Applications of Mathematics 12

January 1999 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		**Sub-Organizers**
1. Problem Solving	A	Problem Set
2. Number	B	Matrices
	C	Financial Decision-Making
3. Patterns and Relations	D	Fractals
	E	Linear Programming
	F	Non-Linear Functions
4. Shape and Space	G	Periodic Functions
	H	Geometry Applications
5. Statistics and Probability	I	Data Analysis
	J	Applications of Probability

Part A: Multiple Choice

Q	K	C	CO	PLO	Q	K	C	CO	PLO
1.	C	K	2	B1	24.	A	H	4	G3
2.	A	U–	2	B3	25.	A	K	4	H1
3.	B	U–	2	B1	26.	C	U–	4	H2
4.	A	U–	2	B2	27.	B	U–	4	H2
5.	B	U+	2	B3	28.	C	U+	4	H1
6.	A	H	2	B3	29.	B	H	4	H2
7.	D	H	2	B3, H1	30.	B	H	4	H2
8.	C	U–	2	C2	31.	C	H	4	H2
9.	A	U+	2	C1	32.	D	K	5	I4
10.	B	U+	2	C2	33.	A	U–	5	I2
11.	C	K	3	D2	34.	B	H	5	I2
12.	A	U–	3	D1	35.	B	K	5	J5
13.	C	H	3	D4	36.	B	U–	5	J2
14.	B	U+	3	D4	37.	C	U–	5	J4
15.	B	U+	3	D4	38.	B	U–	5	J1
16.	D	K	3	E3	39.	B	U+	5	J5
17.	D	H	3	E3	40.	C	U+	5	J4
18.	C	K	3	F1	41.	A	H	5	J6
19.	D	U–	3	F2	42.	A	H	5	J3
20.	B	H	3	F2	43.	A	U–	1	A1
21.	D	U–	4	G3	44.	C	U+	1	A2
22.	B	U+	4	G3	45.	D	H	1	A1
23.	B	U+	4	G3					

Multiple Choice = 45 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1a.	1	U+	2	2	C2
1b.	2	U–	1	2	C2
2a.	3	U+	1	5	I2
2b.	4	U+	1	5	I2
2c.	5	U+	1	5	I2
3.	6	U+	2	5	J6
4a.	7	U+	1	4	H2
4b.	8	U+	2	4	H2
5.	9	U+	3	2	B3
6a.	10	U+	2	3	E1
6b.	11	U+	2	3	E4
7.	12	U+	3	3	F3
8a.	13	U+	2	1	A2
8b.	14	U+	2	1	A2, F3

Written Response = 25 marks

Multiple Choice = 45 (45 questions)
Written Response = 25 (8 questions)
EXAMINATION TOTAL = 70 marks

LEGEND:
Q = Question Number | **K** = Keyed Response | **C** = Cognitive Level
B = Score Box Number | **S** = Score | **CO** = Curriculum Organizer
PLO = Prescribed Learning Outcome

PART B: WRITTEN RESPONSE

Value: 25 marks **Suggested Time: 45 minutes**

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

Full marks will NOT be given for the final answer only.

Use the following spreadsheet to answer question 1.

A	B	C	D	E	F
Payment #	Bal. Before Payment	Payment	Payment to Princ.	Interest Payment	Outstanding Princ. Balance
1	20000.00	406.01	239.34	166.67	19760.66
2	19760.66	406.01	241.33	164.67	19519.33
3	19519.33	406.01	243.34	162.66	19275.98
34	10952.10	406.01	314.74	91.27	10637.37
35	10637.37	406.01	317.36	88.64	10320.01
36	10320.01	406.01	320.01	86.00	10000.00

1. A new car valued at \$20 000 may be leased for three years and then bought for its residual value of \$10 000. The spreadsheet above shows the first three and the last three payments of a schedule for the leasing of the car over three years.

   a) What is the total cost of buying the car through this lease-purchase plan? **(2 marks)**

