Principles of Mathematics 12

August 1999 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers	Sub-Organizers
1. Problem Solving	A Problem Set
2. Patterns and Relations	B Sequences and Series
	C Polynomials
	D Logarithms and Exponents
	E Quadratic Relations
	F Quadratic Systems
3. Shape and Space	G Trigonometry
	H Geometry

Part A: Multiple Choice

Q	K	C	CO	PLO	Q	K	C	CO	PLO
1.	A	K	2	C4	24.	B	K	2	B1
2.	C	U	2	C5	25.	D	U	2	B5
3.	D	U	2	C3	26.	D	U	2	B2
4.	B	U	2	C1, A7	27.	B	U	2	B6
5.	D	H	2	C9	28.	A	H	2	B3
6.	B	K	2	E5	29.	A	U	3	G1
7.	A	U	2	E2	30.	C	U	3	G5
8.	D	K	2	F1	31.	D	U	3	G2
9.	D	U	2	E4	32.	C	U	3	G2
10.	C	U	2	E4	33.	D	U	3	G3
11.	C	U	2, 1	F1	34.	B	U	3	G5
12.	C	U	2, 1	F1, A7	35.	D	U	3	G5
13.	A	U	2	F2	36.	A	H	3	G7
14.	B	H	2	E4	37.	D	U	3, 1	G9, A7
15.	A	H	2	E7	38.	D	H	3	G7
16.	B	H	2, 1	F5, C9, A1	39.	A	U	3	H2
17.	A	K	2	D4	40.	B	U	3	H2
18.	B	K	2	D5	41.	C	U	3	H2
19.	C	U	2	D3	42.	C	H	3	H2
20.	B	U	2, 1	D2, A7	43.	D	U	1	A3
21.	A	U	2	D1	44.	B	U	1	A3
22.	C	H	2	D4	45.	D	U	1	A3
23.	C	H	2	D5					

Multiple Choice = 45 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1.	1	U	3	2	B4
2.	2	U	3	2	E6
3.	3	U	3	2	C4
4.	4	U	3	3	G8
5.	5	U	3	2	D5, A7
6.	6	U	3	3	H2
7.	7	U	3	1	A1, A7
8.	8	H	4	3	H4

Written Response = 25 marks

Multiple Choice = 45 (45 questions)
Written Response = 25 (8 questions)
EXAMINATION TOTAL = 70 marks

LEGEND:
Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
CO = Curriculum Organizer
PLO = Prescribed Learning Outcome

PART B: WRITTEN RESPONSE

Value: 25 marks **Suggested Time: 45 minutes**

INSTRUCTIONS:	Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided. If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions. When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution. **Full marks will NOT be given for the final answer only.**

1. In an arithmetic sequence, the 3rd term is 7 and the 6th term is 11. Determine the 1st term of this sequence. **(3 marks)**

Solution

$a + 2d = 7$ ← **1 mark**

$a + 5d = 11$ ← **½ mark**

$3d = 4$

$d = \frac{4}{3}$ ← **½ mark**

$a + 2\left(\frac{4}{3}\right) = 7$ ← **½ mark**

$a = \frac{13}{3}$ ← **½ mark**

Alternate Solution 1

$a + 2d = 7$ ← **1 mark**

$a + 5d = 11$ ← **½ mark**

$-5a - 10d = -35$

$2a + 10d = 22$

} ← **½ mark** for process of getting two equations into one equation for a.

$-3a = -13$ ← **½ mark**

$a = \frac{13}{3}$ ← **½ mark**

1. In an arithmetic sequence, the 3rd term is 7 and the 6th term is 11. Determine the 1st term of this sequence. **(3 marks)**

Alternate Solution 2

$$\_\ \_\ 7\ \_\ \_\ 11$$

$$\begin{array}{cc} \uparrow & \uparrow \\ a & t_4 \end{array}$$

$\leftarrow \frac{1}{2}$ **mark** for method of changing 7 to 1st term, 11 to 4th term.

