Applications of Mathematics 12

June 1998 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers	Sub-Organizers
1. Problem Solving	A
2. Number	B, C
3. Patterns and Relations	D, E, F
4. Shape and Space	G, H
5. Statistics and Probability	I, J

Part A: Multiple Choice

Q	K	C	CO	PLO
1.	C	K	2	B1
2.	B	U–	2	B1
3.	C	U–	2	B2
4.	C	U–	2	B1, B2
5.	A	U+	2	B2
6.	C	H	2	B3
7.	A	H	2	B3
8.	B	U–	2	C2
9.	C	U–	2	C1
10.	B	H	2	C1
11.	D	K	3	D2
12.	B	U–	3	D1
13.	B	U+	3	D3
14.	A	U+	3	D4
15.	D	H	3	D1
16.	A	K	3	E3
17.	D	U–	3	E1
18.	A	K	3	F2
19.	D	U+	3	F2
20.	D	U+	3	F3
21.	C	H	3	F2
22.	A	K	4	G3
23.	C	U–	4	G3
24.	C	U+	4	G4
25.	B	H	4	G3, 4
26.	A	U–	4	H2
27.	B	U–	4	H1
28.	D	U–	4	H2
29.	D	U+	4	H2
30.	D	U+	4	H2
31.	B	H	4	H2
32.	C	H	4	H2
33.	C	K	5	I4
34.	B	U+	5	I3
35.	D	K	5	J3
36.	A	U–	5	J5
37.	B	U–	5	J4
38.	A	U–	5	J7
39.	B	U+	5	J1
40.	A	H	5	J6
41.	D	H	5	J5
42.	B	U–	1	A1
43.	C	U+	1	A1
44.	D	U+	1	A1
45.	B	H	1, 5	A2, H2

Multiple Choice = 45 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1.	1	U–	2	5	I2
2.	2	U–	2	3	F1, 3
3.	3	U+	3	2	B3
4a.	4	U+	1	4	H2
4b.	5	U+	2	4	H2
5.	6	U+	3	2	C2
6a.	7	U+	3	3	E3, 4
6b.	8	U+	1	3	E3, 4
7a.	9	U+	1	5	I3
7b.	10	U+	1	5	I3
8a.	11	U+	2	5	J3
8b.	12	U+	1	5	J3
9.	13	H	3	1	A2

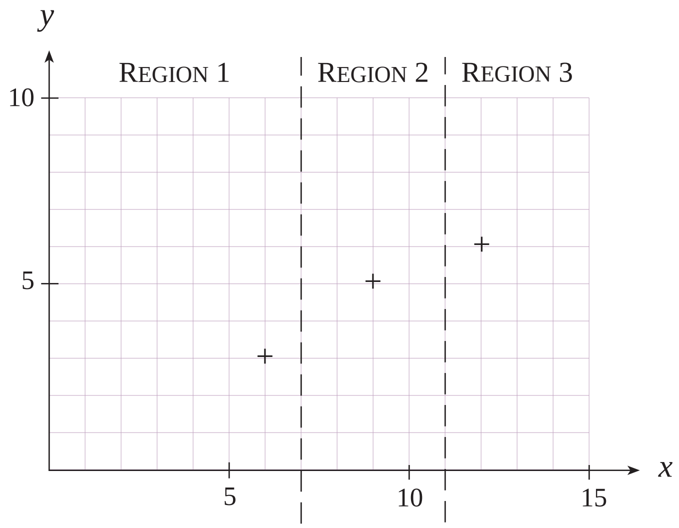
Written Response = 25 marks

Multiple Choice = 45 (45 questions)
Written Response = 25 (9 questions)
EXAMINATION TOTAL = 70 marks

LEGEND:

Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
CO = Curriculum Organizer
PLO = Prescribed Learning Outcome

Use the following graph to answer question 1.



1. In order to find the line of best fit for a set of data, the following three summary points were determined to represent the data set:

$$\underset{\text{Region 1}}{(6,\ 3)}\ ,\quad \underset{\text{Region 2}}{(9,\ 5)}\ ,\quad \text{and}\quad \underset{\text{Region 3}}{(12,\ 6)}$$

Find the equation of the median-median line. **(2 marks)**

SOLUTION:

