Applications of Mathematics 12

June 2003 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers	Sub-Organizers
A. Problem Solving	A Problem Set, Case Studies
B. Number	B Number Operations
C. Patterns and Relations	C Variables and Equations
D. Patterns and Relations	D Patterns
E. Shape and Space	E Measurement
F. Shape and Space	F 3-D Objects and 2-D Shapes
G. Statistics and Probability	G Chance and Uncertainty

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	A	K	1.5	B	B1	19.	A	U	1.5	E	E2
2.	B	K	1.5	B	B1	20.	B	U	1.5	E	E2
3.	D	U	1.5	B	B1	21.	C	U	1.5	E	E2
4.	A	U	1.5	B	B1	22.	D	H	1.5	E, A	E1; A1
5.	D	U	1.5	B	B1	23.	D	K	1.5	F	F1
6.	B	H	1.5	B	B2	24.	D	U	1.5	F	F3
7.	D	U	1.5	C	C2	25.	A	K	1.5	G	G8
8.	B	U	1.5	C	C3	26.	A	K	1.5	G	G8
9.	B	U	1.5	C	C3	27.	B	U	1.5	G	G5
10.	D	K	1.5	D	D1	28.	A	U	1.5	G	G5
11.	C	U	1.5	D	D1	29.	D	U	1.5	G	G8
12.	C	U	1.5	D	D3	30.	C	H	1.5	G, A	G4; A1
13.	C	U	1.5	D	D3	31.	B	H	1.5	G	G5
14.	A	U	1.5	D	D3	32.	A	H	1.5	G	G8
15.	B	U	1.5	D	D6	33.	C	U	1.5	G	G1
16.	D	H	1.5	D	D6	34.	B	U	1.5	G	G2
17.	C	H	1.5	D	D6	35.	B	H	1.5	G	G2
18.	C	U	1.5	E	E3	36.	C	H	1.5	G	G2

Multiple Choice = 54 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1a.	1	U	3	B, G	B1; G8
1b.	2	U	3	B, G	B1; G8
2.	3	U	4	E	E1
3a.	4	U	3	D	D3
3b.	5	U	2	D	D3
4a.	6	U	2	G	G5
4b.	7	U	2	G	G5
4c.	8	U	2	G	G5

Written Response = 21 marks

Part C: Case Studies

Q	B	C	S	CO	PLO
5a.	9	U–	2	C	C3
5b.	10	U+	5	C	C3
5c.	11	U+	3	C	C3
5d.	12	U+	2	C	C3
5e.	13	H	1	C	C3
6a.	14	U+	4	F, A	F1, 3, 4; A1, 4, 9
6b.	15	U+	5	F, A	F1, 3, 4; A1, 4, 9
6c.	16	H	3	F, A	F1, 3, 4; A1, 4, 9

Case Study = 25 marks

Multiple Choice = 54 (36 questions)
Written Response = 21 (4 questions)
Case Studies = 25 (2 questions)
EXAMINATION TOTAL = 100 marks

LEGEND:

Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
CO = Curriculum Organizer
PLO = Prescribed Learning Outcome

PART B: WRITTEN RESPONSE

Value: 21 marks **Suggested Time: 30 minutes**

1. A basketball team is playing in a tournament. The probability that they will win their first game is 70%. Their coach has observed that if the team wins a game, the probability that they will win their next game is 80%. However, if the team loses a game, the probability that they will win their next game falls to 40%.

   a) Write the initial probability matrix and the transition matrix which could be used to predict the team's wins and losses in future games. **(3 marks)**

Solution

$$P = \begin{matrix} \text{W} & \text{L} \end{matrix} \begin{bmatrix} 0.7 & 0.3 \end{bmatrix} \quad \leftarrow \textbf{1 mark}$$

$$T = \textbf{From} \begin{matrix} \\ \text{W} \\ \text{L} \end{matrix} \begin{matrix} \textbf{To} \\ \text{W} \quad \text{L} \end{matrix} \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix} \begin{matrix} \\ \leftarrow \textbf{1 mark} \\ \leftarrow \textbf{1 mark} \end{matrix}$$

b) Determine the probability the team will lose its third game. **(3 marks)**

Solution

$$PT^2 = \begin{matrix} \text{W} & \text{L} \end{matrix} \begin{bmatrix} 0.672 & 0.328 \end{bmatrix} \quad \leftarrow \textbf{1 mark}$$

