

# Mathematics 12

## August 1998 Provincial Examination

### ANSWER KEY / SCORING GUIDE

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- Topics:**
1. Trigonometry
  2. Quadratic Relations
  3. Exponential and Logarithmic Functions
  4. Polynomial Functions
  5. Sequences and Series
  6. Introduction to Calculus
  7. Geometry
  8. Problem Solving

#### Part A: Multiple Choice

Q	C	T	K	S	ILO		Q	C	T	K	S	ILO
1.	K	2	B	1	17		26.	K	4	B	1	36
2.	U	2	A	1	14		27.	K	4	A	1	40
3.	U	2	A	1	17		28.	U	4	B	1	41
4.	U	2	C	1	16		29.	U	4	C	1	40
5.	U	2	B	1	11		30.	U	4	A	1	37
6.	U	2	A	1	15		31.	U	4	D	1	39
7.	U	2	D	1	18		32.	U	4	B	1	43
8.	U	2	D	1	17		33.	H	4	A	1	38
9.	H	2	A	1	20		34.	U	5	B	1	45
10.	H	2	B	1	17		35.	U	5	C	1	46
11.	U	1	C	1	01		36.	U	5	B	1	47
12.	U	1	B	1	02		37.	U	5	B	1	46
13.	K	1	C	1	04		38.	H	5	B	1	46
14.	U	1	D	1	05		39.	U	6	C	1	51
15.	U	1	A	1	08		40.	U	6	B	1	50
16.	U	1	D	1	06, 09		41.	K	6	C	1	56
17.	H	1	D	1	09		42.	U	6	A	1	57
18.	H	1	B	1	08		43.	U	6	B	1	53
19.	K	3	D	1	28		44.	U	6	B	1	61
20.	U	3	C	1	29		45.	H	6	C	1	58
21.	U	3	D	1	32		46.	U	7	B	1	63
22.	U	3	A	1	31		47.	U	7	C	1	63
23.	U	3	A	1	26		48.	U	8	D	1	64
24.	U	3	D	1	27		49.	U	8	C	1	64
25.	H	3	C	1	24		50.	U	8	B	1	64

**Part B: Written Response**

<b>Q</b>	<b>B</b>	<b>C</b>	<b>T</b>	<b>S</b>	<b>ILO</b>	<b>Q</b>	<b>B</b>	<b>C</b>	<b>T</b>	<b>S</b>	<b>ILO</b>
1.	1	U	5	2	46	4.	5	U	3	3	32
2a.	2	U	6	2	60	5.	6	U	1	3	09
2b.	3	U	6	1	60	6.	7	H	8	2	64
3.	4	U	2	3	21	7.	8	H	7	4	63

Multiple Choice = 50 (50 questions)

Written Response = 20 (7 questions)

**Total = 70 marks**

**LEGEND:**

**Q** = Question Number

**C** = Cognitive Level

**T** = Topic

**K** = Keyed Response

**S** = Score

**ILO** = Intended Learning Outcome

**B** = Score Box Number

**PART B: WRITTEN RESPONSE**

**Value: 20 marks**

**Suggested Time: 45 minutes**

**INSTRUCTIONS:** Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

**Full marks will NOT be given for the final answer only.**

1. Find four geometric means between 96 and 729.

**(2 marks)**

**Solution:**

$$96 \quad \_ \_ \_ \_ \quad 729$$

$$a = 96$$

$$t_n = ar^{n-1}$$

$$n = 6 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$729 = 96r^5 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$t_n = 729$$

$$7.59375 = r^5$$

$$(7.59375)^{\frac{1}{5}} = r$$

$$1.5 = r \quad \leftarrow \frac{1}{2} \text{ mark}$$

$\therefore$  four geometric means are: 144, 216, 324, 486  $\leftarrow \frac{1}{2} \text{ mark}$

2. A particle moves along the  $x$ -axis so that its position at time  $t$  is given by  $x(t) = 4t^3 - 33t^2 + 30t + 12$ , where  $t$  is measured in seconds and  $x$  is measured in metres.