Solution

$$\begin{array}{rl} 406.01 & \leftarrow \tfrac{1}{2}\ \textbf{mark} \\ \times\ 36 & \leftarrow \tfrac{1}{2}\ \textbf{mark} \\ \hline 14\,616.36 & \\ +\ 10\,000 & \leftarrow \tfrac{1}{2}\ \textbf{mark} \\ \hline \$\,24\,616.36 & \leftarrow \tfrac{1}{2}\ \textbf{mark} \end{array}$$

b) This car could have been purchased outright with the same interest rate and 36 monthly payments of $645.34. How much is saved by using this option? **(1 mark)**

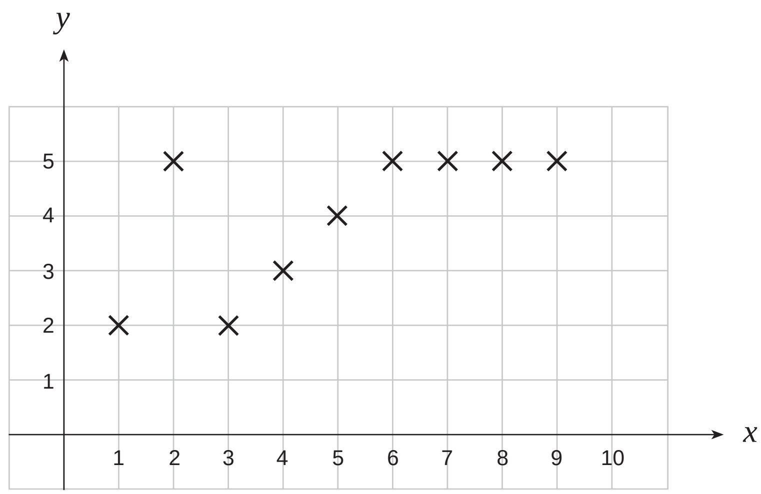
Solution

$$\begin{array}{r} \$645.34 \\ \times\ 36 \\ \hline 23\,232.24 \\ -\ 24\,616.36 \\ \hline -1\,384.12 \end{array}$$

23 232.24 $\leftarrow \frac{1}{2}$ **mark**

$1 384.12 is saved using this option. $\leftarrow \frac{1}{2}$ **mark**

2. A scatter plot of data points is shown below.



a) Determine the coordinates of the three summary points used to find the median-median line of best fit. **(1 mark)**

Solution

$(2,\ 2)$, $(5,\ 4)$, $(8,\ 5)$ $\leftarrow$ **1 mark**

b) Determine the equation of the median-median line of best fit. **(1 mark)**

Solution

Using $(2,\ 2)$ and $(8,\ 5)$ $\rightarrow$ $y = \frac{1}{2}x + 1$

Using slope $= \frac{1}{2}$ and $(5,\ 4)$ $\rightarrow$ $y = \frac{1}{2}x + \frac{3}{2}$

} Slope of $\frac{1}{2}$ $\leftarrow$ **½ mark**

$\therefore$ move up $\frac{1}{6}$ of unit along y-axis

Equation is $y = \frac{1}{2}x + \frac{7}{6}$ $\leftarrow$ **½ mark**

Alternate Solution

Using the median/median STAT feature on a graphing calculator, students could obtain the answer. } $\leftarrow$ **1 mark**

c) Using the equation from part b), determine the value of y when the x-value is 18. **(1 mark)**

Solution

Substituting $y = \frac{1}{2}(18) + \frac{7}{6}$

$y = \frac{61}{6}$ or $y \approx 10.17$ $\leftarrow$ **1 mark**

3. A factory produces light bulbs with a mean lifetime of 900 hours and a standard deviation of 100 hours. If a random sample of 36 of these light bulbs is selected, construct a 95% confidence interval for the lifetime of these bulbs. **(2 marks)**

Solution

$$900 - \frac{1.96(100)}{\sqrt{36}} < \mu < 900 + \frac{1.96(100)}{\sqrt{36}}$$ ← **1 mark**

$$867 < \mu < 933$$ ← **1 mark**

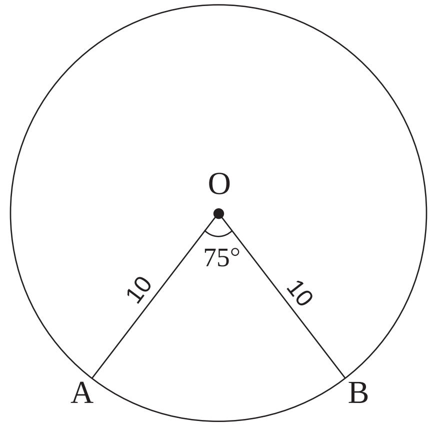
Alternate Solution

Using STAT features on a graphing calculator, students could obtain the answer.

