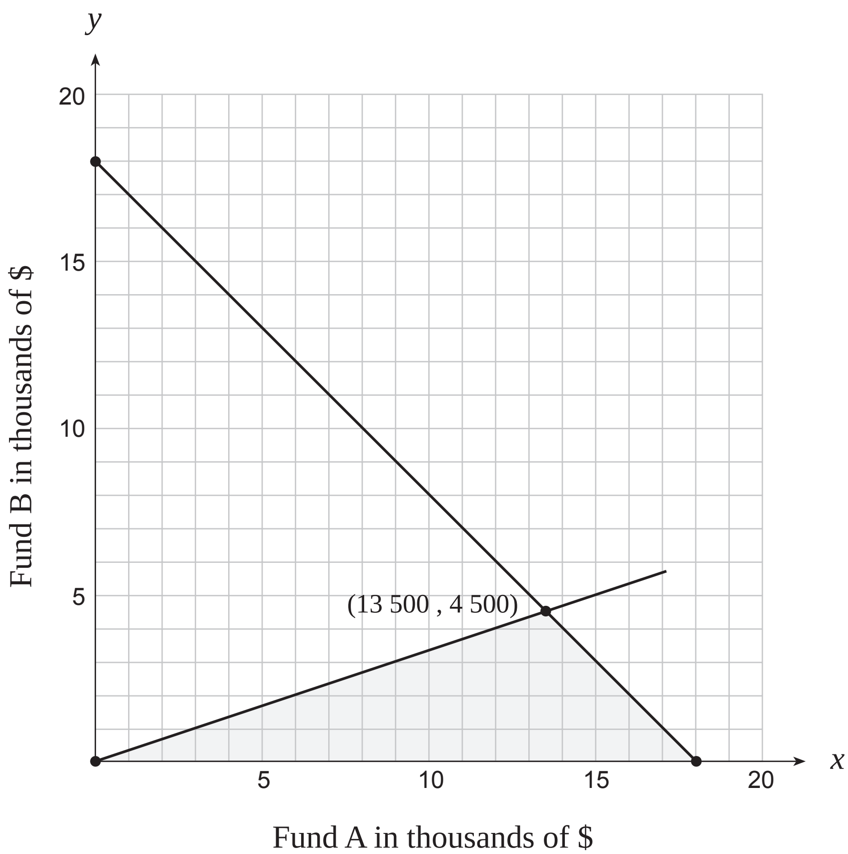
$x \ge 0$, $y \ge 0$ } optional

Objective function:

$R = 0.08x + 0.12y$ ← **1 mark**

b) Graph the feasible region obtained by these constraints.	**(1 mark)**

Solution



Marks for graph:

$\frac{1}{2}$ **mark** shading

$\frac{1}{2}$ **mark** intersection

c) Determine the maximum return on his investment given these constraints.	**(1 mark)**

Solution

Check vertex points in objective function. $\leftarrow \frac{1}{2}$ **mark**

$(0,\ 0)$ minimum	$R = 0$	
$(13\,500,\ 4\,500)$	$R = 1\,620$	$\leftarrow \frac{1}{2}$ **mark**
$(18\,000,\ 0)$	$R = 1\,440$	

3. A new light rapid transit system has just started. Each month, 90% of the commuters who use light rapid transit will continue to do so, while 20% of those who travel by car will switch to light rapid transit. At present, 15% of all commuters use light-rapid transit, while the rest use cars.

   a) After two months, what percent of commuters will be using cars? **(2 marks)**

Solution

1 mark for matrix

$\frac{1}{2}$ **mark** for raising to 2^{nd} power
↓

$$[.15 \quad .85]\begin{bmatrix} .90 & .10 \\ .20 & .80 \end{bmatrix}^2 = [.4135 \quad .5865]$$

↑
approximately 59% will use cars ← $\frac{1}{2}$ **mark**

b) If the pattern continues, what percent of commuters will be using light rapid transit in the long run? **(1 mark)**

Solution

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} .9 & .1 \\ .2 & .8 \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix}$$

$$\left.\begin{aligned} .9x + .2y &= x \\ .1x + .8y &= y \end{aligned}\right\} .1x = .2y$$

$x = 2y$ $\leftarrow \frac{1}{2}$ **mark**

But $x + y = 1$

So $2y + y = 1$

$3y = 1$

$y = \frac{1}{3}$

$x = \frac{2}{3}$ $\leftarrow \frac{1}{2}$ **mark**

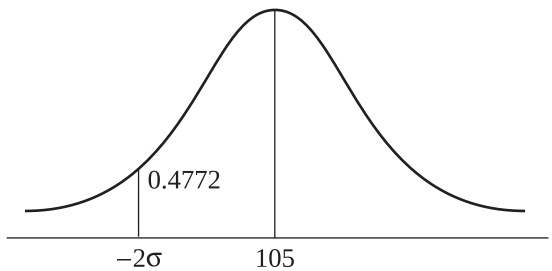
67% will use the light rapid transit system.

