Principles of Mathematics 12

June 1999 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		Sub-Organizers	
1.	Problem Solving	A	Problem Set
2.	Patterns and Relations	B	Sequences and Series
		C	Polynomials
		D	Logarithms and Exponents
		E	Quadratic Relations
		F	Quadratic Systems
3.	Shape and Space	G	Trigonometry
		H	Geometry

Part A: Multiple Choice

Q	K	C	CO	PLO
1.	A	U	2	C4
2.	A	U	2	C6
3.	C	U	2	C2, A7
4.	D	U	2	C5
5.	B	U	2	C7
6.	A	H	2	C1
7.	B	U	2	E2
8.	C	U	2	F5
9.	D	K	2	E6
10.	D	K	2	E5
11.	C	U	2	E6
12.	A	U	2	F1
13.	D	H	2	F3
14.	D	H	2	E5
15.	A	K	2	B1
16.	B	U	2	B6
17.	A	U	2	B4
18.	C	U	2	B2
19.	D	U	2	B4
20.	D	U	2	B4
21.	A	H	2	B4
22.	B	K	2	D4
23.	A	U	2	D5
24.	C	U	2	D5
25.	A	K	2	D2
26.	B	U	2	D5
27.	B	H	2	D5
28.	D	H	2	D2
29.	B	U	3	G1
30.	D	K	3	G5
31.	B	U	3	G6
32.	D	U	3	G2
33.	B	U	3	G3
34.	D	U	3	G9, A7
35.	C	U	3	G9
36.	D	U	3	G7
37.	A	H	3	G5
38.	D	H	2, 3	B4, G7
39.	C	U	3	H1
40.	D	U	3	H2
41.	C	U	3	H2
42.	A	H	3	H3
43.	C	H	1	A3, 7
44.	B	U	1	A3
45.	C	U	1	A3

Multiple Choice = 45 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1.	1	U	3	2	F2
2.	2	U	3	2	E7
3.	3	U	3	3	G8
4.	4	U	3	2	C9
5.	5	U	3	2	D5
6.	6	U	3	3	H4
7.	7	U	3	1	A3
8.	8	H	4	3	H4

Written Response = 25 marks

Multiple Choice = 45 (45 questions)
Written Response = 25 (8 questions)
EXAMINATION TOTAL = 70 marks

LEGEND:
Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
CO = Curriculum Organizer
PLO = Prescribed Learning Outcome

PART B: WRITTEN RESPONSE

Value: 25 marks **Suggested Time: 45 minutes**

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

If, in a justification, you refer to information produced by the calculator, this information must be presented clearly in the response. For example, if a graph is used in the solution of the problem, it is important to sketch the graph, showing its general shape and indicating the appropriate window dimensions.

When using the calculator, you should provide a decimal answer that is correct to **at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

Full marks will NOT be given for the final answer only.

1. Solve the following system algebraically. Express all solutions as ordered pairs. **(3 marks)**
$$3x^2 + 4y^2 = 21$$ $$x^2 - 2y = 1$$

Solution

$$x^2 = 2y + 1$$

$$3(2y+1) + 4y^2 = 21 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$6y + 3 + 4y^2 = 21$$

$$4y^2 + 6y - 18 = 0$$

$$2y^2 + 3y - 9 = 0 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$(y+3)(2y-3) = 0$$

$$y = -3,\ \frac{3}{2} \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

	$y = \frac{3}{2}$	$y = -3$	
	$x^2 = 2y+1$	$x^2 = 2y+1$	
	$x^2 = 4$	$x^2 = -5$	
$\frac{1}{2}$ **mark** $\rightarrow$	$x = \pm 2$	$\varnothing$	$\leftarrow \frac{1}{2}$ **mark**

$$\left(2,\ \frac{3}{2}\right),\ \left(-2,\ \frac{3}{2}\right)$$

$\uparrow$

$\frac{1}{2}$ **mark**

1. Solve the following system algebraically. Express all solutions as ordered pairs. **(3 marks)**

$$3x^2 + 4y^2 = 21$$
$$x^2 - 2y = 1$$

Alternate Solution 1

$$3x^2 + 4y^2 = 21 \qquad 3x^2 + 4y^2 = 21$$

$$x^2 - 2y = 1 \quad \rightarrow \quad -3x^2 + 6y = -3 \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$4y^2 + 6y = 18 \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$4y^2 + 6y - 18 = 0$$