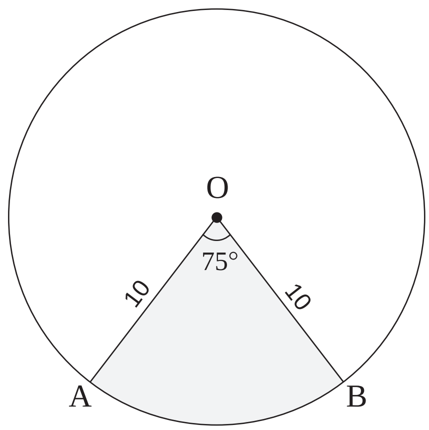
$867.33 < \mu < 932.67$ ← **2 marks**

Use the following diagram to answer question 4.

Given: Circle with centre O
Radius 10 cm
$\angle AOB = 75°$



4. a) Calculate the area of the shaded sector. **(1 mark)**

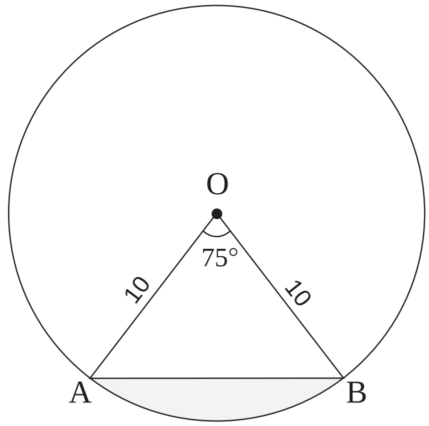


Solution

$$A_1 = \frac{75°}{360°}\pi(10)^2$$
$$= 65.45 \text{ cm}^2$$

← **1 mark**

b) Calculate the area of the shaded segment. **(2 marks)**



Solution

$$\text{Area of } \Delta AOB = \frac{1}{2}(10)(10)\sin 75°$$

$$= 48.30 \text{ cm}^2 \qquad \leftarrow \textbf{1 mark}$$

$$\text{Area of segment} = \text{area of sector} - \text{area of triangle} \qquad \leftarrow \textbf{1 mark}$$

$$= 65.45 - 48.30$$

$$= 17.15 \text{ cm}^2$$

Alternate Solution

$$\text{Angle in radians: } 75 \times \frac{\pi}{180} = \frac{5\pi}{12} \qquad \leftarrow \textbf{1 mark}$$

$$\text{Area of segment} = \frac{1}{2}r^2(\theta - \sin\theta) \qquad \leftarrow \textbf{1 mark}$$

$$= \frac{1}{2}(100)\left(\frac{5\pi}{12} - \sin\frac{5\pi}{12}\right)$$

$$= 17.15355$$

$$= 17.15 \text{ cm}^2$$

5. A car rental agency has one outlet in Vancouver, B.C. and a second outlet in Calgary, Alberta. Each month, 20% of the cars that start the month in Vancouver end up in Calgary, while 10% of the cars that start the month in Calgary end up in Vancouver. At the beginning of business, there are 500 cars in each location. How many cars will be in Vancouver after two months?

