Applications of Mathematics 12

June 2001 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers	Sub-Organizers
1. Problem Solving	A Problem Set
2. Number	B Matrices
	C Financial Decision-Making
3. Patterns and Relations	D Fractals
	E Linear Programming
	F Non-Linear Functions
4. Shape and Space	G Periodic Functions
	H Geometry Applications
5. Statistics and Probability	I Data Analysis
	J Applications of Probability

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	A	K	1.5	2	B2	16.	B	U	1.5	4	G3
2.	D	U	1.5	2	B2	17.	A	H	1.5	4	G4
3.	A	U	1.5	2	B3	18.	D	K	1.5	4	H2
4.	D	U	1.5	2	B3	19.	C	H	1.5	4	H2
5.	B	H	1.5	2	B3	20.	A	K	1.5	5	I4
6.	C	U	1.5	2	C1	21.	B	H	1.5	5	I2
7.	A	H	1.5	2	C2	22.	A	K	1.5	5	J5
8.	B	U	1.5	3	D4	23.	B	U	1.5	5	J5
9.	A	U	1.5	3	D3	24.	B	U	1.5	5	J3
10.	C	H	1.5	3	D3	25.	D	U	1.5	5	J4
11.	D	K	1.5	3	E2	26.	D	U	1.5	5	J6
12.	C	H	1.5	3	E3	27.	B	H	1.5	5	J1
13.	B	K	1.5	3	F2	28.	D	U	1.5	1	A1
14.	C	U	1.5	3	F2	29.	C	U	1.5	1	A1
15.	C	U	1.5	4	G3	30.	C	H	1.5	1	A1

Multiple Choice = 45 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1a.	1	U	2	5	I2, 3
1b.	2	U	1	5	I2, 3
1c.	3	U	2	5	I2, 3
2a.	4	U	3	2	B3
2b.	5	H	3	2	B3
3.	6	U	6	3	F3
4.	7	U	6	3	E4
5a.	8	U	5	1	A1
5b.	9	U	1	1	A1
6.	10	U	6	5	J6

Written Response = 35 marks

Part C: Case Study

Q	B	C	S	CO	PLO
7a.	11	U	3	4, 1	H2, A2
7b.	12	U	3	4, 1	H2, A2
7c.	13	U	3	4, 1	H2, A2
7d.	14	U	5	4, 1	H2, A2
7e.	15	U	2	1, 2	A2, C2
7f.	16	H	4	1, 2	A2, C2

Case Study = 20 marks

Multiple Choice = 45 (30 questions)
Written Response = 35 (6 questions)
Case Study = 20 (1 question)
EXAMINATION TOTAL = 100 marks

LEGEND:

Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
CO = Curriculum Organizer
PLO = Prescribed Learning Outcome

PART B: WRITTEN RESPONSE

Value: 35 marks **Suggested Time: 45 minutes**

1. The table below shows the scores for five students on a mathematics exam consisting of a non-calculator and a calculator section. Each section of the exam is scored out of 100.

Student	Non-Calculator Section Score	Calculator Section Score
A	75	86
B	60	54
C	83	91
D	65	64
E	87	95

a) Determine an equation for a least squares regression line for this data. **(2 marks)**

Solution

Enter the table as STAT lists on the graphing calculator. The required equation is:

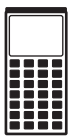
$y = 1.5\overline{30}x - 35.\overline{24}$ ***or*** $y = 0.62x + 25.295$

↑ ↑ ↑ ↑

1 mark each **1 mark each**

b) Determine the correlation coefficient. **(1 mark)**

Solution



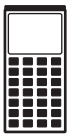
The correlation coefficient, with diagnostics on, is given on the calculator screen.

$r = 0.9775$ ← **1 mark**

$r^2 = 0.955$ ← **$\frac{1}{2}$ mark**

c) Use the regression equation from part a) to predict the score a student could expect to receive on the non-calculator section if he scores 78 on the calculator section. **(2 marks)**

Solution



Paste the regression equation as Y_1

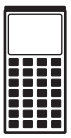
Enter $Y_2 = 78$ ← **1 mark**

Solve for the point of intersection:

$x = 74$ $y = 78$

∴ the student could expect to score 74 on the non-calculator section. ← **1 mark**

Alternate Solution



Substitute $y = 78$ in the regression equation.