$P_1(6,\ 3)$ and $P_3(12,\ 6)$

$$m = \frac{6-3}{12-6} = \frac{3}{6} = 0.5 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$y-3 = 0.5(x-6)$$

$$y = 0.5x$$

Points 1 and 3 are on the line $y = 0.5x$ ← **½ mark**

$P_2(9,\ 5)$

Parallel to this line and passing through the second point is

$$y-5 = 0.5(x-9)$$

$$y = 0.5x + 0.5 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$\frac{1}{3}$ of the way from $y = 0.5x$ to $0.5x + 0.5$ is

$$y = 0.5x + 0.1\overline{6}$$

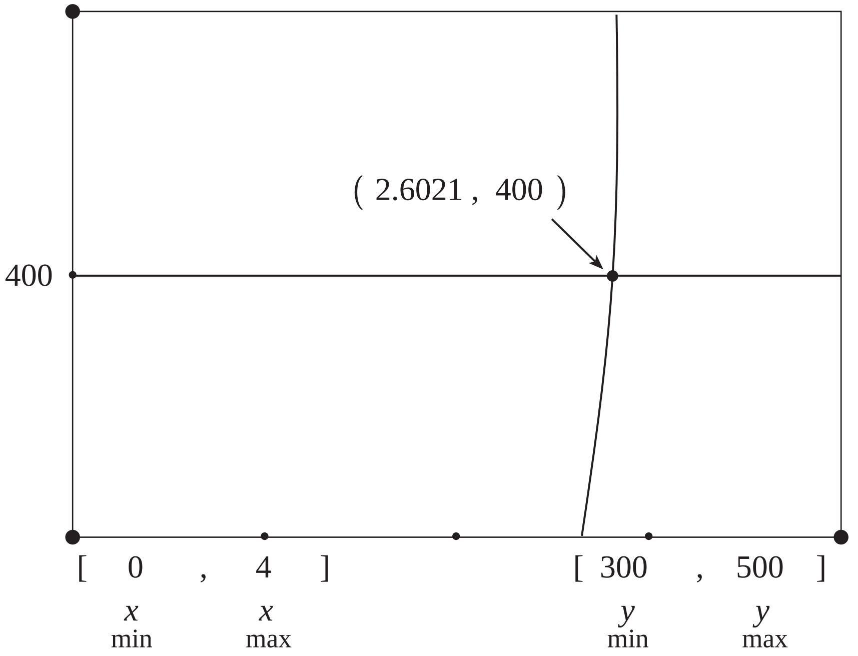
or $$y = \frac{1}{2}x + \frac{1}{6} \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

NOTE: Most students will use the statistical functions on the graphing calculator and provide the equation only.

2. Determine x such that $10^x = 400$. (Accurate to 4 decimal places.) **(2 marks)**

SOLUTION:

If a graph is used, sketch the graph, indicate appropriate window dimensions and explain clearly how your solution is derived from the graph.



$\frac{1}{2}$ **mark for graph**

To express 400 as a power of 10, zoom in on the graph until you read the x value for a y value of 400. $y = 10^{2.6021}$

OR

Graph $y = 400$ with $y = 10^x$ and find the intersection point.

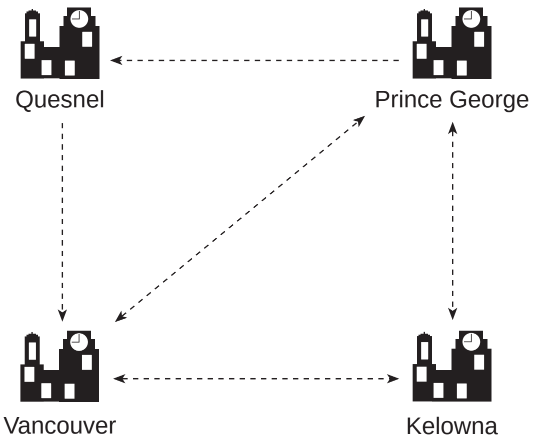
OR

Graph $y = 10^x - 400$ and find the zero.

← $\frac{1}{2}$ **mark**

Answer: $x = 2.6021$ ← **1 mark**

Use the following diagram to answer question 3.



3. The diagram above illustrates direct flights, between four cities, offered by a certain airline. Complete the following network matrix and determine all situations where a traveller can go from one city to a different city having exactly two routes with one stopover on each route.