$$2y^2 + 3y - 9 = 0$$

$$(2y - 3)(y + 3) = 0$$

$$y = \frac{3}{2} \qquad y = -3 \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$y = \frac{3}{2}$	$y = -3$
$x^2 = 2y + 1$	$x^2 = 2y + 1$
$x^2 = 4$	$x^2 = -5$
$\tfrac{1}{2}$ **mark** $\rightarrow$ $x = \pm 2$	$\varnothing$ $\leftarrow \tfrac{1}{2}$ **mark**

$$\left(2, \frac{3}{2}\right), \left(-2, \frac{3}{2}\right)$$

$\uparrow$ $\tfrac{1}{2}$ **mark**

1. Solve the following system algebraically. Express all solutions as ordered pairs. **(3 marks)**

$$3x^2 + 4y^2 = 21$$
$$x^2 - 2y = 1$$

Alternate Solution 2

$$3x^2 + 4y^2 = 21$$

$$x^2 - 2y = 1 \rightarrow y = \frac{x^2 - 1}{2}$$

$$3x^2 + 4\left(\frac{x^2 - 1}{2}\right)^2 = 21 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$3x^2 + 4\left(\frac{x^4 - 2x^2 + 1}{4}\right) = 21$$

$$3x^2 + x^4 - 2x^2 + 1 = 21$$

$$x^4 + x^2 - 20 = 0 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$\left(x^2 + 5\right)\left(x^2 - 4\right) = 0$$

$$\varnothing \qquad x = \pm 2$$

$$\uparrow \qquad \uparrow$$

$\frac{1}{2}$ **mark** $\frac{1}{2}$ **mark**

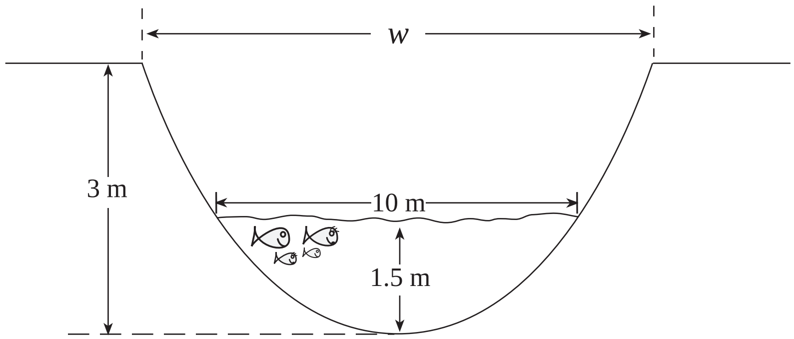
$$y = \frac{(\pm 2)^2 - 1}{2} = \frac{3}{2} \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$\left(2, \frac{3}{2}\right), \left(-2, \frac{3}{2}\right)$$

$$\uparrow$$

$\frac{1}{2}$ **mark**

2. The cross section of a drainage ditch is parabolic in shape, as shown in the diagram below. When the width of the water surface is 10 metres, the maximum depth of the water is 1.5 metres. Determine the width of the water, w, when the maximum depth is 3 metres. **(3 marks)**



Solution

$$y = ax^2 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$1.5 = a(5)^2 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$0.06 = a \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$y = 0.06x^2$$

$$3 = 0.06x^2 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$50 = x^2$$

$$5\sqrt{2} = x \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$\therefore$ the width of the surface is $2\left(5\sqrt{2}\right) = 10\sqrt{2} \approx 14.14$ m $\leftarrow \tfrac{1}{2}$ **mark**

3. Prove the identity: **(3 marks)**

$$\frac{\sin\theta+\tan\theta}{1+\cos\theta} = \frac{\sin 2\theta}{2\cos^2\theta}$$

Solution

$$\frac{\sin\theta+\tan\theta}{1+\cos\theta} = \frac{\sin 2\theta}{2\cos^2\theta}$$

LEFT SIDE	RIGHT SIDE
$\frac{1}{2}$ **mark** → $= \dfrac{\sin\theta+\dfrac{\sin\theta}{\cos\theta}}{1+\cos\theta}$	$= \dfrac{2\sin\theta\cos\theta}{2\cos^2\theta}$ ← $\frac{1}{2}$ **mark**
$\frac{1}{2}$ **mark** → $= \dfrac{\dfrac{\sin\theta\cos\theta+\sin\theta}{\cos\theta}}{1+\cos\theta}$	$= \dfrac{\sin\theta}{\cos\theta}$ ← $\frac{1}{2}$ **mark**
$\frac{1}{2}$ **mark** → $= \dfrac{\sin\theta(\cos\theta+1)}{\cos\theta}\cdot\dfrac{1}{1+\cos\theta}$	
$\frac{1}{2}$ **mark** → $= \dfrac{\sin\theta}{\cos\theta}$	