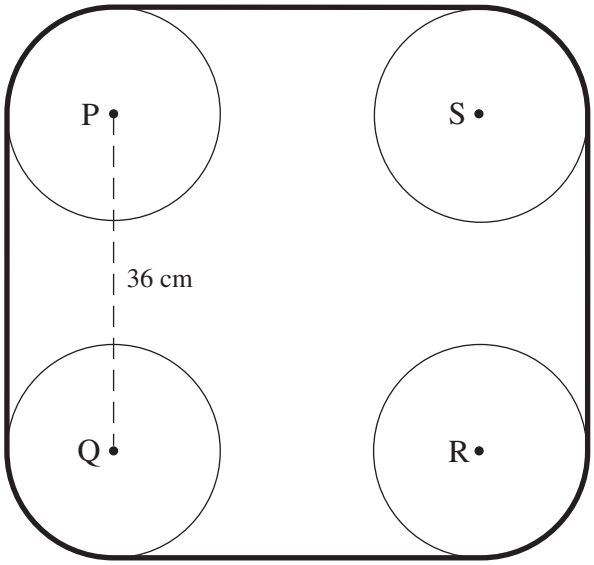
$\uparrow$

$\mathbf{1\frac{1}{2}}$ **marks**

$\therefore$ the probability is $0.328 \approx 0.33$ $\leftarrow \mathbf{\frac{1}{2}}$ **mark**

2. Four pulleys, each with a diameter of 12 cm, are connected by a belt, as shown in the diagram. The centres of pulleys P, Q, R, and S form a square. The distance between centres P and Q is 36 cm. Determine the total length of the belt around the outside of the pulleys. **(4 marks)**

Solution



Total length between the centres $PQ + QR + RS + SP = 4(36) = 144$ cm. ← **1 mark**

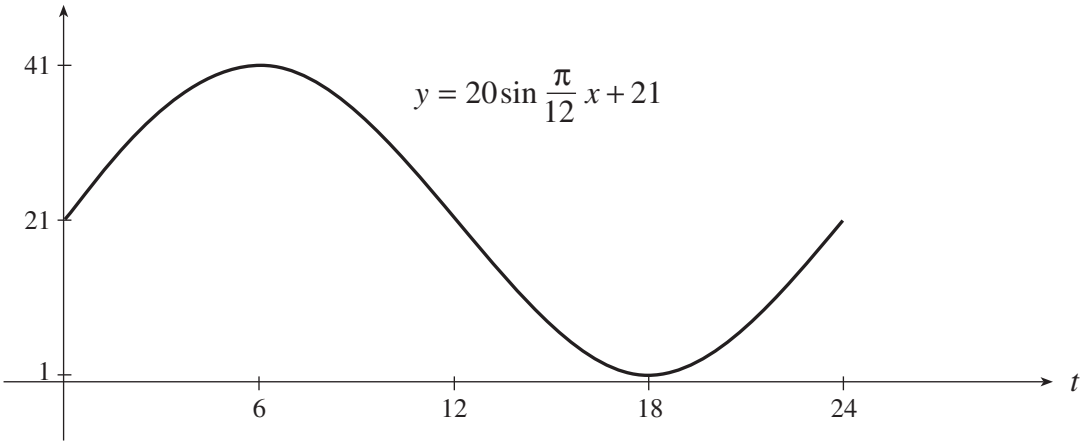
The curved part of the belt equals the circumference of one circle, which is 12π cm. ← **2 marks**

The total length of the belt is $144 + 12\pi$ or 181.70 cm. ← **1 mark**

3. The height, h, in metres of a Ferris wheel seat above the ground varies sinusoidally with time. The radius of the Ferris wheel is 20 m, the wheel makes one rotation every 24 s, and the lowest height of the seat is 1 m above the ground. Bill is riding this Ferris wheel, which stops periodically to let other riders on. At one stop, Bill is at a height of 21 m above the ground. When the ride resumes, Bill moves upward.

   a) Determine a sinusoidal function to describe Bill's height above the ground as a function of time. **(3 marks)**

Solution



$\frac{1}{2}$ mark given for the graph if there was no work after it.