(3 marks)

Solution

The present situation is described by the matrix $\begin{bmatrix} 500 & 500 \end{bmatrix} = \begin{bmatrix} \text{cars in Vancouver} & \text{cars in Calgary} \end{bmatrix}$ $\Big\} \leftarrow \frac{1}{2}$ **mark**

The transition matrix shows the change

$$\textbf{From} \begin{matrix} V \\ C \end{matrix} \overset{\textbf{To} \atop V \quad C}{\begin{bmatrix} .8 & .2 \\ .1 & .9 \end{bmatrix}}$$

$\Big\} \leftarrow \frac{1}{2}$ **mark**

Multiply $\begin{bmatrix} 500 & 500 \end{bmatrix} \begin{bmatrix} .8 & .2 \\ .1 & .9 \end{bmatrix} = \overset{V \quad\quad C}{\begin{bmatrix} 450 & 550 \end{bmatrix}}$ $\Big\} \leftarrow 1$ **mark**

Multiply $\begin{bmatrix} 450 & 550 \end{bmatrix} \begin{bmatrix} .8 & .2 \\ .1 & .9 \end{bmatrix} = \overset{V \quad\quad C}{\begin{bmatrix} 415 & 585 \end{bmatrix}}$ $\Big\} \leftarrow \frac{1}{2}$ **mark**

$\therefore$ After 2 months, there will be 415 cars in Vancouver. $\} \leftarrow \frac{1}{2}$ **mark**

OR

Multiply $\begin{bmatrix} 500 & 500 \end{bmatrix} \begin{bmatrix} .8 & .2 \\ .1 & .9 \end{bmatrix}^2 = \overset{V \quad\quad C}{\begin{bmatrix} 415 & 585 \end{bmatrix}}$ $\Big\} \leftarrow 1\frac{1}{2}$ **marks**

$\therefore$ After 2 months, there will be 415 cars in Vancouver. $\} \leftarrow \frac{1}{2}$ **mark**

6. The high school ski club plans to rent buses and vans for a school trip to Blackcomb Mountain. Each bus will take 40 students, needs 3 chaperones, and costs $1 200 to rent. Each van takes 8 students, needs 1 chaperone, and costs $100 to rent. The planners must accommodate at least 400 students and at most 36 chaperones.

   a) If b represents the number of buses to be rented and v represents the number of vans to be rented, list the constraints and the objective function needed to determine the minimum rental cost, C. **(2 marks)**

Solution

	Number of passengers per vehicle		
	BUS	VAN	TOTAL NUMBER OF PASSENGERS
Students	40	8	400
Chaperones	3	1	36
Cost per vehicle	$1 200	$100	

$b \geq 0$

$v \geq 0$

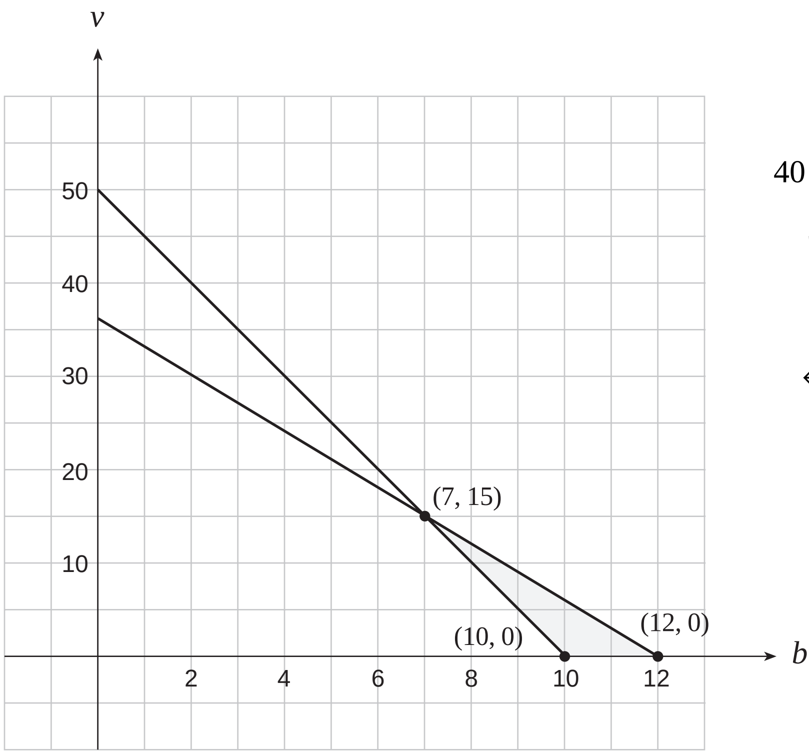
$40b + 8v \geq 400$ ← **½ mark**

$3b + v \leq 36$ ← **½ mark**

$C = 1\,200b + 100v$ ← **1 mark**

b) Graph the feasible region and determine the minimum amount the ski club should budget for transportation costs. **(2 marks)**