LS = RS

3. Prove the identity: **(3 marks)**

$$\frac{\sin\theta + \tan\theta}{1+\cos\theta} = \frac{\sin 2\theta}{2\cos^2\theta}$$

Alternate Solution 1

$$\frac{\sin\theta + \tan\theta}{1+\cos\theta} = \frac{\sin 2\theta}{2\cos^2\theta}$$

LEFT SIDE	RIGHT SIDE
$\frac{1}{2}$ **mark** → $= \dfrac{\sin\theta + \dfrac{\sin\theta}{\cos\theta}}{1+\cos\theta}$	$= \dfrac{2\sin\theta\cos\theta}{2\cos^2\theta}$ ← $\frac{1}{2}$ **mark**
$\frac{1}{2}$ **mark** → $= \dfrac{\left(\sin\theta + \dfrac{\sin\theta}{\cos\theta}\right)\cos\theta}{(1+\cos\theta)\cos\theta}$	$= \dfrac{\sin\theta}{\cos\theta}$ ← $\frac{1}{2}$ **mark**
$= \dfrac{\sin\theta\cos\theta + \sin\theta}{(1+\cos\theta)\cos\theta}$	
$\frac{1}{2}$ **mark** → $= \dfrac{\sin\theta(\cos\theta + 1)}{(1+\cos\theta)\cos\theta}$	
$\frac{1}{2}$ **mark** → $= \dfrac{\sin\theta}{\cos\theta}$	

LS = RS

3. Prove the identity: **(3 marks)**

$$\frac{\sin\theta + \tan\theta}{1 + \cos\theta} = \frac{\sin 2\theta}{2\cos^2\theta}$$

Alternate Solution 2

$$\frac{\sin\theta + \tan\theta}{1 + \cos\theta} = \frac{\sin 2\theta}{2\cos^2\theta}$$

LEFT SIDE	RIGHT SIDE
	$= \frac{2\sin\theta\cos\theta}{2\cos^2\theta}$ $\leftarrow$ **$\frac{1}{2}$ mark**
	$= \frac{\sin\theta}{\cos\theta}$ $\leftarrow$ **$\frac{1}{2}$ mark**
	$= \frac{\frac{\sin\theta}{\cos\theta}(1+\cos\theta)}{(1+\cos\theta)}$ $\leftarrow$ **1 mark**
	$= \frac{\frac{\sin\theta}{\cos\theta} + \sin\theta}{1+\cos\theta}$ $\leftarrow$ **$\frac{1}{2}$ mark**
	$= \frac{\sin\theta + \tan\theta}{1+\cos\theta}$ $\leftarrow$ **$\frac{1}{2}$ mark**

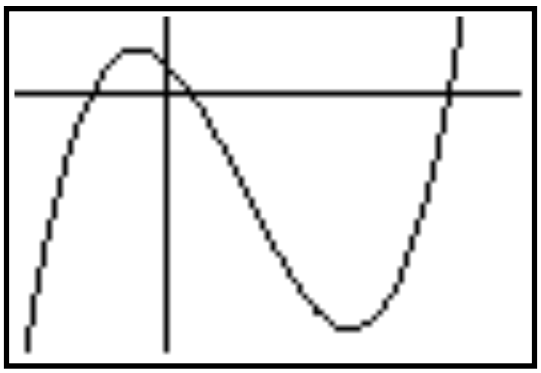
LS = RS

4. Solve the following inequality using a graphing calculator. **(3 marks)**

$$x^3 - 8x^2 > 18x - 20$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window. The solution may be given in algebraic form or shown on a number line.

Solution



$Y_1 = x^3 - 8x^2 - 18x + 20$ ← $\frac{1}{2}$ **mark** for equation

← $\frac{1}{2}$ **mark** for graph

$x\ [-5,\ 12]$ $y\ [-175,\ 50]$ ← $\frac{1}{2}$ **mark** for window dimensions

The solution to this inequality can be found when $Y_1 > 0$ or where the graph of Y_1 is above the *x*-axis.