Period $= \frac{2\pi}{b} = 24 \rightarrow b = \frac{\pi}{12}$ ← **1 mark**

Amp = 20 ← **$\frac{1}{2}$ mark**

Vertical displacement = 21 ← **$\frac{1}{2}$ mark**

Phase shift = 0

$y = a\sin b(x - c) + d$

$y = 20\sin\frac{\pi}{12}x + 21$ ← **1 mark**

b) Determine Bill's height above the ground 15 s after the Ferris wheel starts moving again.
(2 marks)

Solution

By calculator when x is 15, $y = 6.85786\ldots \approx 6.86$ m

↑ **1 mark** ↑ **1 mark**

1 mark given if answer was done with calculator in degrees.

4. A multiple-choice test has five questions with four choices for each question. A student guesses at all the answers without reading any of the questions.

a) What is the probability she will get all the answers correct? **(2 marks)**

Solution

$\left(\frac{1}{4}\right)^5 = \frac{1}{1024} = 0.0009765625$ ← **2 marks**

b) What is the probability she will get exactly three answers correct? **(2 marks)**

Solution

$\text{binompdf}\left(5,\ 0.25,\ 3\right) = \frac{45}{512} \approx 0.0879 \approx 8.79\%$ ← **2 marks**

or

${}_5C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2 = \frac{45}{512} \approx 0.08789\ldots \approx 8.79\%$ ← **2 marks**

c) What is the probability she will get three or more answers correct? **(2 marks)**

Solution

$1 - \text{binomcdf}\left(5,\ 0.25,\ 2\right) = \frac{53}{512} \approx 0.1035 \approx 10.35\%$ ← **2 marks**

or

$1 - \left({}_5C_0\left(\frac{3}{4}\right)^5 + {}_5C_1\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 + {}_5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3 \right) = \frac{53}{512} \approx 0.1035 \approx 10.35\%$ ← **2 marks**

or

$\binom{5}{3}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2 + \binom{5}{4}\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right) + \binom{5}{5}\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^0 = \frac{53}{512} \approx 0.1035 \approx 10.35\%$ ← **2 marks**

PART C: CASE STUDIES

Value: 25 marks **Suggested Time: 30 minutes**

5. Megan is buying a car and has agreed on the purchase price of $18 000. She will receive $5 500 as a trade-in value for her old car. She will only have to pay one tax (PST at 7.5%) on the amount after the trade-in has been deducted.

   a) What is the cost of her new car, including taxes, after trade-in? **(2 marks)**

Solution

$$(18\,000 - 5\,500)(1.075) = \$13\,437.50$$

↑ **1 mark** ↑ **½ mark** ↑ **½ mark**

b) Megan has an additional $1 000 which she intends to use as a down payment. She is trying to choose between a 2-year or a 3-year loan to finance the balance of her purchase. Given that the bank offers a rate of 4% per annum compounded semi-annually, with payments at the end of each month, complete the table below. **(5 marks)**

Solution

	2-year loan	**3-year loan**	
Loan amount after down payment	$12 437.50	$12 437.50	← **½ mark each**
Monthly payment	539.92	367.02	← **1 mark each**
Total of all payments	12 958.08	13 212.72	← **½ mark each**
Interest paid	520.58	775.22	← **½ mark each**

c) Megan calculates her operating costs as follows:

- driving to and from work = 48 km per day for 200 days per year
- pleasure = 6 000 km per year
- gas consumption = 8.3 litres per 100 km
- cost of gas = 62.9 cents per litre

Determine her average monthly fuel cost. **(3 marks)**

Solution

$(48 \times 200) + 6\,000 = 15\,600$ km ← **1 mark**

$\frac{15\,600}{100}(8.3)(0.629) = \$814.43$ per year

∴ yearly cost is \$814.43 ← **1 mark**

monthly cost is \$67.87 ← **1 mark**

d) Megan's insurance will cost \$1 485 per year. If Megan chooses the 3-year loan, determine her total **monthly** cost to operate her vehicle (including her loan, fuel and insurance).
(2 marks)

Solution

Total monthly cost for Megan's loan, fuel and insurance:

$367.02 + 67.87 + \frac{1485}{12} = \$558.64$ ← **2 marks**

e) Give two other factors Megan might include in determining the cost of owning and operating her car for a year. **(1 mark)**