Solution



$40b + 8v \geq 400 \quad (\text{or} \quad 5b + v \geq 50)$

$3b + v \leq 36$

← **1 mark**

Test:

$(10,\ 0)\quad C = 1\,200(10)$
$\qquad = \$12\,000$

$(12,\ 0)\quad C = 1\,200(12)$
$\qquad = \$14\,400$

$(7,\ 15)\quad C = 1\,200(7) + 100(15)$
$\qquad = \$9\,900$

← $\frac{1}{2}$ **mark**

Cost: $9 900 ← $\frac{1}{2}$ **mark**

7. A storage box is in the shape of a rectangular prism. It has a volume of $20\ \text{m}^3$, a height of 2 m and a length that is 1 metre greater than its width. What are the dimensions of the rectangular base of the box? (Answer to the nearest 0.1 m.) **(3 marks)**

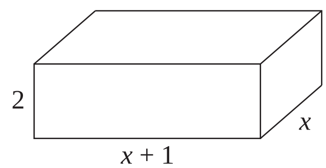
If providing a graphical solution, state the function(s) used, sketch the graph, indicate appropriate window dimensions and clearly explain how your solution is derived from the graph.

Solution

$V = \ell wh$

$20 = 2x(x+1)$ ← **1 mark**

$2x^2 + 2x - 20 = 0$



$Y_1 = 2x^2 + 2x - 20$

← **1 mark** for graph

$x\ [-4.7,\ 4.7]$ $y\ [-3.1,\ 10]$

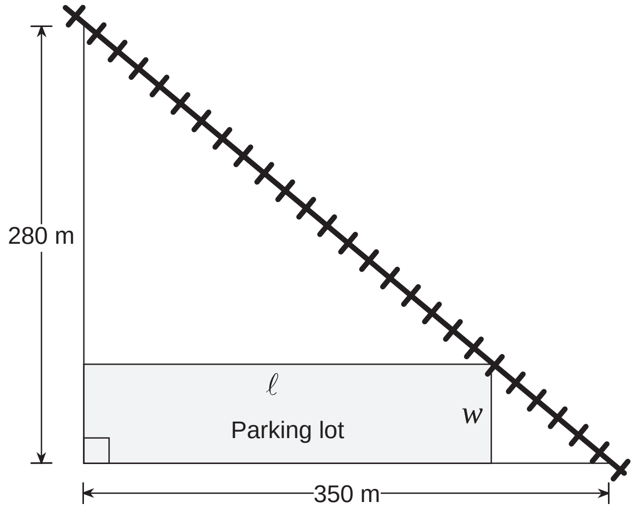
Determine the **positive** value of x that gives a zero for the function.

$x \approx 2.7 \rightarrow x + 1 = 3.7$

↑ **½ mark** ↑ **½ mark**

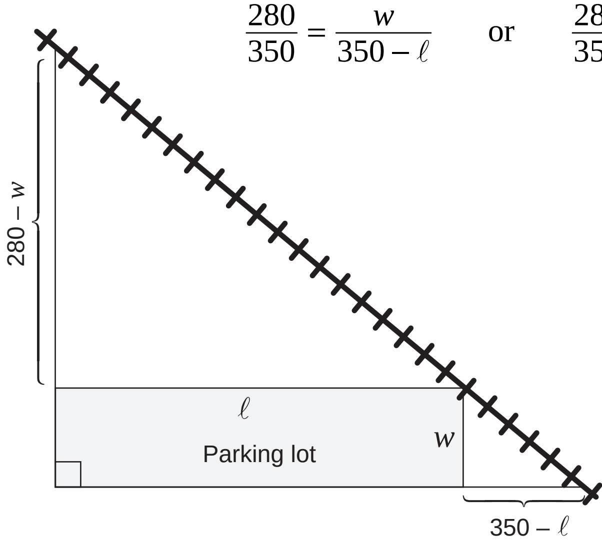
Base of box should measure $2.7\,\text{m} \times 3.7\,\text{m}$