Alternate Solution

$$\begin{bmatrix} .9 & .1 \\ .2 & .8 \end{bmatrix}^{100} = \begin{bmatrix} .\bar{6} & .\bar{3} \\ .\bar{6} & .\bar{3} \end{bmatrix}$$ $\leftarrow$ **1 mark**

4. A company that makes chocolate bars sells one type whose mass is supposed to be 100 g. The company knows that the masses of their bars follow a normal distribution with a mean of 105 g and a standard deviation of 2.5 g. If they make 40 000 of these bars a day, how many have mass less than 100 g ? **(3 marks)**

Solution



$z_{100} = \dfrac{100-105}{2.5} = -2$ ← **1 mark**

$z = 2 \rightarrow 0.4772$ from table ← **½ mark**

$0.5 - 0.4772 = 0.0228$ ← **1 mark**

$0.0228 \times 40\ 000 = 912$ bars ← **½ mark**

$\therefore$ 912 bars are less than 100 g

Alternate Solution

Using the normal cdf feature on a graphing calculator:

normal cdf $(0,\ 100,\ 105,\ 2.5) \times 40\ 000 = 910$ bars

Note: Because of rounding of z-scores, the answers are slightly different. Both are acceptable.

5. A learning curve describes the rate at which a skill can be learned. Assume the equation

$$n = -153\log\left(1 - \frac{w}{90}\right)$$

describes the number of practice sessions, n, it will take to reach a skill level of w words per minute on a computer keyboard.

a) Determine how many sessions will be needed to reach a level of 60 words per minute.

(1 mark)

Solution

Substitute a value of $w = 60$ in the formula and calculate n.

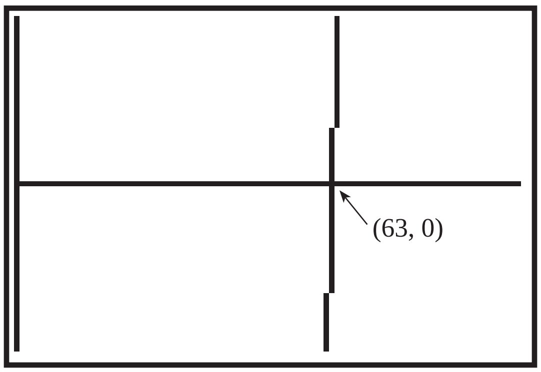
$n = -153\log\left(1 - \frac{60}{90}\right)$ ← $\frac{1}{2}$ **mark**

= 73 sessions are needed ← $\frac{1}{2}$ **mark**

b) Determine the skill level in words per minute after 80 sessions. **(2 marks)**

If providing a graphical solution, state the function(s) used, sketch the graph, indicate appropriate window dimensions and clearly explain how your solution is derived from the graph.

Solution



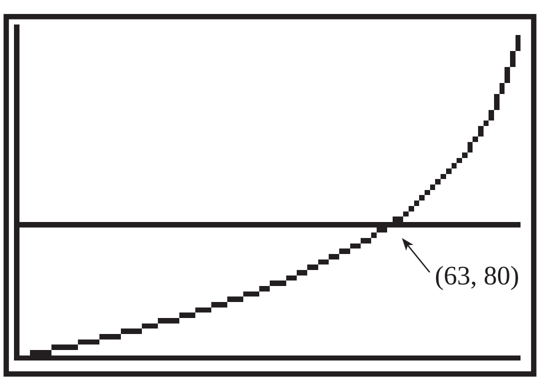
$x\ [0,\ 100] \qquad y\ [-2,\ 2]$

$Y_1 = -153\log\left(1-\frac{x}{90}\right)-80$ ← **1 mark**

← **½ mark** for graph

$\therefore$ the skill level will be 63 words per minute ← **½ mark**

Alternate Solution 1



$x\ [0,\ 85] \qquad y\ [0,\ 200]$

$Y_1 = -153\log\left(1-\frac{x}{90}\right)$

$Y_2 = 80$

} ← **1 mark**

Point of intersection: (63, 80) ← **1 mark**

$\therefore$ the skill level will be 63 words per minute

b) Determine the skill level in words per minute after 80 sessions. **(2 marks)**

If providing a graphical solution, state the function(s) used, sketch the graph, indicate appropriate window dimensions and clearly explain how your solution is derived from the graph.