a) Determine the velocity, in m/s, of the particle at time  $t = 2$  seconds. **(2 marks)**

**Solution:**

$$x(t) = 4t^3 - 33t^2 + 30t + 12$$

$$v(t) = x'(t) = 12t^2 - 66t + 30 \quad \leftarrow \mathbf{1 \text{ mark}}$$

$$v(2) = 12(2)^2 - 66(2) + 30 \quad \leftarrow \frac{1}{2} \mathbf{mark}$$

$$= -54 \text{ m/s} \quad \leftarrow \frac{1}{2} \mathbf{mark}$$

b) Determine the time(s), in seconds, when the particle is stationary. **(1 mark)**

**Solution:**

$$v(t) = 0 \quad \leftarrow \frac{1}{2} \mathbf{mark}$$

$$12t^2 - 66t + 30 = 0$$

$$2t^2 - 11t + 5 = 0$$

$$(2t - 1)(t - 5) = 0$$

$$\left. \begin{array}{l} t = \frac{1}{2} \text{ s} \\ t = 5 \text{ s} \end{array} \right\} \leftarrow \frac{1}{2} \mathbf{mark}$$

**Note to markers:** No deduction for missing units.

3. Graph the solution of the following system:

(3 marks)

$$\frac{x^2}{25} + \frac{y^2}{9} < 1$$

$$x^2 + 4x + y^2 \geq 12$$

**Solution:**

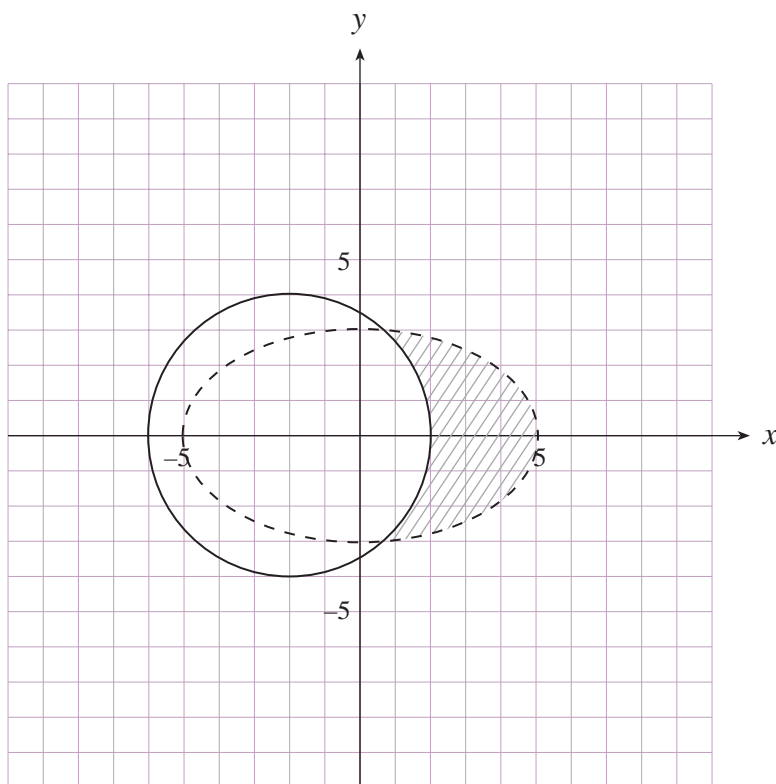
$$\frac{x^2}{25} + \frac{y^2}{9} < 1$$

$$x^2 + 4x + y^2 \geq 12$$

$$x^2 + 4x + 4 + y^2 \geq 12 + 4$$

$$(x + 2)^2 + y^2 \geq 16$$

} ← 1 mark



Ellipse graph ← 1 mark

Circle graph ←  $\frac{1}{2}$  mark

Shading ←  $\frac{1}{2}$  mark

4. Solve for  $x$ :  $500 = 2.5(7.04)^{\frac{x}{3}}$  (Accurate to at least 2 decimal places.)

**(3 marks)**

**Solution:**

$$500 = 2.5(7.04)^{\frac{x}{3}}$$

$$\frac{500}{2.5} = 7.04^{\frac{x}{3}} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$200 = 7.04^{\frac{x}{3}}$$