$t_n = a + (n-1)d$

$11 = 7 + 3d$ $\leftarrow$ **1 mark**

$4 = 3d$

$\frac{4}{3} = d$ $\leftarrow \frac{1}{2}$ **mark**

$a = 7 - \frac{4}{3} - \frac{4}{3}$ $\leftarrow \frac{1}{2}$ **mark**

$a = \frac{13}{3}$ $\leftarrow \frac{1}{2}$ **mark**

2. Change the following equation to standard form. **(3 marks)**

$$4x^2 - 9y^2 - 16x - 18y - 29 = 0$$

Solution

$$4x^2 - 16x - 9y^2 - 18y - 29 = 0$$

½ mark ↓ **½ mark** ↓

$$4\left(x^2 - 4x\right) - 9\left(y^2 + 2y\right) = 29$$

1 mark → $4\left(x^2 - 4x + 4\right) - 9\left(y^2 + 2y + 1\right) = 29 + 16 - 9$

½ mark → $4(x-2)^2 - 9(y+1)^2 = 36$

$$\frac{4(x-2)^2}{36} - \frac{9(y+1)^2}{36} = \frac{36}{36}$$

½ mark → $\dfrac{(x-2)^2}{9} - \dfrac{(y+1)^2}{4} = 1$

3. If $x+2$ is a factor of the polynomial $P(x)=2x^3+kx^2-32x-4k^2$, determine all possible values of k. **(3 marks)**

Solution

$$P(-2)=0 \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$P(-2)=-16+4k+64-4k^2=0 \quad \leftarrow \textbf{1 mark}$$

$$4k^2-4k-48=0 \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$k^2-k-12=0$$

$$(k-4)(k+3)=0$$

$$k=4\ ,\quad k=-3$$

$$\uparrow \qquad\qquad \uparrow$$

$$\tfrac{1}{2}\textbf{ mark} \quad \tfrac{1}{2}\textbf{ mark}$$

Alternate Solution

-2	2	k	-32	$-4k^2$	
		-4	$8-2k$	$48+4k$	← $\tfrac{1}{2}$ **mark**
	2	$-4+k$	$-24-2k$	0	← $\tfrac{1}{2}$ **mark**

$$-4k^2+4k+48=0 \quad \leftarrow \textbf{1 mark}$$

$$k^2-k-12=0$$

$$(k-4)(k+3)=0$$

$$k=4\ ,\quad k=-3$$

$$\uparrow \qquad\qquad \uparrow$$

$$\tfrac{1}{2}\textbf{ mark} \quad \tfrac{1}{2}\textbf{ mark}$$

4. Prove the identity: **(3 marks)**

$$\frac{\sin 2\theta}{\cos\theta} + \frac{\cos 2\theta}{\sin\theta} = \csc\theta$$

Solution

$$\frac{\sin 2\theta}{\cos\theta} + \frac{\cos 2\theta}{\sin\theta} = \csc\theta$$

	LEFT SIDE	RIGHT SIDE
1 mark →	$\frac{2\sin\theta\cos\theta}{\cos\theta} + \frac{1 - 2\sin^2\theta}{\sin\theta}$	
1 mark →	$2\sin\theta + \frac{1}{\sin\theta} - 2\sin\theta$	
$\frac{1}{2}$ mark →	$\frac{1}{\sin\theta}$	
$\frac{1}{2}$ mark →	$\csc\theta$	

LS = RS

4. Prove the identity: **(3 marks)**

$$\frac{\sin 2\theta}{\cos\theta} + \frac{\cos 2\theta}{\sin\theta} = \csc\theta$$

Alternate Solution 1

$$\frac{\sin 2\theta}{\cos\theta} + \frac{\cos 2\theta}{\sin\theta} = \csc\theta$$

LEFT SIDE		RIGHT SIDE
1 mark →	$\dfrac{2\sin\theta\cos\theta}{\cos\theta} + \dfrac{2\cos^2\theta - 1}{\sin\theta}$	
$\frac{1}{2}$ **mark** →	$2\sin\theta + \dfrac{2\cos^2\theta - 1}{\sin\theta}$	
$\frac{1}{2}$ **mark** →	$\dfrac{2\sin^2\theta + 2\cos^2\theta - 1}{\sin\theta}$	
	$\dfrac{2\left(\sin^2\theta + \cos^2\theta\right) - 1}{\sin\theta}$	
$\frac{1}{2}$ **mark** →	$\dfrac{2-1}{\sin\theta}$	
	$\dfrac{1}{\sin\theta}$	
$\frac{1}{2}$ **mark** →	$\csc\theta$	

LS = RS

4. Prove the identity:	**(3 marks)**

$$\frac{\sin 2\theta}{\cos\theta} + \frac{\cos 2\theta}{\sin\theta} = \csc\theta$$