Alternate Solution 2

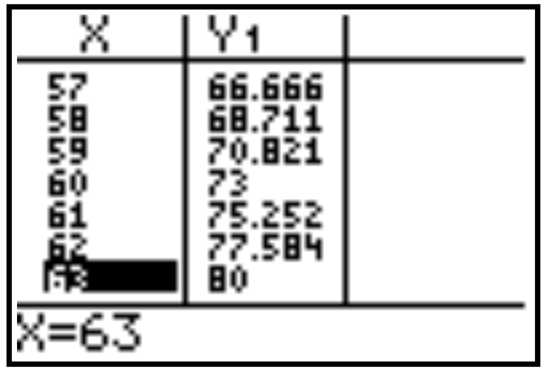
Enter equation $Y_1 = -153\log\left(1 - \frac{x}{90}\right)$

Set table: Start at 50

$\Delta\text{Tbl} = 1$

Display table

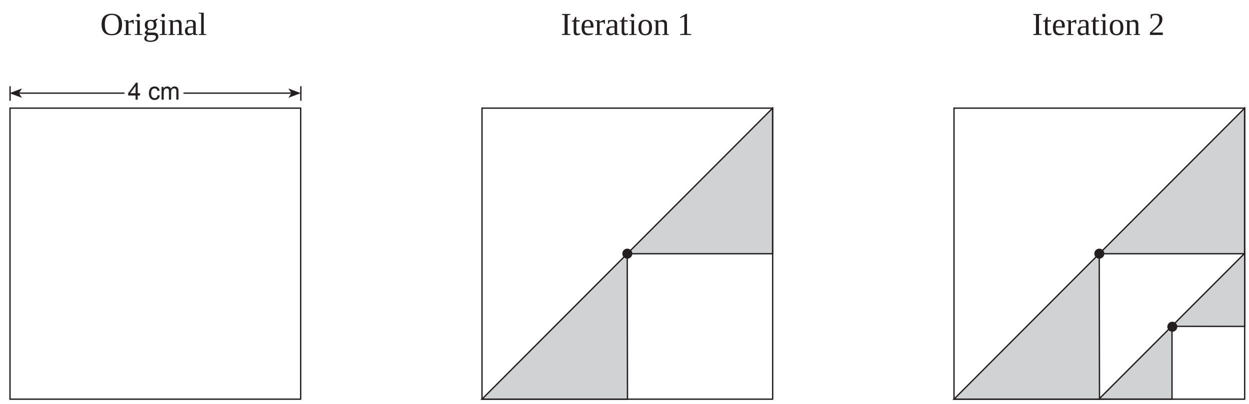
Scroll to the pair 63, 80
Skill level = 63 wpm. } ← **1 mark**



← **1 mark**

6. A fractal is created as follows:

A square is drawn with sides 4 cm. A diagonal is constructed and a new square is created below the diagonal using the midpoint of the diagonal as one corner. The two small triangles formed below the diagonal are shaded. This process is continued with the smaller square.



a) If this process is continued, what is the total area of the **shaded** region of the 4th iteration?

(2 marks)

Solution

Area of the first triangles shaded is $\frac{1}{4}$ of the square

$$\left(4+1+\frac{1}{4}+\frac{1}{16}\right)=\frac{85}{16}=5.3125\text{ cm}^2$$

↑ **1 mark** for pattern

↑ **1 mark** for answer

b) If this process continues without end, determine the area of the shaded region. **(1 mark)**