(3 marks)

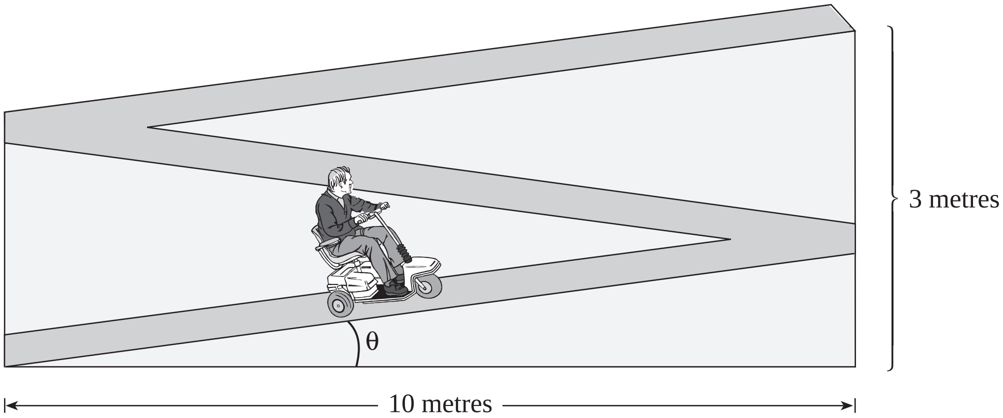
SOLUTION:

$$\boldsymbol{F} = \textbf{From}\ \begin{array}{cc} & \textbf{To} \\ & \begin{array}{cccc} \mathrm{V} & \mathrm{Q} & \mathrm{K} & \mathrm{P} \end{array} \\ \begin{array}{c} \mathrm{V} \\ \mathrm{Q} \\ \mathrm{K} \\ \mathrm{P} \end{array} & \left[\begin{array}{cccc} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{array}\right] \end{array} \left.\vphantom{\begin{array}{c}1\\1\\1\\1\\1\\1\end{array}}\right\} \leftarrow \textbf{1 mark} \qquad 1 \text{ wrong} - \tfrac{1}{2}\ \textbf{mark}$$

$$\boldsymbol{F}^2 = \textbf{From}\ \begin{array}{cc} & \textbf{To} \\ & \begin{array}{cccc} \mathrm{V} & \mathrm{Q} & \mathrm{K} & \mathrm{P} \end{array} \\ \begin{array}{c} \mathrm{V} \\ \mathrm{Q} \\ \mathrm{K} \\ \mathrm{P} \end{array} & \left[\begin{array}{cccc} 2 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ \textcircled{2} & 0 & 1 & 2 \end{array}\right] \end{array} \left.\vphantom{\begin{array}{c}1\\1\\1\\1\\1\\1\end{array}}\right\} \leftarrow \textbf{1 mark}$$

Prince George to Vancouver will allow the traveller to have exactly two routes and one stopover. ← **1 mark**

4. A wheelchair ramp is to be constructed in three sections, with a uniform rise throughout, as shown in the diagram. Over its entire distance, the ramp must rise 3 m. The horizontal distance covered is 10 m.



a) At what angle, θ, must the ramp be constructed from the horizontal? (Accurate to the nearest tenth of a degree.) **(1 mark)**

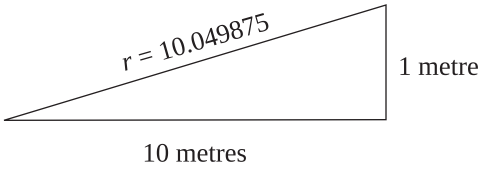
SOLUTION:

$\tan\theta = \frac{1}{10}$ ← ½ **mark** $\qquad \cos\theta = \frac{10}{\sqrt{101}} \qquad \sin\theta = \frac{1}{\sqrt{101}}$

$\theta = 5.71°$ ← ½ **mark**

b) If the cost of constructing the ramp is \$400 per lineal metre, determine the cost of the ramp to the nearest dollar. **(2 marks)**

SOLUTION:



$r^2 = 10^2 + 1^2$ ← ½ **mark**

$r = 10.04987562$ ← ½ **mark**

$3r = 30.14962686$ ← ½ **mark** for concept

$\times \quad \$400.00$

$\$12\ 059.85$

$\$12\ 060.00$ ← ½ **mark**

5. How much should a parent invest now, at 8% compounded quarterly, to accumulate \$24 000 in 18 years, for their child's post-secondary education? **(3 marks)**

SOLUTION:

$$A = P\left(1+\frac{r}{n}\right)^{nt} \qquad\qquad A = P(1+i)^t$$

$$\overset{\frac{1}{2}\textbf{ mark}}{\overset{\downarrow}{24\,000}} = P\overset{\textbf{1 mark}}{\overset{\downarrow}{\left(1+\frac{0.08}{4}\right)}}^{4(18)} \overset{\frac{1}{2}\textbf{ mark}}{\hookleftarrow} \qquad\qquad 24\ 000 = P(1.02)^{72}$$

$$P = \$5\,767.65 \qquad \leftarrow \textbf{1 mark}$$

$\therefore$ The parent needs to invest \$5 767.65

TVM Solve:

$$N = 72 \longrightarrow \$80.05$$

$$N = 216 \longrightarrow \$\ 8.91$$

$$24\ 000 = P(1.08)^{18} \longrightarrow 6005.98 \quad \leftarrow \textbf{1}\tfrac{1}{2}\textbf{ marks}$$