Alternate Solution 2

$$\frac{\sin 2\theta}{\cos\theta} + \frac{\cos 2\theta}{\sin\theta} = \csc\theta$$

LEFT SIDE		RIGHT SIDE
1 mark →	$\frac{2\sin\theta\cos\theta}{\cos\theta} + \frac{\cos^2\theta - \sin^2\theta}{\sin\theta}$	
½ mark →	$2\sin\theta + \frac{\cos^2\theta - \sin^2\theta}{\sin\theta}$	
½ mark →	$\frac{2\sin^2\theta + \cos^2\theta - \sin^2\theta}{\sin\theta}$	
	$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$	
½ mark →	$\frac{1}{\sin\theta}$	
½ mark →	$\csc\theta$	

LS = RS

4. Prove the identity: **(3 marks)**

$$\frac{\sin 2\theta}{\cos\theta} + \frac{\cos 2\theta}{\sin\theta} = \csc\theta$$

Alternate Solution 3

$$\frac{\sin 2\theta}{\cos\theta} + \frac{\cos 2\theta}{\sin\theta} = \csc\theta$$

	LEFT SIDE	RIGHT SIDE
1 mark →	$\dfrac{\sin 2\theta \sin\theta + \cos 2\theta \cos\theta}{\cos\theta \sin\theta}$	
1 mark →	$\dfrac{\cos(2\theta - \theta)}{\cos\theta \sin\theta}$	
$\frac{1}{2}$ mark →	$\dfrac{\cos\theta}{\cos\theta \sin\theta}$	
	$\dfrac{1}{\sin\theta}$	
$\frac{1}{2}$ mark →	$\csc\theta$	

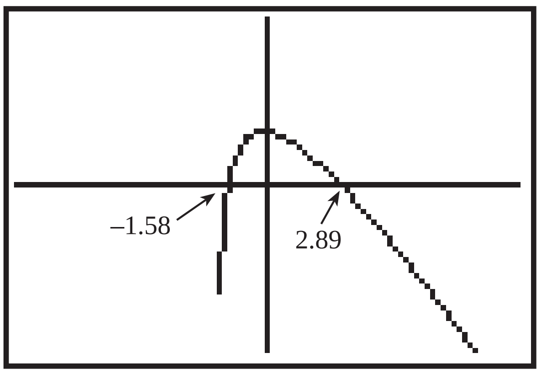
LS = RS

5. Solve the following equation using a graphing calculator. **(3 marks)**

$$\log_3(x+2) = \frac{1}{2}x$$

Sketch the graph in the viewing window below and state the function(s) used in your graph. Indicate appropriate window dimensions that will provide enough of the graph so that all intersection points or all zeros are visible.

Solution



$x\ [-10,\ 10]$ $y\ [-2,\ 2]$

$x = -1.58\ ,\ 2.89$

↑

1½ marks for solution

½ mark for change of base

↓

$$Y_1 = \frac{\log(x+2)}{\log 3} - \frac{1}{2}x$$

← **½ mark** for manipulation of equation

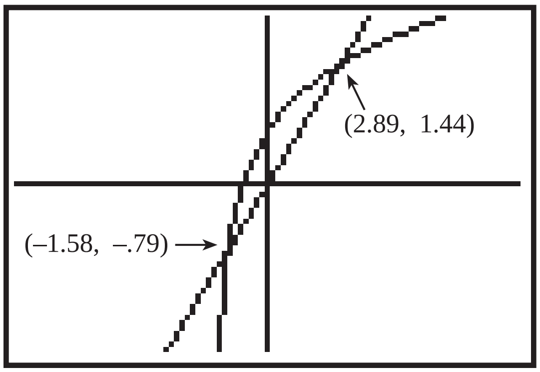
← **½ mark** for graph, showing 2 roots and appropriate window dimensions

5. Solve the following equation using a graphing calculator. **(3 marks)**

$$\log_3(x+2) = \frac{1}{2}x$$

Sketch the graph in the viewing window below and state the function(s) used in your graph. Indicate appropriate window dimensions that will provide enough of the graph so that all intersection points or all zeros are visible.

