

Applications of Mathematics 12

June 2004 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers	Sub-Organizers
A. Problem Solving	A Integrated Throughout
B. Number	B Number Operations I
C. Number	C Number Operations II
D. Patterns and Relations	D Patterns
E. Shape and Space	E Measurement
F. Shape and Space	F 3-D Objects and 2-D Shapes
G. Statistics and Probability	G Chance and Uncertainty

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	D	K	1.5	B	B1	21.	A	U	1.5	E	E3
2.	A	U	1.5	B	B2	22.	A	U	1.5	E	E2
3.	C	U	1.5	B	B1	23.	D	U	1.5	E	E1
4.	A	U	1.5	B	B2	24.	D	U	1.5	E	E1
5.	B	H	1.5	B	B1	25.	C	H	1.5	E	E2
6.	B	H	1.5	B	B2	26.	B	H	1.5	E	E1, 4
7.	D	K	1.5	C	C2	27.	D	K	1.5	F	F1
8.	D	U	1.5	C	C4	28.	D	U	1.5	F	F2
9.	C	U	1.5	C	C3	29.	A	U	1.5	F	F2, F3
10.	B	U	1.5	C	C4	30.	B	H	1.5	F	F4
11.	C	U	1.5	C	C4	31.	A	H	1.5	F	F4
12.	D	H	1.5	C	C1	32.	A	K	1.5	G	G5
13.	C	K	1.5	D	D4	33.	C	K	1.5	G	G2
14.	D	K	1.5	D	D1	34.	A	U	1.5	G	G1
15.	D	U	1.5	D	D4	35.	A	U	1.5	G	G8
16.	C	U	1.5	D	D4	36.	B	U	1.5	G	G4
17.	B	U	1.5	D	D1	37.	C	U	1.5	G	G5
18.	C	U	1.5	D	D6	38.	B	H	1.5	G	G8
19.	B	H	1.5	D	D4	39.	D	H	1.5	G	G8
20.	B	H	1.5	D	D5, D6	40.	B	H	1.5	G	G2

Multiple Choice = 60 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1a.	1	U	2	C	C1, C2
1b.	2	U	1	C	C1, C2
1c.	3	U	2	C	C1, C2
2a.	4	U	1	G	G2
2b.	5	U	2	G	G2
2c.	6	U	2	G	G2
3a.	7	U	3	B	B1, B2
3b.	8	U	2	B	B1, B2
4a.	9	U	2	D	D2
4b.	10	U	3	D	D2
5a.	11	U	1	E	E2, E1
5b.	12	U	2	E	E2, E1
5c.	13	U	2	E	E2, E1
6a.	14	U	3	F	F3
6b.	15	U	2	F	F3

Written Response = 30 marks

Multiple Choice = 60 (40 questions)

Written Response = 30 (6 questions)

EXAMINATION TOTAL = 90 marks

LEGEND:

Q = Question Number

B = Score Box Number

PLO = Prescribed Learning Outcome

K = Keyed Response

S = Score

C = Cognitive Level

CO = Curriculum Organizer

PART B: WRITTEN RESPONSE

Value: 30 marks

Suggested Time: 45 minutes

1. A couple plans to purchase a townhouse that will cost \$120 000 after all fees and taxes are applied. They plan to provide a down payment of \$25 000. A financial institution is offering a mortgage rate of 4.25% per annum compounded semi-annually.

a) What will the monthly payments be if they choose a 20-year amortization period for the mortgage?

(2 marks)

Solution



Using TVM Solver:

20-year mortgage:

$$N = 240 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$I = 4.25\%$$

$$PV = -95\,000 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 2$$

PMT : END

$$\text{Solve for PMT} = 586.39 \quad \leftarrow \mathbf{1 \text{ mark}}$$

b) What will the monthly payments be if they choose a 10-year amortization period for the mortgage?

(1 mark)

Solution



Using TVM Solver:

10-year mortgage:

$$N = 120 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$I = 4.25\%$$

$$PV = -95\,000$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 2$$

PMT : END

$$\text{Solve for PMT} = 971.47 \quad \leftarrow \frac{1}{2} \text{ mark}$$

c) How much would they save if the mortgage is paid off in 10 years instead of 20 years?

(2 marks)

Solution

Total amount paid for a 20-year mortgage:

$$240(586.39) = \$140\,733.60 \quad \leftarrow \frac{1}{2} \text{ mark}$$

Total amount paid for a 10-year mortgage:

$$120(971.47) = \$116\,576.40 \quad \leftarrow \frac{1}{2} \text{ mark}$$

Difference:

$$= 140\,733.60 - 116\,576.40 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$= \$24\,157.20 \quad \leftarrow \frac{1}{2} \text{ mark}$$

1 $\frac{1}{2}$ marks was deducted if only the difference of the mortgage was found (ie. $971.47 - 586.39$).