$\therefore\ -2.48 < x < 0.83$ or $x > 9.65$ ← $\frac{1}{2}$ **mark** for correct zeros

↑ ↑

$\frac{1}{2}$ **mark** $\frac{1}{2}$ **mark** ← **marks for analysis of regions on the number line**

OR

↓ ↓

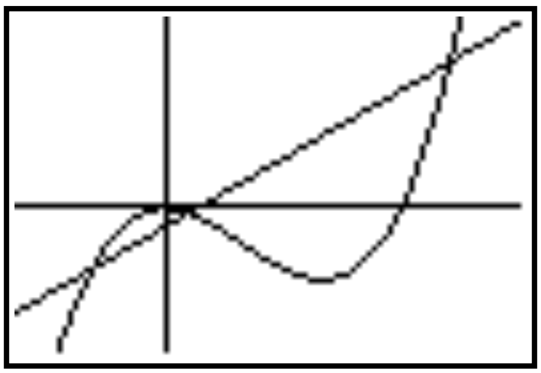
← **number line solution**

4. Solve the following inequality using a graphing calculator. **(3 marks)**

$$x^3 - 8x^2 > 18x - 20$$

Sketch the graph in the viewing window below and indicate appropriate window dimensions. State the function(s) used in your graph. Ensure that the relative maximum and relative minimum points of the function(s) are visible within the viewing window. The solution may be given in algebraic form or shown on a number line.

Alternate Solution



$Y_1 = x^3 - 8x^2$

$Y_2 = 18x - 20$

← **1 mark** for graph $\left(\frac{1}{2}\right.$ **mark** for each function$\left.\right)$

$x\ [-5,\ 12]$ $y\ [-150,\ 200]$

← $\frac{1}{2}$ **mark** for window dimensions

The solution to this inequality can be found when $Y_1 > Y_2$ or where the graph of Y_1 is above the graph of Y_2.

$\therefore\ -2.48 < x < 0.83$ or $x > 9.65$

← $\frac{1}{2}$ **mark** for correct zeros

↑ ↑

$\frac{1}{2}$ **mark** $\frac{1}{2}$ **mark**

← **marks for analysis of regions on the number line**

OR

↓ ↓

–2.48 0.83 9.65

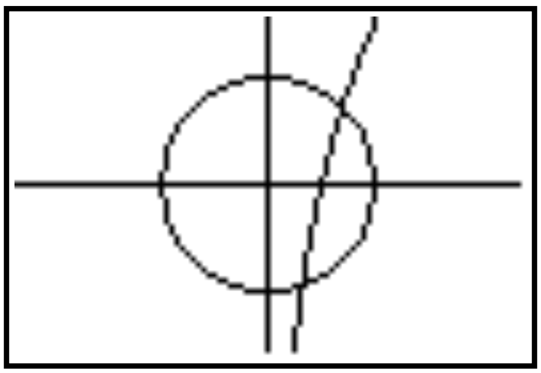
← **number line solution**

5. Solve the following system using a graphing calculator. **(3 marks)**

$$x^2 + y^2 = 4$$
$$y = 10 \log x$$

Sketch the graph in the viewing window below. State the function(s) that you entered to obtain your graph and your solution. Indicate the dimensions of the viewing window that will show enough of the graph so that recognizable characteristics of the function(s) and all intersection points are visible.

Solution



$Y_1 = \sqrt{(4-x)^2}$

$Y_2 = -Y_1$ } ← $\frac{1}{2}$ **mark** for equations

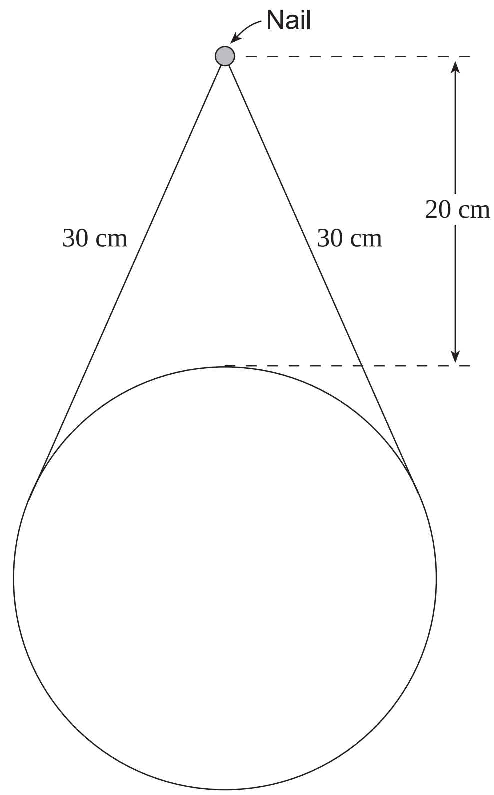
$Y_3 = 10\log(x)$

← **1 mark** for graph

$x \; [-4.7, \; 4.7]$ $\quad$ $y \; [-3.1, \; 3.1]$ $\quad$ ← **deduct** $\frac{1}{2}$ **mark** if window dimenions not given