Alternate Solution 1



x $[-10, 10]$ y $[-2, 2]$

$\frac{1}{2}$ **mark** for change of base

↓

$$Y_1 = \frac{\log(x+2)}{\log 3}$$

$$Y_2 = \frac{1}{2}x$$

← $\frac{1}{2}$ **mark** for each graph, showing 2 points of intersection and window dimensions

$$x = \underbrace{-1.58 \quad , \quad 2.89}$$

↑

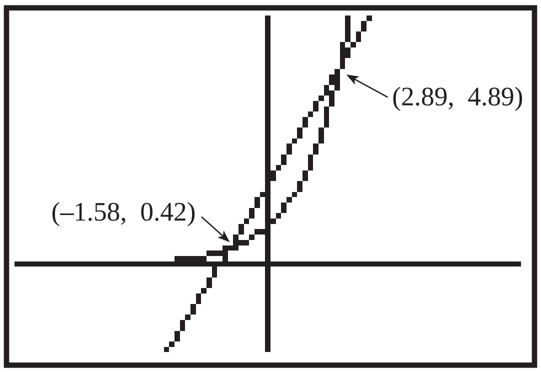
1$\frac{1}{2}$ marks for solution

5. Solve the following equation using a graphing calculator. **(3 marks)**

$$\log_3(x+2) = \frac{1}{2}x$$

Sketch the graph in the viewing window below and state the function(s) used in your graph. Indicate appropriate window dimensions that will provide enough of the graph so that all intersection points or all zeros are visible.

Alternate Solution 2



$x\ [-10,\ 10]$ $y\ [-2,\ 6]$

$Y_1 = x + 2$

$Y_2 = 3^{\frac{1}{2}x}$

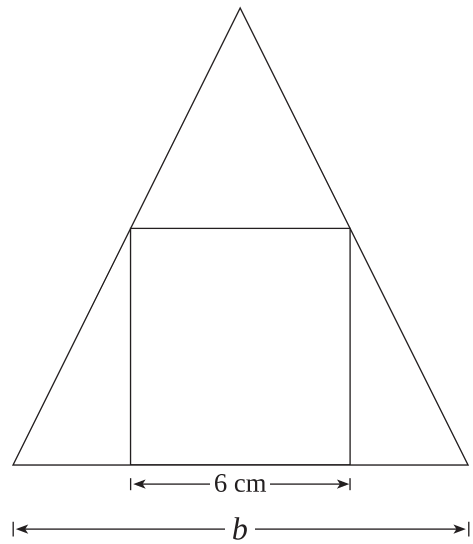
← $\frac{1}{2}$ **mark** for each equation

← $\frac{1}{2}$ **mark** for each graph, with appropriate window dimensions

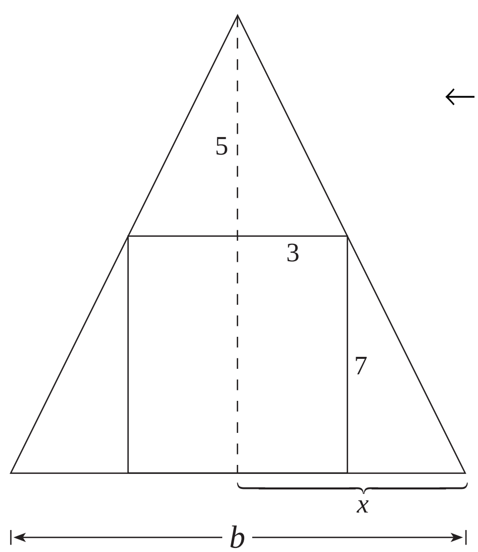
$x = -1.58\ ,\ 2.89$

↑ **1 mark** for solution

6. A rectangle with width 6 cm and height 7 cm is inscribed in an isosceles triangle of height 12 cm, as shown in the diagram below. Determine the length of base b of the isosceles triangle. **(3 marks)**



Solution



← **1 mark** for labelling information given on diagram.

Similar triangles $\Rightarrow \frac{5}{3} = \frac{12}{x}$ ← **1 mark**

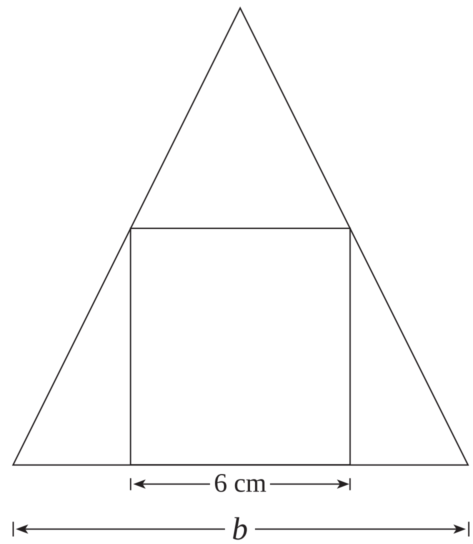
$\Rightarrow x = \frac{36}{5}$ or 7.2 ← **½ mark**