2. The weights of males between the ages of 18 and 35 years in a certain city is normally distributed with a mean of 75.8 kg and a standard deviation of 3.8 kg.

a) What is the z -score for a weight of 85 kg?

(1 mark)

 **Solution**

$$\begin{aligned} z &= \frac{x - \mu}{\sigma} \\ &= \frac{85 - 75.8}{3.8} && \leftarrow \frac{1}{2} \text{ mark} \\ &= 2.42 && \leftarrow \frac{1}{2} \text{ mark} \end{aligned}$$

b) What percent of this age group of males weighs more than 80 kg?

(2 marks)

 **Solution**



$$\text{ShadeNorm}(80, 1E99, 75.8, 3.8) = 0.1345228$$

OR

$$\begin{aligned} \text{normalcdf}(80, 1E99, 75.8, 3.8) &= 0.1345228 \\ &\approx 13.45\% \end{aligned}$$

c) What percentage of this group weighs between 60 and 70 kg?

(2 marks)

 **Solution**



$$\text{ShadeNorm}(60, 70, 75.8, 3.8) = 0.0634495$$

OR

$$\begin{aligned} \text{normalcdf}(60, 70, 75.8, 3.8) &= 0.0634495 \\ &\approx 6.34\% \end{aligned}$$

3. Businesses in a certain community use either the postal service or a private courier to send their parcels. At present, 60% use the postal service and 40% use private couriers. A survey shows that 82% of those that use the postal service will continue to do so next year, and that 91% of those that use private couriers will continue using this method.

a) If this trend continues, what percentage of businesses will be using private couriers after 2 years? (Clearly show the initial state matrix and transition matrix used.) **(3 marks)**

 Solution

$$I = \begin{matrix} & \text{PS} & \text{PC} \\ \begin{bmatrix} 60 & 40 \end{bmatrix} & & \end{matrix} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\text{From } T = \begin{matrix} & \text{To} \\ & \text{PS} & \text{PC} \\ \begin{matrix} \text{PS} \\ \text{PC} \end{matrix} & \begin{bmatrix} 0.82 & 0.18 \\ 0.09 & 0.91 \end{bmatrix} \end{matrix} \quad \leftarrow 1 \text{ mark}$$

1 mark

↓

$$IT^2 = \begin{matrix} & \text{PS} & \text{PC} \\ \begin{bmatrix} 47.544\dots & 52.456 \end{bmatrix} & & \end{matrix}$$

∴ approximately 52.46% $\leftarrow \frac{1}{2} \text{ mark}$

b) Given that this trend continues for the long term, what percent of businesses will eventually be using the postal service? **(2 marks)**

 Solution

1 mark for any higher power concept.

$$\downarrow$$

$$IT^{100} = \begin{matrix} & \text{PS} & \text{PC} \\ \begin{bmatrix} 33.333\dots & 66.666\dots \end{bmatrix} & & \end{matrix}$$

↑
 $\frac{1}{2} \text{ mark}$

∴ eventually 33.33% will be using the postal service. $\leftarrow \frac{1}{2} \text{ mark}$

4. On a certain day, the angle of elevation of the sun at the equator is measured at specific times on a 24-hour clock. Data for these measurements is as follows.

Time (h)	Elevation (°)
06:00	0.1
08:00	26.9
10:00	52.3
12:00	66.6
14:00	53.0
16:00	27.8
18:00	0.9

- a) Determine the sine regression equation for this data.

(2 marks)

Solution



Enter data into the calculator: Time as List1 and Elevation as List2

SinReg L1, L2:

$$a = 37.0154\dots \leftarrow \frac{1}{2} \text{ mark}$$

$$b = 0.4013\dots \leftarrow \frac{1}{2} \text{ mark}$$

$$c = 3.0250\dots \leftarrow \frac{1}{2} \text{ mark}$$

$$d = 28.0967\dots \leftarrow \frac{1}{2} \text{ mark}$$

$$y = 37.02 \sin(0.40x + 3.02) + 28.10$$

OR

$$y = 37.015\dots \sin(0.401\dots x + 3.025\dots) + 28.0967\dots$$

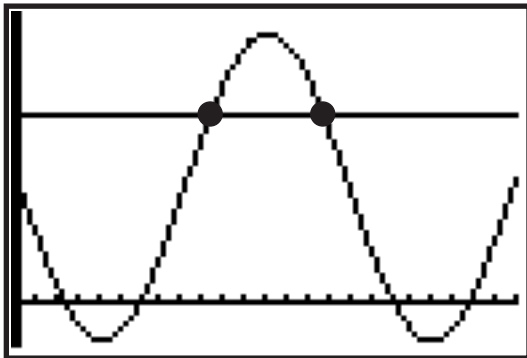
b) On this day, how long was the angle of elevation of the sun greater than 45° ?