Solution

$$\left(4+1+\frac{1}{4}+\ \dots\right) = \frac{4}{1-\frac{1}{4}}$$

↑ $\frac{1}{2}$ **mark** for formula

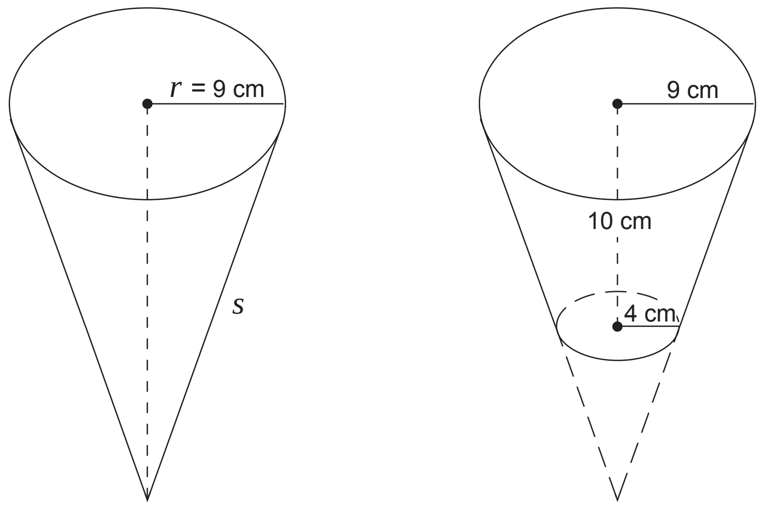
$$= \frac{16}{3}$$

$$= \frac{16}{3}\ \text{cm}^2$$

← $\frac{1}{2}$ **mark** for answer

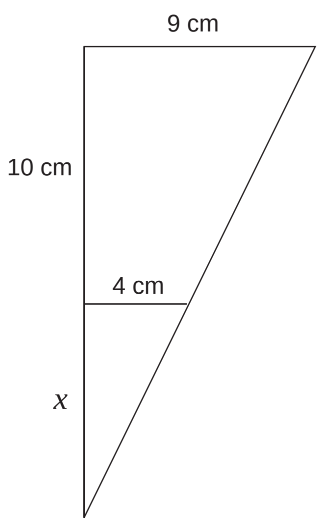
7. A funnel is made from a right circular cone by cutting a small cone from the tip of the larger cone, as shown in the diagram below. Determine the slant height s of the original cone.

(3 marks)



Solution

Original cone:



$$\frac{x}{x+10} = \frac{4}{9}$$ ← **1 mark** ($\frac{1}{2}$ **mark** if wrong ratio)

$$9x = 4x + 40$$

$$5x = 40$$

$$x = 8$$ ← $\frac{1}{2}$ **mark**

$\therefore$ height is 18 cm ← $\frac{1}{2}$ **mark**

Slant height: $9^2 + 18^2 = s^2$ ← $\frac{1}{2}$ **mark**

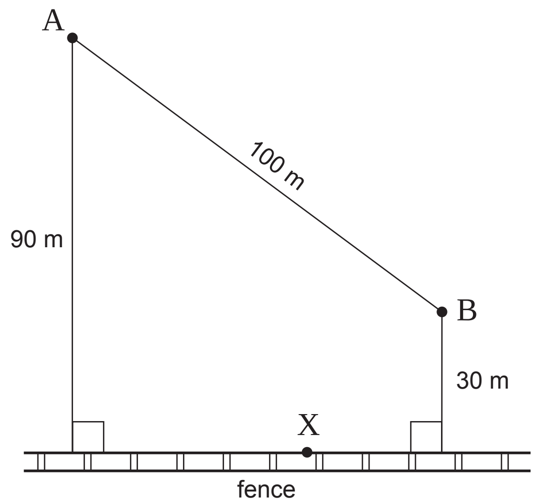
$$405 = s^2$$

$$s = \sqrt{405}$$

$$= 9\sqrt{5}$$ ← $\frac{1}{2}$ **mark**

8. In running a race, participants must start at point A, run to a point X on the fence, and then finish at point B. If the total distance to be run is a minimum, how far is it from A to X ?

(3 marks)



Solution

$100^2 - 60^2 = d^2 \qquad d = 80$ ← $\frac{1}{2}$ **mark**

$\frac{90}{x} = \frac{30}{80 - x}$ ← $\frac{1}{2}$ **mark**

$30x = 90(80 - x)$

$30x = 7\,200 - 90x$

$120x = 7\,200$

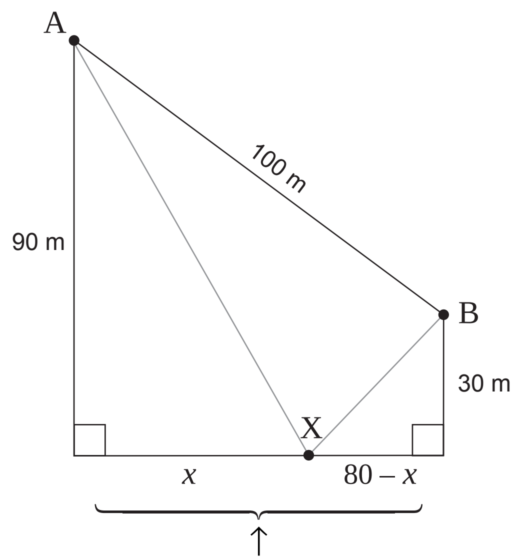
$x = 60$ ← $\frac{1}{2}$ **mark**

$90^2 + x^2 = (\text{AX})^2$ ← $\frac{1}{2}$ **mark**

$90^2 + 60^2 = \left(\text{AX}^2\right)$

$108.17 = \text{AX}$

∴ it is 108.17 m from A to X ← $\frac{1}{2}$ **mark**



$\frac{1}{2}$ **mark**

END OF KEY