$\therefore b = 2x$

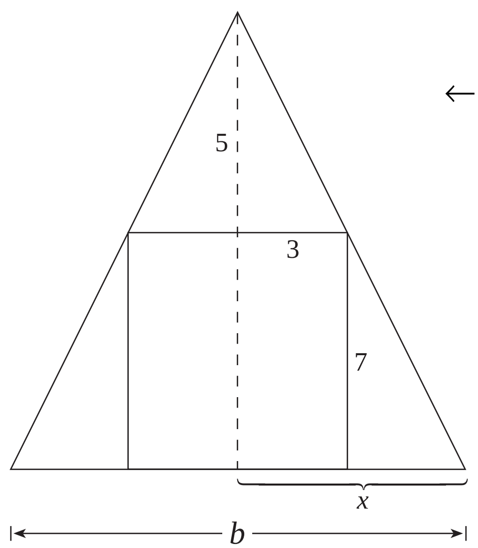
$b = 2(7.2)$

$b = 14.4$ cm ← **½ mark**

6. A rectangle with width 6 cm and height 7 cm is inscribed in an isosceles triangle of height 12 cm, as shown in the diagram below. Determine the length of base b of the isosceles triangle. **(3 marks)**



Alternate Solution 1



← **1 mark** for labelling information given on diagram.

$$\frac{5}{3} = \frac{7}{x-3}$$ ← **1 mark**

$$5x - 15 = 21$$

$$5x = 36$$

$$x = \frac{36}{5} = 7.2$$ ← **½ mark**

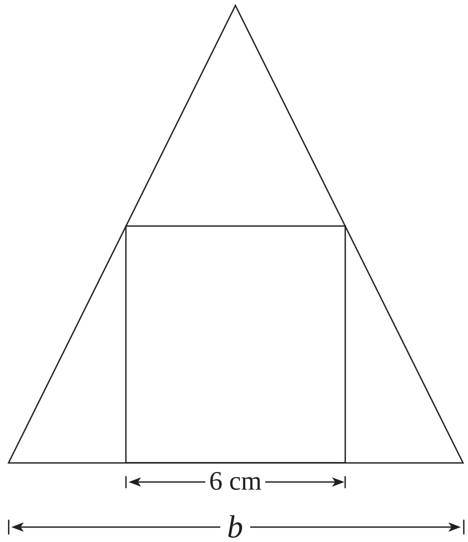
$$\therefore\ b = 2x$$

$$b = 2(7.2)$$

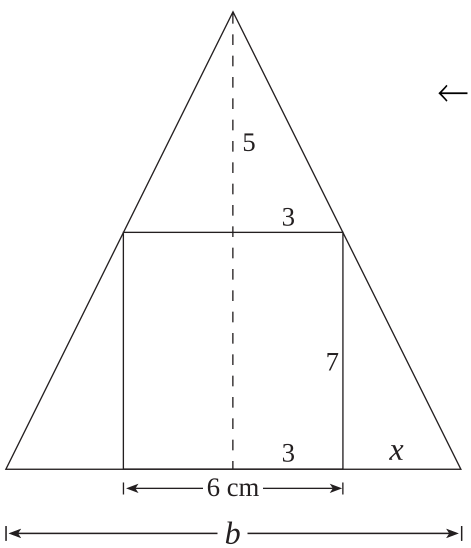
$$b = 14.4 \text{ cm}$$ ← **½ mark**

Note: Students could also use the proportion $\frac{12}{x} = \frac{7}{x-3}$

6. A rectangle with width 6 cm and height 7 cm is inscribed in an isosceles triangle of height 12 cm, as shown in the diagram below. Determine the length of base b of the isosceles triangle. **(3 marks)**



Alternate Solution 2



← **1 mark** for labelling information given on diagram.