(3 marks)

Solution



Enter into the graphing calculator:



x $[0, 24]$

y $[-10, 70]$

$$Y_1 = 37.02 \sin(0.40x + 3.02) + 28.10 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$Y_2 = 45 \quad \leftarrow \frac{1}{2} \text{ mark}$$

Determine the points of intersection:


$$9.30 \text{ h} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$14.77 \text{ h} \quad \leftarrow \frac{1}{2} \text{ mark}$$

9.30 h to 14.77 h = difference of 5.47 h

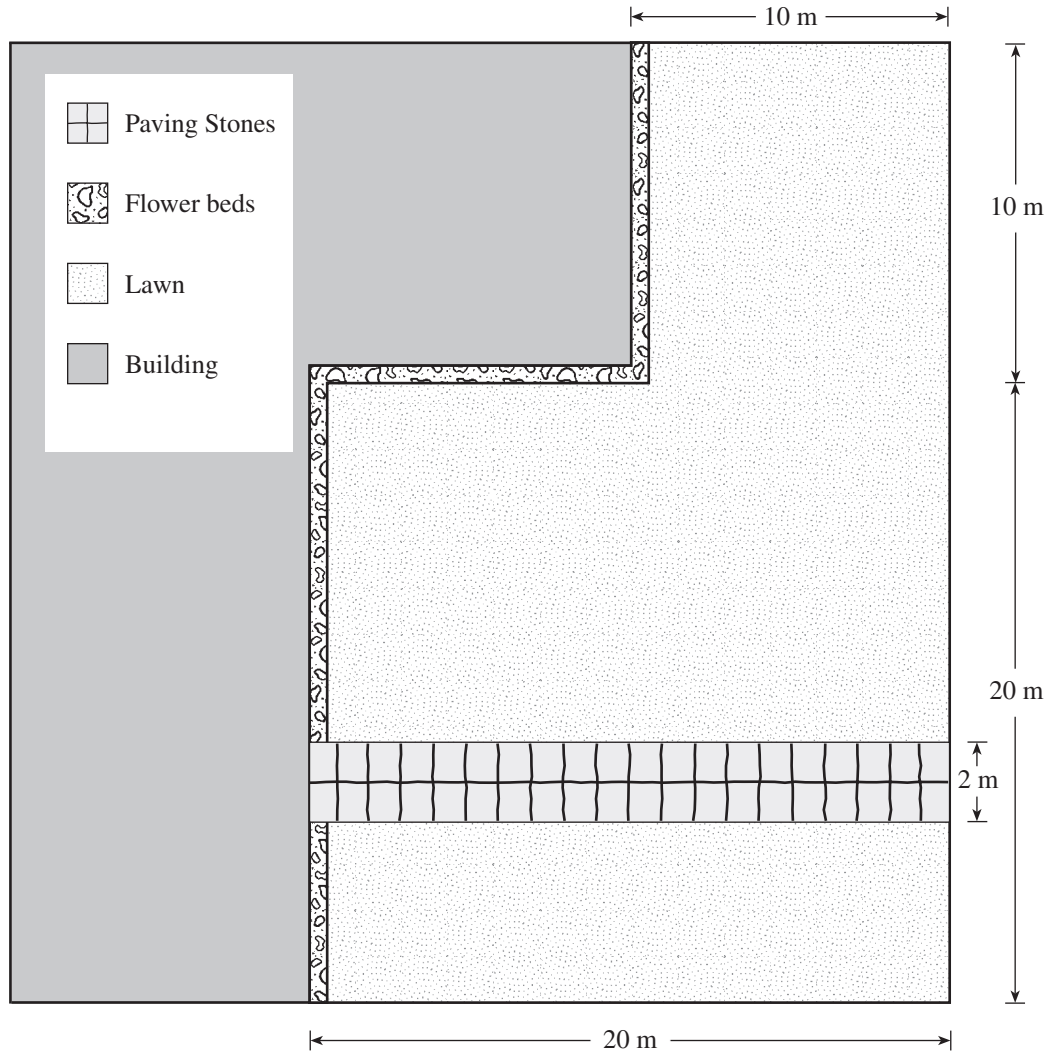
\therefore the altitude of the sun is greater than 45°
for 5.47 h (or 5h and 28 m). \leftarrow 1 mark for

concept subtraction

Note:  If intersection points time coordinates are not rounded to 2 decimal places then the correct answer would be 5.46 h.