$78 = 1.53\overline{03}x - 35.\overline{24}$ ← **1 mark**

$x = \left(78 + 35.\overline{24}\right) \div 1.53\overline{03}$

$= 74$ ← **1 mark**

∴ the student could expect to score 74 on the non-calculator section.

Note: $\frac{1}{2}$ mark if student substituted for x instead of y.

2. A cafe opens its outdoor patio only on days that it does not rain. From past records it was found that when it rained one day, the probability that it would rain the next day was 0.40. When it did not rain one day, the probability of rain the next day was 0.06.

   a) If it rains on Thursday, what is the probability that it will not rain on the following Sunday?
   (3 marks)

Solution

If it is raining on Thursday, the patio is closed. The initial matrix is

$$\begin{matrix} \begin{bmatrix} 1 & 0 \end{bmatrix} \\ \text{rain} \quad \text{no rain} \end{matrix} \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

The transition matrix is

$$\textbf{From} \quad \begin{matrix} \text{rain} \\ \text{no rain} \end{matrix} \begin{bmatrix} 0.40 & 0.60 \\ 0.06 & 0.94 \end{bmatrix} \qquad \leftarrow \textbf{1 mark}$$

$$\underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0.40 & 0.60 \\ 0.06 & 0.94 \end{bmatrix}^3}_{\uparrow \textbf{ 1 mark}} = \underbrace{\begin{bmatrix} 0.12664 & 0.87336 \end{bmatrix}}_{\uparrow \tfrac{1}{2}\textbf{ mark}}$$

$\therefore$ the probability that there will be no rain on Sunday is 0.87.

b) Over the long term, what percentage of days is the patio expected to be open? **(3 marks)**

Solution

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 0.40 & 0.60 \\ 0.06 & 0.94 \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \quad \leftarrow \textbf{1 mark}$$

$$\begin{aligned} .4x + .06y &= x && \leftarrow \tfrac{1}{2} \textbf{ mark} \\ .06y &= .6x \\ 6y &= 60x \\ y &= 10x \\ x + y &= 1 && \leftarrow \tfrac{1}{2} \textbf{ mark} \\ x + 10x &= 1 \\ 11x &= 1 \\ x &= \frac{1}{11}, \quad y = \frac{10}{11} && \leftarrow \tfrac{1}{2} \textbf{ mark} \end{aligned}$$

$\therefore$ the patio will be open 91% of the days. $\leftarrow \tfrac{1}{2}$ **mark**

Alternate Solution

Raise the transition matrix to a large enough power to reach a steady state.

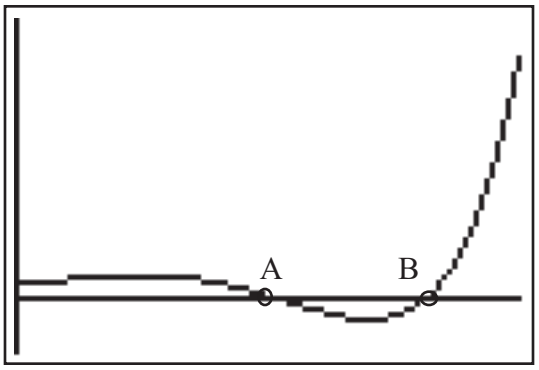
$$\underbrace{\begin{bmatrix} 0.40 & 0.60 \\ 0.06 & 0.94 \end{bmatrix}^{50}}_{\uparrow \textbf{2 marks}} = \begin{bmatrix} 0.\overline{09} & 0.\overline{90} \\ 0.\overline{09} & 0.\overline{90} \end{bmatrix}$$

$\therefore$ the patio will be open 91% of the days. $\leftarrow$ **1 mark**

Note: In this particular question, raising the transition matrix to a power of 23 or greater will result in the steady state as shown on the calculator.