$$\frac{5}{3} = \frac{7}{x}$$ ← **1 mark**

$$x = \frac{21}{5} = 4.2$$ ← **½ mark**

$$\therefore\ b = (3 + 4.2)2 = 14.4 \text{ cm}$$ ← **½ mark**

Note: Students could also use the proportion $\frac{12}{x+3} = \frac{7}{x}$

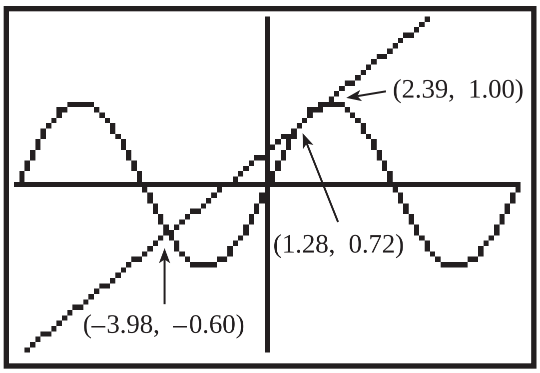
7. Solve the following system using a graphing calculator. Express all solutions as ordered pairs. **(3 marks)**

$$y = \sin \frac{\pi}{5} x$$

$$y = \frac{1}{4} x + 0.4$$

Sketch the graph in the viewing window below. State the functions that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the functions and all intersection points are visible. (*Note:* Graph at least one period of the sine curve.)

Solution



$Y_1 = \sin\left(\frac{\pi}{5} x\right)$

$Y_2 = \frac{1}{4} x + 0.4$

← $\frac{1}{2}$ **mark** for each graph

$x\ [-10,\ 10]$ $y\ [-2,\ 2]$ ← $\frac{1}{2}$ **mark** for window dimensions

$(-3.98,\ -0.60)$, $(1.28,\ 0.72)$, $(2.39,\ 1.00)$

↑ ↑ ↑

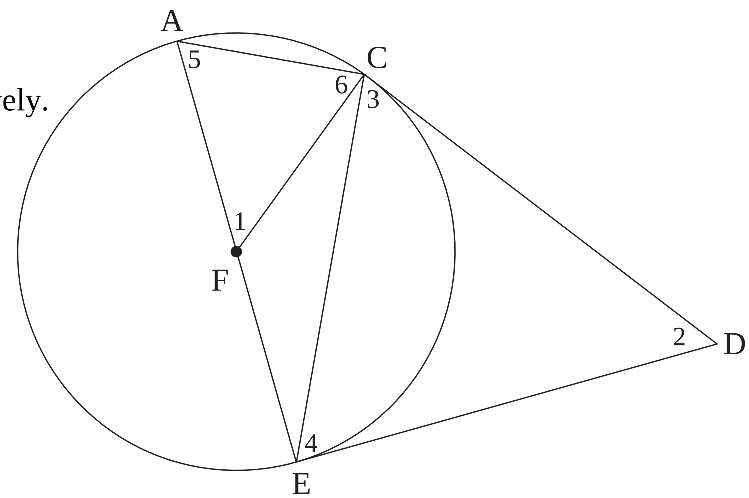
$\frac{1}{2}$ **mark** $\frac{1}{2}$ **mark** $\frac{1}{2}$ **mark**

8. Complete the proof. **(4 marks)**

Given: DC and DE are tangents to the circle at points C and E respectively.
F is the centre
AE is a diameter

Prove: $\angle 1 = \angle 2$

Students are encouraged to number angles.



Solution

Method 1:

PROOF

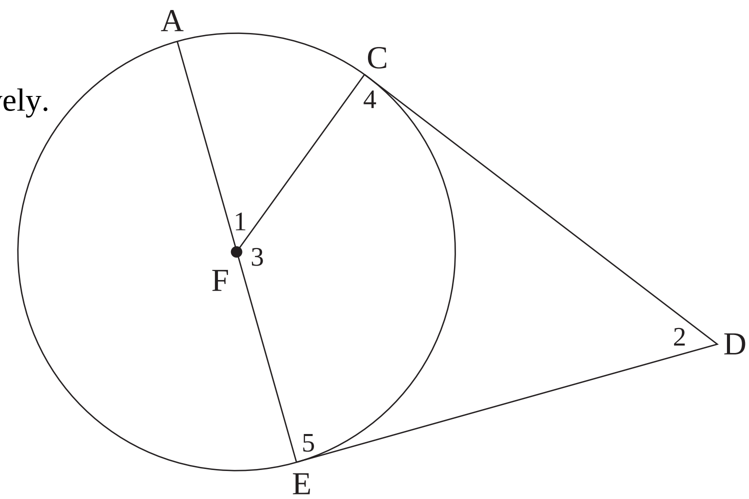
Statement		Reason	
Join AC, CE		construction	
DC and DE are tangents		given	
$\angle 3 = \angle 5$	← $\frac{1}{2}$ **mark**	$\angle$ between tangent and chord	← $\frac{1}{2}$ **mark**
DC = DE	← $\frac{1}{2}$ **mark**	tangents from exterior point are =	← $\frac{1}{2}$ **mark**
$\angle 3 = \angle 4$		$\angle$s opposite = sides are =	← $\frac{1}{2}$ **mark**
F is the centre		given	
AF = CF		radii =	
$\angle 5 = \angle 6$		$\angle$s opposite = sides are =	← $\frac{1}{2}$ **mark**
$\angle 6 = \angle 4$		substitution	← $\frac{1}{2}$ **mark**
$\angle 1 = \angle 2$		3rd $\angle$s of Δs are =	← $\frac{1}{2}$ **mark**