8. A straight section of railroad tracks crosses two highways 350 m and 280 m from an intersection, as shown in the diagram below. A parking lot is to be constructed at the intersection.



a) Express the width of the parking lot, w, in terms of the length, ℓ. **(2 marks)**

Solution



$$\frac{280}{350}=\frac{w}{350-\ell} \quad \text{or} \quad \frac{280}{350}=\frac{280-w}{\ell} \quad \text{or} \quad \frac{280-w}{\ell}=\frac{w}{350-\ell}$$ ← **1 mark**

all give $280\ell+350w=98\ 000$ ← $\frac{1}{2}$ **mark**

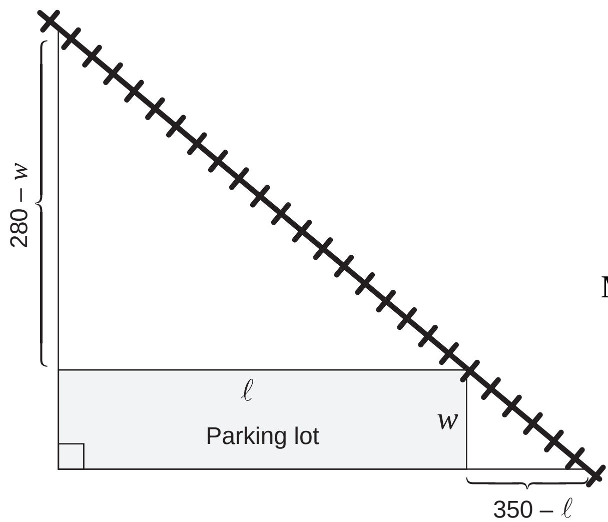
$$4\ell+5w=1\ 400$$

$$w=280-\ .8\ell$$ ← $\frac{1}{2}$ **mark**

b) Find the dimensions of the largest rectangular parking lot that can be constructed between the highways and the railroad. **(2 marks)**

If providing a graphical solution, state the function(s) used, sketch the graph, indicate appropriate window dimensions and clearly explain how your solution is derived from the graph.

Solution



$A = \ell w$ ← $\frac{1}{2}$ **mark**

$= \ell(280 - 0.8\ell)$

$= 280\ell - 0.8\ell^2$ ← $\frac{1}{2}$ **mark**

Max occurs in graph of $y = 280x - 0.8x^2$

when $x = 175$ ← $\frac{1}{2}$ **mark**

$w = 280 - 0.8\ell$

$= 280 - 0.8(175)$

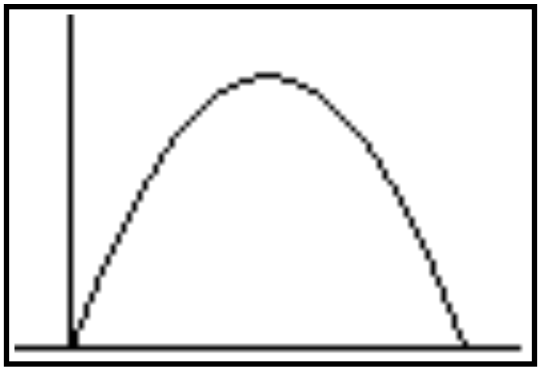
$= 140$ ← $\frac{1}{2}$ **mark**

$\therefore$ dimensions are $175 \text{ m} \times 140 \text{ m}$

b) Find the dimensions of the largest rectangular parking lot that can be constructed between the highways and the railroad. **(2 marks)**

If providing a graphical solution, state the function(s) used, sketch the graph, indicate appropriate window dimensions and clearly explain how your solution is derived from the graph.

Alternate Solution



$Y_1 = 280x - .8x^2$ ← **½ mark** for equation

$x\ [-50,\ 400]$ $y\ [-200,\ 30\ 000]$ ← **½ mark** for window dimensions

Maximum value occurs when $x = 175$ ← **½ mark**

$$\begin{aligned} w &= 280 - 0.8\ell \\ &= 280 - 0.8(175) \\ &= 140 \end{aligned}$$ ← **½ mark**

END OF KEY