3. Over an 18-hour period, the temperature, T, of a chemical reaction is given by the function $T = 100(2)^{\frac{x}{3}} - x^3$, where x represents the number of hours since the reaction began.
For how many **minutes** was the temperature below zero? **(6 marks)**

Solution



$Y_1 = 100(2)^{\frac{x}{3}} - x^3$

← **2 marks for graph with functions and windows**

x $[0,\ 20]$ y $[-500,\ 2\,500]$

Determine zeroes A 10.1176…
B 16.347… } ← **2 marks**

Time below zero $16.347\ldots - 10.1176\ldots$
$= 6.2294\ldots$ hours } ← **1 mark**

$= 373.77$ minutes
$= 374$ minutes } ← **1 mark**

4. A manufacturing company has an order for 6 000 boxes of stove bolts and 3 200 boxes of lag screws. Two of its factories produce both bolts and screws at the same time. Details of each factory's operations are shown in the table below. **(6 marks)**

	Stove Bolts (boxes per day)	**Lag Screws (boxes per day)**	**Factory Operating Costs Per Day**
Factory A	600	400	\$1 500
Factory B	1 000	400	\$2 000

List the constraints and the objective function, then solve the linear programming problem to determine the number of days each factory should be used to minimize the cost to fill the order.

Solution

Days for Factory A = a
Days for Factory B = b

$$\left.\begin{array}{c} a \geq 0,\ b \geq 0 \\ 600a + 1\,000b \geq 6\,000 \rightarrow 3a + 5b \geq 30 \\ 400a + 400b \geq 3\,200 \rightarrow a + b \geq 8 \end{array}\right\} \leftarrow \textbf{1 mark}$$

$$C = 1\,500a + 2\,000b \quad \leftarrow \textbf{1 mark}$$

Test corner points

$(0,\ 8) \quad \rightarrow \$16\,000$

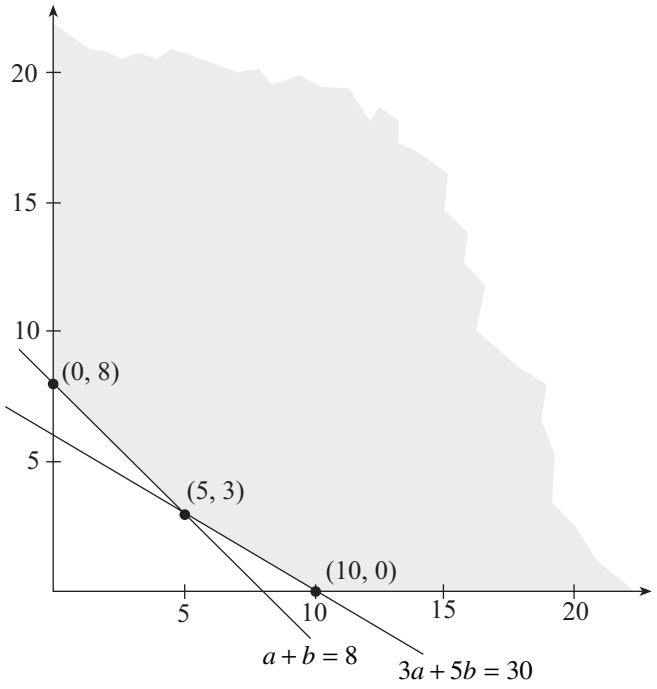
$\boxed{(5,\ 3) \quad \rightarrow \$13\,500}$

$(10,\ 0) \quad \rightarrow \$15\,000$

$\therefore$ Factory A should be used for 5 days and Factory B should be used for 3 days.

2 marks for values at corner points and answer

2 marks for graph with corner points identified



5. A car rental company can rent out all of its 100 cars in one day if it charges \$30 per day for each car. For each \$2 increase in price, the number of rentals decreases by 5. The average cost of maintenance for each car rented is \$10 per day.

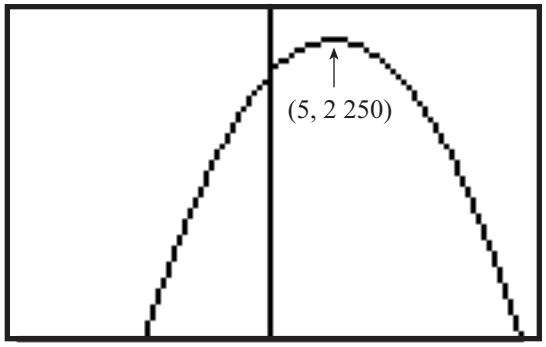
a) What should the car rental company charge per day to maximize profits? **(5 marks)**

Solution

Let x represent the number of \$2 increases.

$$\text{Profit} = (30 + 2x)(100 - 5x) - 10(100 - 5x)$$
$$= 2\ 000 + 100x - 10x^2$$

← **2 marks**



Graphical solution:
$Y_1 = 2\ 000 + 100x - 10x^2$

← **2 marks** for graph and window

$x\ [-20,\ 20]$ $y\ [0,\ 2\ 500]$

Maximum profit occurs when $x = 5$, or an increase of \$10
∴ the company needs to charge \$40/day. ← **1 mark**

b) What will be the maximum profit in one day? **(1 mark)**

Solution

The maximum profit will be \$2 250. ← **1 mark**

5. A car rental company can rent out all of its 100 cars in one day if it charges $30 per day for each car. For each $2 increase in price, the number of rentals decreases by 5. The average cost of maintenance for each car rented is $10 per day.