8. Complete the proof. **(4 marks)**

Given: DC and DE are tangents to the circle at points C and E respectively.
F is the centre
AE is a diameter

Prove: $\angle 1 = \angle 2$

Students are encouraged to number angles.



Alternate Solution

Method 1:

PROOF

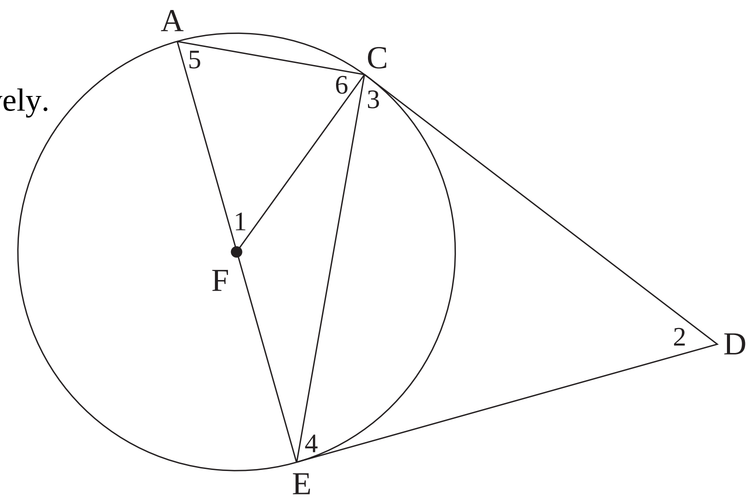
Statement		Reason	
DE and DC are tangents		given	
F is the centre		given	
$\angle 4 = \angle 5 = 90°$	← **1 mark**	tangent $\perp$ radius	← $\frac{1}{2}$ **mark**
$\angle 2 + \angle 3 = 180°$	← $\frac{1}{2}$ **mark**	$\angle$ sum of quadrilateral	← $\frac{1}{2}$ **mark**
$\angle 1 + \angle 3 = 180°$	← $\frac{1}{2}$ **mark**	$\angle$s on a line are supplementary	← $\frac{1}{2}$ **mark**
$\angle 1 = \angle 2$		supplements of $=$ $\angle$s are $=$	← $\frac{1}{2}$ **mark**

8. Complete the proof. **(4 marks)**

Given: DC and DE are tangents to the circle at points C and E respectively.
F is the centre
AE is a diameter

Prove: $\angle 1 = \angle 2$

Students are encouraged to number angles.



Solution

Method 2:

Join AC and CE

$\frac{1}{2}$ **mark** → CD = DE since tangents from external point are = ← $\frac{1}{2}$ **mark**

$\therefore$ $\angle 3 = \angle 4 = x$ since $\angle$s opposite = sides are = ← $\frac{1}{2}$ **mark**

$\frac{1}{2}$ **mark** → $\angle 3 = \angle 5 = x$ since $\angle$ between tangent and chord ← $\frac{1}{2}$ **mark**

and $\angle 6 = \angle 5 = x$ since $\angle$s opposite = radii are = ← $\frac{1}{2}$ **mark**

$\therefore$ $\angle 1 = 180° - 2x$, $\angle 2 = 180° - 2x$ (sum of $\angle$s of Δs) ← $\frac{1}{2}$ **mark**