$\therefore$ the solutions are: $(1.39, \; 1.44)$, $(0.65, \; -1.89)$ ← **1** $\frac{1}{2}$ **marks** for solutions

6. A circular mirror is suspended from a nail by two wires, each of length 30 cm, which are tangent to the mirror. If the nail is 20 cm from the top of the mirror, determine the diameter of the mirror. **(3 marks)**



Solution

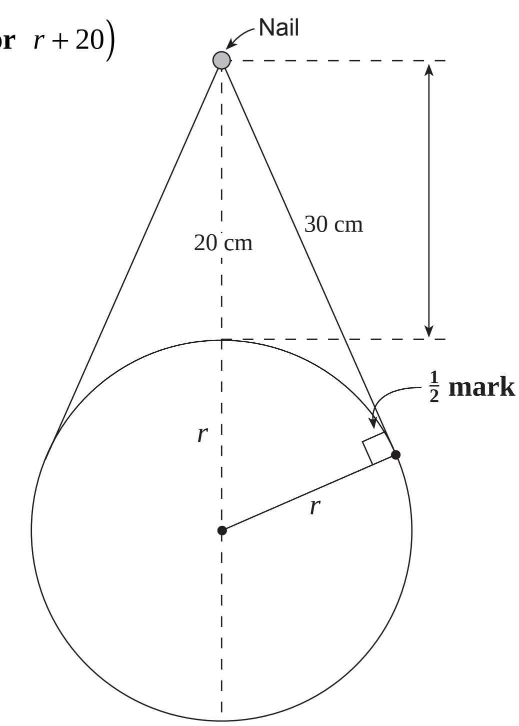
$r^2 + 30^2 = (r+20)^2$ ← **1 mark** ($\frac{1}{2}$ **for eqn.** $\frac{1}{2}$ **for** $r+20$)

$r^2 + 900 = r^2 + 40r + 400$ ← $\frac{1}{2}$ **mark**

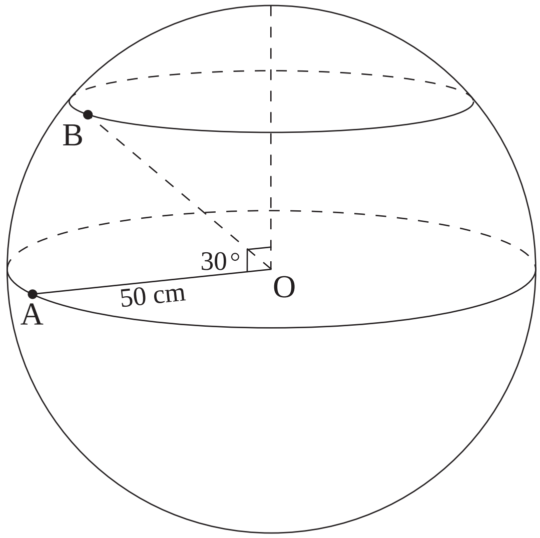
$500 = 40r$

$r = \frac{25}{2}$ ← $\frac{1}{2}$ **mark**

Diameter = 25 cm ← $\frac{1}{2}$ **mark**



7. Two circles are drawn around a sphere with centre O and radius 50 cm. The centre circle passing through point A is at $0°$ latitude, and a smaller circle passing through point B is at $30°$ latitude, as shown in the diagram below. Determine the circumference of the circle at point B. **(3 marks)**