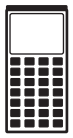
a) What should the car rental company charge per day to maximize profits? **(5 marks)**

Alternate Solution 1

Rental Charge ($)	No. of cars	Revenue ($)	Maintenance Costs ($)	Profit ($)
30	100	3 000	1 000	2 000
32	95	3 040	950	2 090
34	90	3 060	900	2 160
36	85	3 060	850	2 210
38	80	3 040	800	2 240
40	75	3 000	750	2 250
42	70	2 940	700	2 240
44	65	2 860	650	2 210

For full marks student needs to show a fairly complete table, along with an explanation that because of the symmetry of a quadratic function, the maximum occurs when the rental charge is $40/car per day.

Alternate Solution 2



List under L_1 on calculator "Rental Charges"
List under L_2 on calculator "Profit"

STAT CALC QuadReg, L_1, L_2, Y_1

$Y_1 = -2.5x^2 + 200x - 1\,750$

find maximum of $(40,\ 2\,250)$

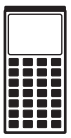
b) What will be the maximum profit in one day? **(1 mark)**

Solution

From the table, the maximum profit will be $2 250. ← **1 mark**

6. A multiple-choice test has 50 questions. For students who have prepared for the test, the probability that they will answer any question correctly is 0.75. A student who correctly answers 35 to 39 questions (inclusive) will receive a grade of B. What percentage of students who have prepared for the test will receive a grade of B? **(6 marks)**

Solution



Using the binomial cdf on a graphing calculator, ← **1 mark**

$\frac{1}{2}$ **mark**

$$\text{binomcdf}(50,\ 0.75,\ 39) - \text{binomcdf}(50,\ 0.75,\ 34) = 0.5747$$

$\frac{1}{2}$ **mark each** $\frac{1}{2}$ **mark each** **1 mark**

∴ 57% of these students will receive a grade of B on this examination. ← $\frac{1}{2}$ **mark**

6. A multiple-choice test has 50 questions. For students who have prepared for the test, the probability that they will answer any question correctly is 0.75. A student who correctly answers 35 to 39 questions (inclusive) will receive a grade of B. What percentage of students who have prepared for the test will receive a grade of B? **(6 marks)**

Alternate Solution

Using the normal approximation to the binomial,

$\mu = 50(0.75) = 37.5$ ← **1 mark**

$\sigma = \sqrt{50(0.75)(0.25)}$ ← **1 mark**

$= \sqrt{9.375}$

At $x = 34.5$ $\quad z = \dfrac{34.5 - 37.5}{\sqrt{9.375}} = -0.9798$ ← **1 mark**

area is $0.3365\ldots$ ← $\frac{1}{2}$ **mark**

At $x = 39.5$ $\quad z = \dfrac{39.5 - 37.5}{\sqrt{9.375}} = 0.6532$ ← **1 mark**

area is 0.2422 ← $\frac{1}{2}$ **mark**

The area between the two z-scores is 0.58 ← **1 mark**

$\therefore$ 58% of these students will receive a grade of B on this examination.