5. Some landscaping needs to be done to the front of an office building with dimensions as shown in the diagram. The cost of each item to be used is listed in the following table.

Item	Price
Paving Stones (1 m by 1 m)	\$7.50 each
Top Soil	\$20/m ³
Lawn Seed	\$15/100 m ²



a) Determine the cost of the paving stones.

(1 mark)

 **Solution**

$$20 \times 2 = 40 \text{ m}^2$$

Need 40 @ \$7.50 each

$$= 40(7.50) = \$300 \quad \leftarrow \text{1 mark}$$

Full marks for approximate answers.

b) Determine the cost of the flower beds if the top soil is 40 cm deep and the flower beds are 50 cm wide.

(2 marks)

 **Solution**

$$\text{Area of flower beds can be found in various ways} = 19 \text{ m}^2 \quad \leftarrow \text{1 mark}$$

$$\text{Volume of topsoil is } 19(0.4) = 7.6 \text{ m}^3 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\text{Cost of topsoil in flower beds is } 7.6(\$20) = \$152 \quad \leftarrow \frac{1}{2} \text{ mark}$$

c) Determine the cost of preparing the lawn if it requires 10 cm of topsoil plus the seeding.

(2 marks)

 **Solution**

$$\begin{aligned} \text{Area of lawn} &= (19.5 \times 18) + (9.5 \times 10) \\ &= 446 \text{ m}^2 \end{aligned}$$

$$\text{Cost of seeding} = 4.46(15) = \$66.90$$

$$\text{Volume of topsoil} = 4.46(0.10) = 44.60 \text{ m}^3$$

$$\text{Cost of topsoil} = 44.6(20) = \$892$$

$$\text{Cost of preparing lawns} = \$892 + \$66.90 = \$958.90$$

} ← 1 mark

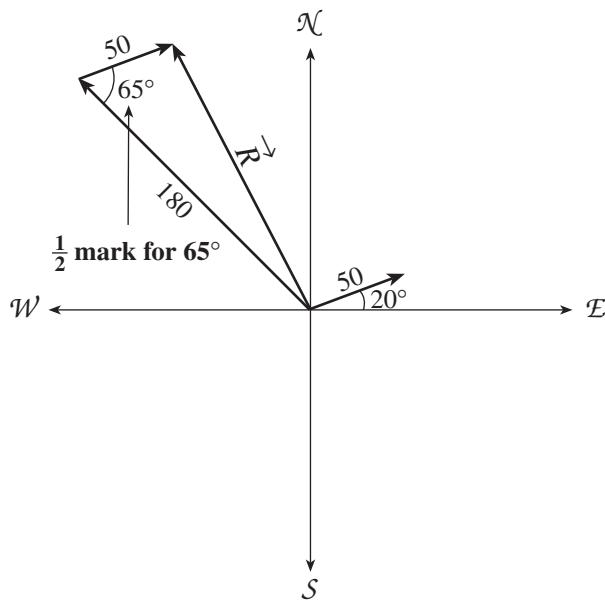
} ← 1 mark

6. An aircraft is cruising at 180 km/h at $[315^\circ]$. The wind is blowing towards $[070^\circ]$ at a speed of 50 km/h.

a) Use the axes provided to sketch the vectors and solve for the magnitude of the resultant velocity vector. (Answer to the nearest km/h.)

(3 marks)

Solution



$$(\vec{R})^2 = 50^2 + 180^2 - 2(50)(180)\cos 65^\circ \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$= 27\,292.87129 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\vec{R} = 165.21$$

Answer = 165 km/h

$\leftarrow \frac{1}{2} \text{ mark}$

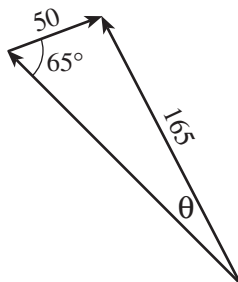
$\leftarrow 1 \text{ mark for vectors without } 65^\circ$

Full marks for precise scale diagram.

b) Determine the direction of the resultant vector. (Answer to the nearest degree.)

(2 marks)

Solution



$$\frac{\sin 65^\circ}{165} = \frac{\sin \theta}{50} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\sin \theta = \frac{50 \sin 65^\circ}{165} = 0.2746... \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\theta = \sin^{-1}(0.2746...) = 15.9... \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\therefore \text{the direction is } 315^\circ + 16^\circ = [331^\circ] \quad \leftarrow \frac{1}{2} \text{ mark}$$

Full marks for students that correctly broke the vectors into components.

END OF KEY