Solution

$\frac{1}{2}$ **mark** each for any **two** reasonable answers

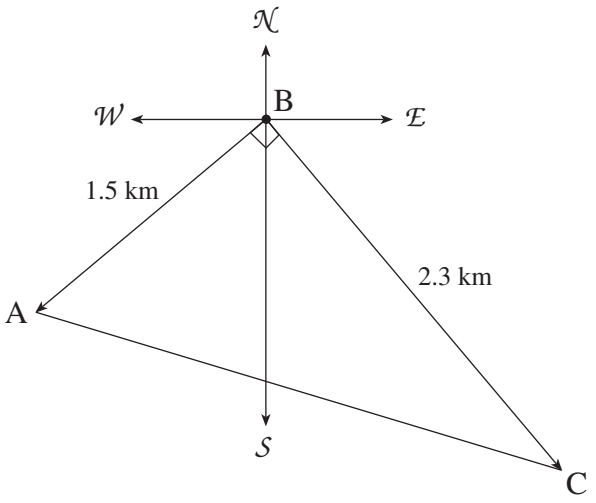
- parking
- new tires
- accidents
- major repairs
- oil changes/lubrication
- car washes

6. The navigation buoys in Georgian Bay are located at the corners of a right triangle.

- Buoy C is 2.3 km at a bearing of 140° from buoy B.
- Buoy A is 1.5 km at a bearing of 230° from buoy B.
- Buoy B is at the right angle of the triangle.

a) Draw a vector diagram showing the relative location of each of the three buoys. Include appropriate distances. Then determine the direction (bearing) from buoy A to buoy B. **(4 marks)**

Solution



← **3 marks**

Bearing from buoy A to buoy B is 50°. ← **1 mark**

b) Determine the distance, to the nearest 0.1 km, and the direction (bearing), to the nearest degree, from buoy A to buoy C. **(5 marks)**

Solution

$$AC^2 = AB^2 + BC^2 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$= 1.5^2 + 2.3^2 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$= 7.54 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$AC = \sqrt{7.54} \approx 2.7 \text{ km} \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$\tan A = \frac{2.3}{1.5} \qquad \leftarrow \textbf{1 mark}$$

$$\angle A = 56.8886\ldots^\circ \approx 57^\circ \qquad \leftarrow \textbf{1 mark}$$

$\therefore$ the direction from buoy A to buoy C is 107°. $\leftarrow$ **1 mark**

Alternate Solution

$$AC^2 = AB^2 + BC^2 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$= 1.5^2 + 2.3^2 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$= 7.54 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$AC = \sqrt{7.54} \approx 2.7 \text{ km} \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$\cos A = \frac{AB^2 + AC^2 - BC^2}{2(AB)(AC)} \qquad \leftarrow \textbf{1 mark}$$

$$= \frac{1.5^2 + \sqrt{7.54}^2 - 2.3^2}{2(1.5)\left(\sqrt{7.54}\right)}$$

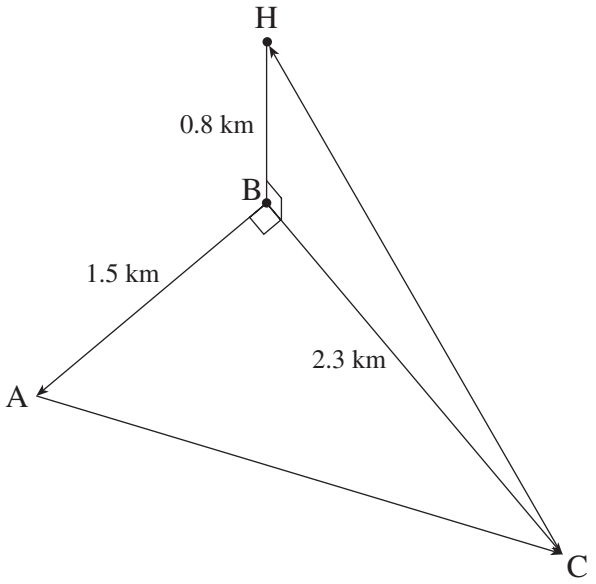
$$= 0.546267\ldots$$

$$\angle A = 56.8886\ldots^\circ \approx 57^\circ \qquad \leftarrow \textbf{1 mark}$$

$\therefore$ the direction from buoy A to buoy C is 107°. $\leftarrow$ **1 mark**

c) A helicopter is hovering at a height of 0.8 km directly above buoy B. Determine the angle of elevation (angle from the horizontal) from buoy C to the helicopter. **(3 marks)**

Solution



1 mark for diagram (illustrating helicopter at a right angle to the horizontal).

Angle above the horizon:

In ΔHBC,

$\tan C = \frac{0.8}{2.3}$ ← **1 mark (2 marks if no diagram)**

$C = 19.2°$

$\approx 19°$ ← **1 mark**

The helicopter is at 19° above the horizontal.

END OF KEY