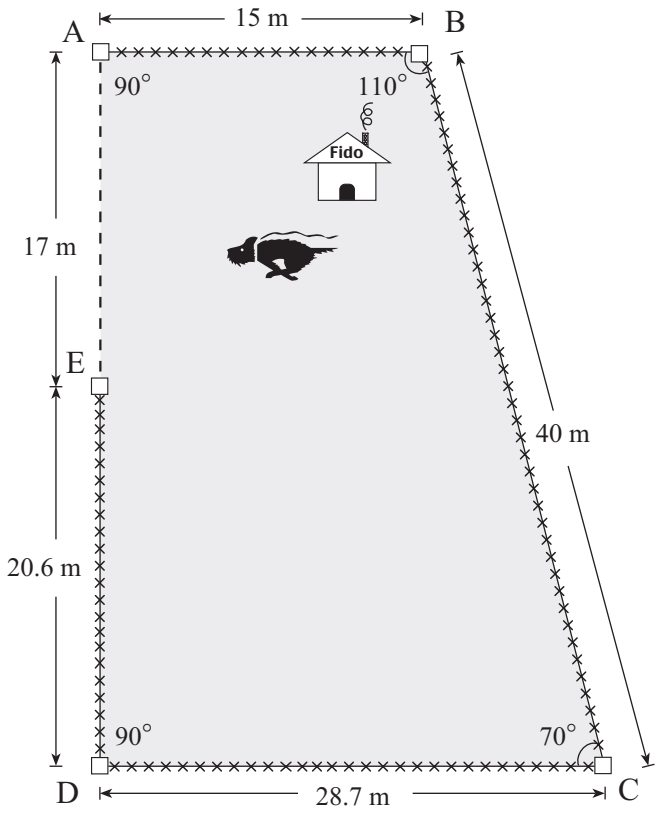
Note: If a student used 35 and 39 for normal approximation instead of 34.5 and 39.5, they obtained 48.08% and received 5 out of 6 marks.

PART C: CASE STUDY

Value: 20 marks **Suggested Time: 30 minutes**

7. An owner of a corner lot is going to fence and seed his backyard. The perimeter of the backyard is shown with dimensions and angles provided. The fence is to be built along the perimeter shown by ××××. The 17 m section from A to E shown by – – – – is unfenced.

   - Posts are already in place at locations A, B, C, D, and E, and are shown in the diagram as □.
   - Ignore the width of the posts in your calculations.
   - All costs given in this problem include appropriate taxes.



a) More posts need to be placed along the perimeter to support the fence to be built. If there is to be no more than 3 m between posts, and the cost of each post is $3.75, determine the minimum cost for the posts. (Ignore the width of the posts in your calculations.) **(3 marks)**

Solution

Number of posts is $4 + 13 + 9 + 6 = 32$

$\underbrace{\uparrow \quad \uparrow \quad \uparrow \quad \uparrow}$

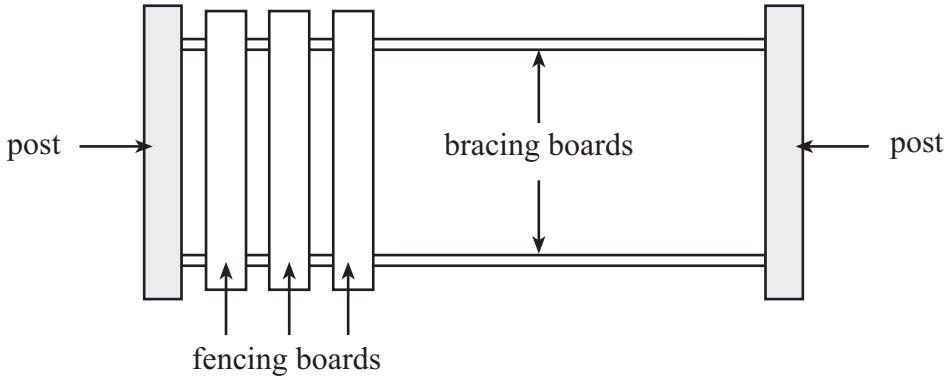
$\frac{1}{2}$ mark each (2 marks in total)

32 posts required @ $3.75 = $120.00

$\uparrow$ $\uparrow$

$\frac{1}{2}$ mark **$\frac{1}{2}$ mark**

Use the following diagram to answer questions 7b) and c).



b) Between each post, 2 horizontal bracing boards are attached, onto which the fencing boards are nailed, as shown in the diagram.