FV 24 000 ← $\frac{1}{2}$ **mark**

I 8%

$$\left.\begin{array}{ll}\text{N} & 18\\ \text{C/Y} & 4\end{array}\right\} \leftarrow \tfrac{1}{2}\textbf{ mark}$$

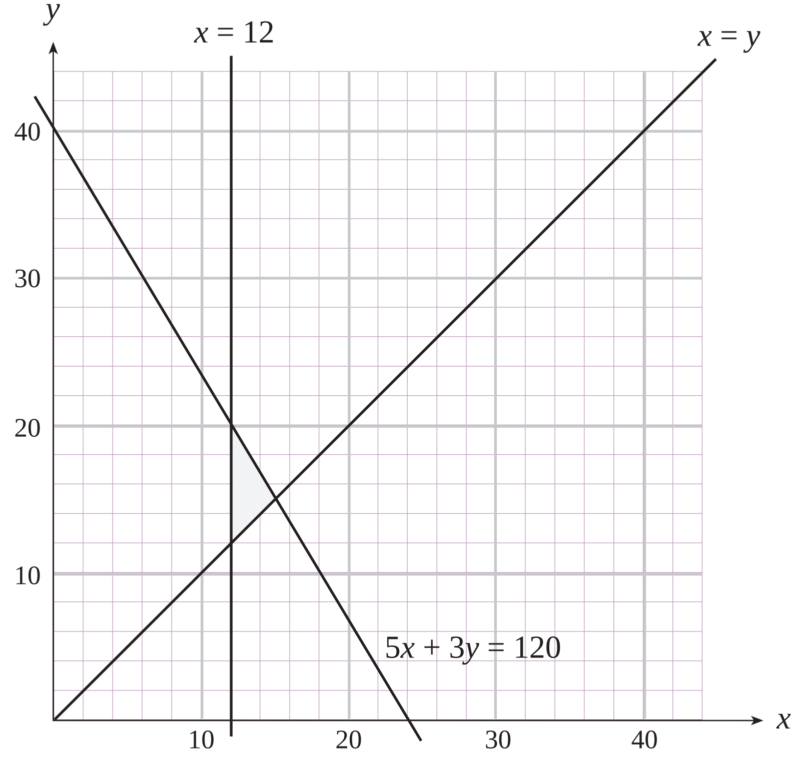
Using TVM Solver:

1st error	deduct **1 mark**
Others	deduct $\frac{1}{2}$ **mark**

6. An engineering firm is hiring x engineers at \$50 000 per year and y assistants at \$30 000 per year. The budget for these salaries is \$1 200 000 per year. There must be at least 12 engineers and at least one assistant for each engineer.

   a) Determine a system of inequalities that must be satisfied by the numbers of engineers and assistants and plot these inequalities on the grid provided. **(3 marks)**

SOLUTION:



3 marks

$\left(\begin{array}{l}\mathbf{\tfrac{1}{2}\ mark\ for\ each\ line} \\ \mathbf{\tfrac{1}{2}\ mark\ for\ shading}\end{array}\right)$

$50\ 000x + 30\ 000y \leq 1\ 200\ 000$ **or** $[5x + 3y \leq 120]$ $\leftarrow \frac{1}{2}$ **mark**

$x \geq 12$ $\leftarrow \frac{1}{2}$ **mark**

$y \geq x$ $\leftarrow \frac{1}{2}$ **mark**

$y \geq 12$

b) Determine the maximum number of engineers that can be hired. **(1 mark)**

SOLUTION:

Vertices of the feasible region:

$$\left.\begin{array}{l}(12,\ 12) \\ (12,\ 20) \\ (15,\ 15)\end{array}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

From the graph, the maximum number of engineers that can be hired is 15. $\leftarrow \frac{1}{2}$ **mark**

7. Statistics Canada lists the value of RRSP holdings by Canadians (in millions of dollars) for various years as shown in the table below.

YEAR	MILLIONS OF $
1970	187
1980	3 355
1985	7 769
1990	13 645
1994	16 072

a) Determine the least squares linear regression line for this data. **(1 mark)**

SOLUTION:

Use graphing calculator to determine the line as:

Domain entered as year:	Domain entered as number of years past 1970:	
$y \approx 695.317\ldots\ x - 1371164.4375\ldots$	$y \approx 695.317\ldots\ x - 1389.7758\ldots$	← **1 mark**

b) Calculate the projected value of RRSP holdings by Canadians in the year 2000. **(1 mark)**

SOLUTION:

Determine the value for year 2000 using any of a number of calculator methods.