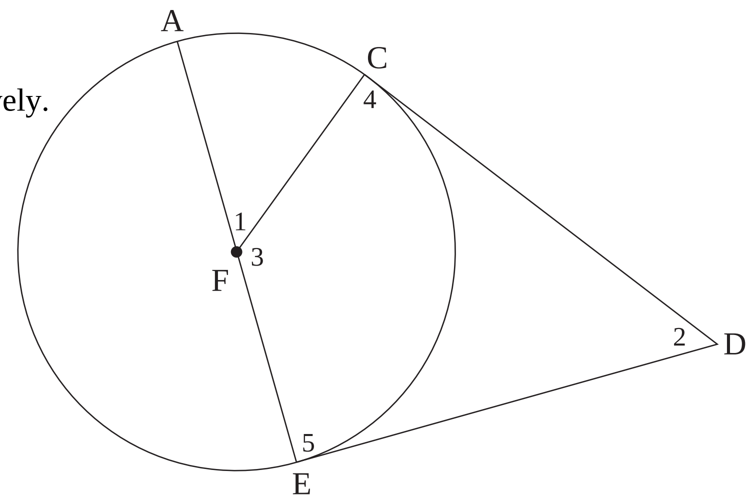
$\Rightarrow \angle 1 = \angle 2$ ← $\frac{1}{2}$ **mark**

8. Complete the proof. **(4 marks)**

Given: DC and DE are tangents to the circle at points C and E respectively.
F is the centre
AE is a diameter

Prove: $\angle 1 = \angle 2$

Students are encouraged to number angles.



Alternate Solution 1

Method 2:

$\frac{1}{2}$ **mark** → $\angle 3 = 180° - \angle 1$, $\angle$s on a line are supplementary ← $\frac{1}{2}$ **mark**

1 mark → $\angle 4 = 90°$, $\angle 5 = 90°$, tangent ⊥ radius ← $\frac{1}{2}$ **mark**

$\therefore\ 180° - \angle 1 + 90° + 90° + \angle 2 = 360°$, sum of $\angle$s of quadrilateral ← $\frac{1}{2}$ **mark**

↑ $\frac{1}{2}$ **mark**

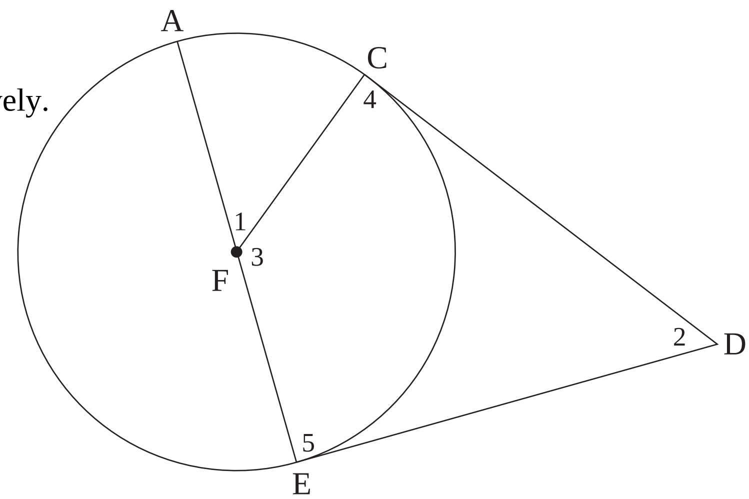
$\Rightarrow\ \angle 1 = \angle 2$ ← $\frac{1}{2}$ **mark**

8. Complete the proof. **(4 marks)**

Given: DC and DE are tangents to the circle at points C and E respectively.
F is the centre
AE is a diameter

Prove: $\angle 1 = \angle 2$

Students are encouraged to number angles.



Alternate Solution 2

Method 2:

$\frac{1}{2}$ **mark** ↓

Since DC and DE are tangents, and tangents are ⊥ to radius, $\angle 4 = \angle 5 = 90°$ ← **1 mark**

$\angle 2 + \angle 4 + \angle 5 + \angle 3 = 360°$, sum of $\angle$s of quadrilateral ← $\frac{1}{2}$ **mark**

$\Rightarrow \angle 2 + 90° + 90° + \angle 3 = 360°$

$\angle 2 + \angle 3 = 180°$ ← $\frac{1}{2}$ **mark**

$\frac{1}{2}$ **mark** → but $\angle 1 + \angle 3 = 180°$ since $\angle$s on a line are supplementary ← $\frac{1}{2}$ **mark**

$\Rightarrow \angle 1 + \angle 3 = \angle 2 + \angle 3$

$\Rightarrow \angle 1 = \angle 2$ ← $\frac{1}{2}$ **mark**

END OF KEY