The bracing boards are bought in 3-metre lengths and cut to fit between each post. Each 3-metre bracing board costs \$2.25. Determine the cost of all the bracing boards needed to build the fence. **(3 marks)**

Solution

Number of bracing boards: $10 + 28 + 20 + 14 = 72$

$\underbrace{\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow}$

$\frac{1}{2}$ mark each **($2\frac{1}{2}$ marks in total)**

Bracing boards: $72 \times \$2.25 = \$162.00$ ← **$\frac{1}{2}$ mark**

c) Fencing boards are nailed to the bracing boards. The fencing boards are approximately 15 cm wide and cost \$1.10 per board. A 10 cm space is left between adjacent fencing boards. Determine the cost of all the fencing boards needed to build the fence. **(3 marks)**

Solution

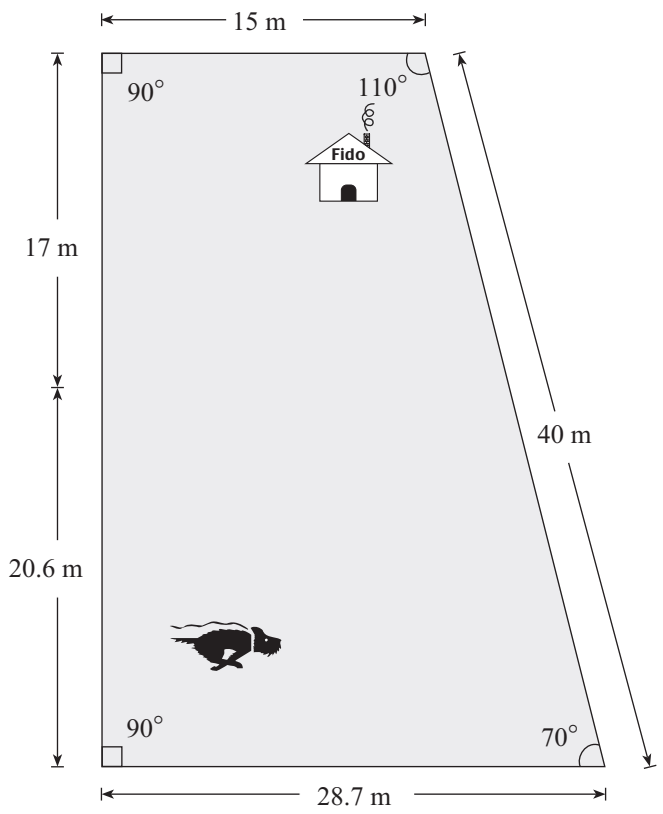
Number of fencing boards: $60 + 160 + 115 + 83 = 418$ **or** $\frac{104.3}{0.25} \sim 418$ ← **$2\frac{1}{2}$ marks**

$\underbrace{\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow}$

$\frac{1}{2}$ mark each **($2\frac{1}{2}$ marks in total)**

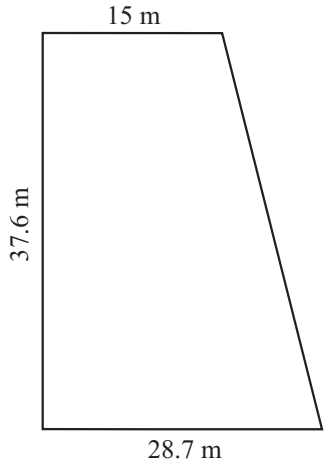
Fencing boards: $418 \times \$1.10 = \$459.80$ ← **$\frac{1}{2}$ mark**

Use the following diagram to answer question 7d).



d) Determine the area of the backyard and then determine the cost of seeding the backyard if it costs approximately \$15 per 100 m^2 for seed and fertilizer. **(5 marks)**

Solution



$$\text{Area} = \frac{1}{2}h(b_1 + b_2) \quad \leftarrow \textbf{1 mark}$$

$$= \frac{1}{2}(37.6)(15 + 28.7) \quad \leftarrow \textbf{1 mark}$$

$$= 821.56 \text{ m}^2 \quad \leftarrow \textbf{1 mark}$$

Seeding Cost:

$$\frac{821.56}{100} \times \$15 \quad \leftarrow \textbf{1 mark}$$

$$= \$123.23 \quad \leftarrow \textbf{1 mark}$$

e) If the owner does the work himself instead of paying a company $2 000 to do the entire job (labour and materials for fencing and seeding), approximately how much money is saved?

(2 marks)

Solution

$120 + 162 + 459.80 + 123.23 = \$865.03$ ← **1 mark**

$\$2\,000 - 865.03 = \$1\,134.97$ ← **1 mark**

f) What factors should the owner consider when determining either to do the work himself or to hire a company? Are there other expenses that the owner may have if he does the work himself?

Solution

Other expenses include: nails, driving costs to obtain materials, buying or renting tools if they are not owned. Other factors are: time, availability of the owner, patience of learning new skills, how quickly the job needs to be done. ← **4 marks**

END OF KEY