$19 470 million dollars ← **1 mark**

8. The number of salmon n caught in a stream over the past 15 years is as follows:

NUMBER CAUGHT (n)	YEARS
$500 \le n < 1\,500$	4
$1\,500 \le n < 2\,500$	8
$2\,500 \le n < 3\,500$	2
$3\,500 \le n < 4\,500$	1

a) What is the mean of the number of salmon caught? **(2 marks)**

SOLUTION:

$\frac{1}{2}$ **mark for midpoint of interval**

$$\left.\begin{aligned} 1\,000 \times 4 &= 4\,000 \\ 2\,000 \times 8 &= 16\,000 \\ 3\,000 \times 2 &= 6\,000 \\ 4\,000 \times 1 &= \underline{4\,000} \end{aligned}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\underbrace{\frac{30\,000}{15}}_{\substack{\uparrow \\ \frac{1}{2}\ \textbf{mark}}} = 2\,000 \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

OR Graphing Calculator ← **2 marks**

b) What is the standard deviation of the number of salmon caught? **(1 mark)**

SOLUTION:

$$\sigma = \sqrt{\frac{1}{15}\left(4\times 1\ 000^2 + 8\times 2\ 000^2 + 2\times 3\ 000^2 + 4\ 000^2\right) - 2\ 000^2}$$

$$\approx 816.50 \qquad \leftarrow \textbf{1 mark}$$

OR

$$\sigma = \sqrt{\frac{1}{15}\left(4(1\ 000-2\ 000)^2 + 8(2\ 000-2\ 000)^2 + 2(3\ 000-2\ 000)^2 + 1(4\ 000-2\ 000)^2\right)}$$

$$\approx 816.50 \qquad \leftarrow \textbf{1 mark}$$

OR Graphing Calculator ← **1 mark**

9. The revenue R received by a company selling calculators is given by $R = xp$, where x is the number of calculators sold at price p in dollars. If $x = 4\ 000 - 25p$, find the price p that would generate the maximum revenue. **(3 marks)**

SOLUTION:

$$R = xp$$

$$R = \left(4\ 000 - 25p\right)p$$

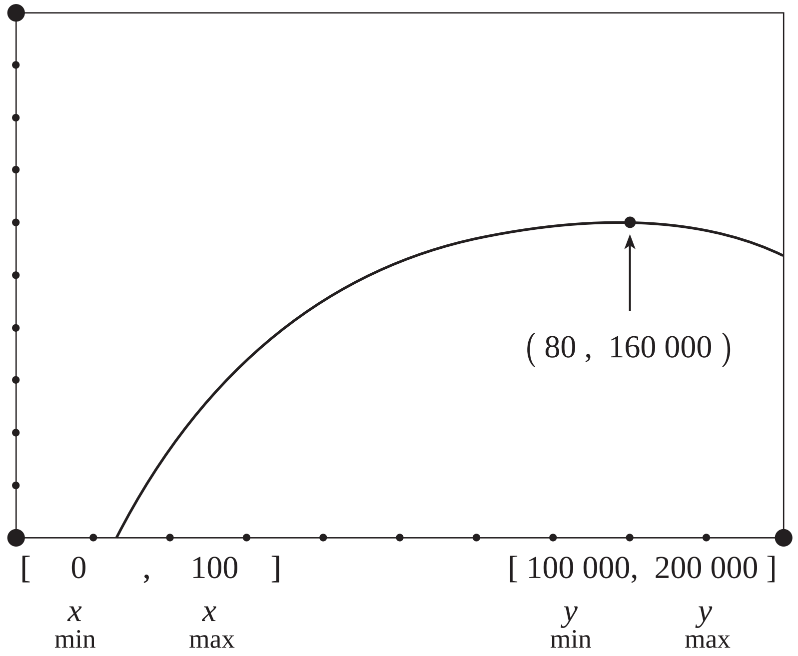
$$R = -25p^2 + 4\ 000p$$

← **1 mark**

Graph with graphing calculator

$$y = -25x^2 + 4\ 000x$$

If a graph is used, sketch the graph, indicate appropriate window dimensions and explain clearly how your solution is derived from the graph.



← **1 mark**

Find max at

$x = 80 \quad y = \$160\ 000$

← **1 mark**

$\therefore x = \$80$ would be the price generating maximum revenue.

END OF KEY