Solution

recognize $OB = 50$ cm ← **½ mark**

$\cos 30° = \frac{r}{50}$ ← **1 mark**

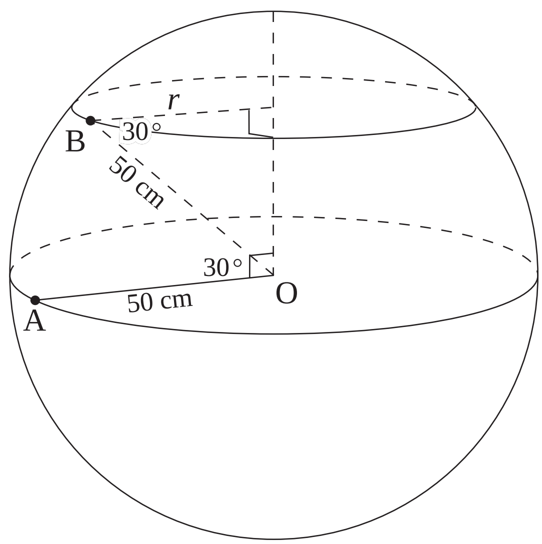
$r = \frac{\sqrt{3}}{2} \times 50$

$= 25\sqrt{3} \doteq 43.30$ ← **½ mark**

so, $C = 2\pi r$

$= 50\pi\sqrt{3}$

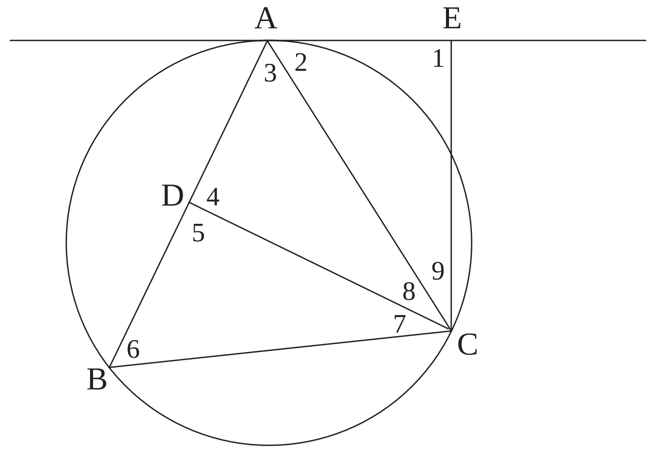
$\doteq 272.07$ cm ← **1 mark**



8. Complete the proof. **(4 marks)**

Given: AE is tangent to the circle at A
CE ⊥ AE
CD ⊥ AB
AC = BC

Prove: CE = CD



Solution

Method 1:

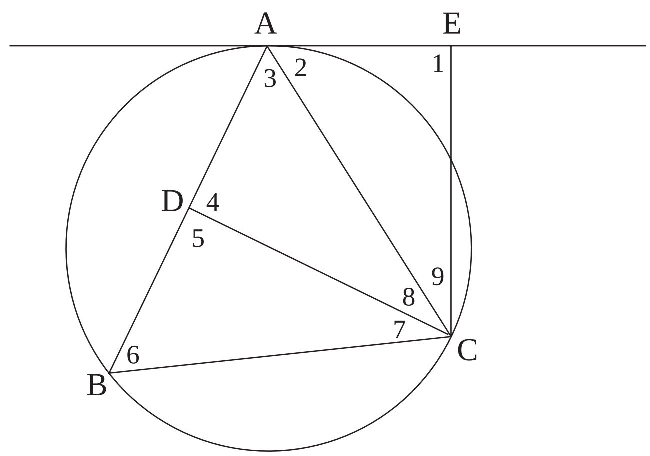
PROOF

	Statement	Reason
	AE is tangent	given
1 mark →	$\angle 6 = \angle 2$	∠ between tangent and chord
	CE ⊥ AE , CD ⊥ AB	given
1 mark →	$\angle 1 = 90°$, $\angle 5 = 90°$	definition of ⊥
½ mark →	$\angle 1 = \angle 5$	both = 90°
	AC = BC	given
½ mark →	$\Delta AEC \cong \Delta BDC$	AAS ← **½ mark**
½ mark →	CE = CD	CPCTC

8. Complete the proof. **(4 marks)**

Given: AE is tangent to the circle at A
CE ⊥ AE
CD ⊥ AB
AC = BC

Prove: CE = CD



Solution

Method 2:

Since AE is tangent, $\angle 2 = \angle 6$ by $\angle$ between tangent and chord ← **1 mark**

Since AC = BC, $\angle 6 = \angle 3$ by $\angle$s opposite = sides are =
$\Rightarrow$ $\angle 2 = \angle 3$ both = $\angle 6$ } ← **$\frac{1}{2}$ mark**

Since CE ⊥ AE and CD ⊥ AB, $\angle 1 = \angle 4 = 90°$, ← **1 mark**

and since AC = AC is common, then $\Delta ADC \cong \Delta AEC$ by AAS ← **1 mark**

$\therefore$ CE = CD ← **$\frac{1}{2}$ mark**

END OF KEY