$$\log 200 = \frac{x}{3} \log 7.04 \quad \leftarrow \mathbf{1 \text{ mark}}$$

$$\frac{3 \log 200}{\log 7.04} = x \quad \leftarrow \mathbf{1 \text{ mark}}$$

$$8.14 = x \quad \leftarrow \frac{1}{2} \text{ mark}$$

5. Solve for  $x$ ,  $0 \leq x < 2\pi$ .

$$2 \sin x = 3 \sin x \cos x$$

(Accurate to at least 2 decimal places.)

**(3 marks)**

**Solution:**

$$2 \sin x = 3 \sin x \cos x$$

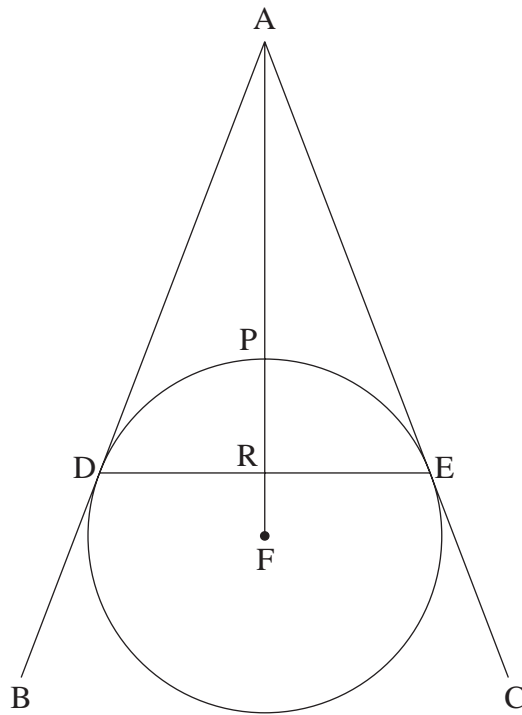
$$2 \sin x - 3 \sin x \cos x = 0 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\sin x(2 - 3 \cos x) = 0 \quad \leftarrow \frac{1}{2} \text{ mark}$$

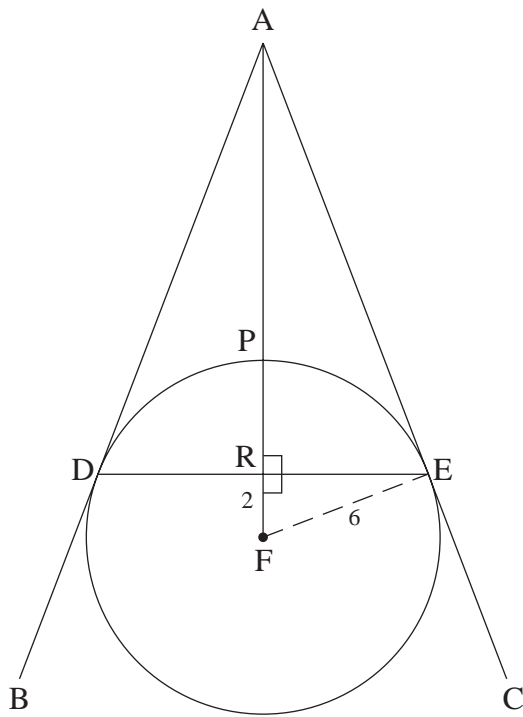
$$\frac{1}{2} \text{ mark} \rightarrow \sin x = 0 \quad \text{or} \quad \cos x = \frac{2}{3} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\frac{1}{2} \text{ mark} \rightarrow x = 0, \pi \quad \text{or} \quad x = 0.84, 5.44 \quad \leftarrow \frac{1}{2} \text{ mark}$$

6. The given circle has centre F and tangents AB and AC at D and E respectively. A, P, R, F are collinear and chord DE is perpendicular to AF. If the radius is 6 and FR is 2, determine the length of AR. (2 marks)



**Solution:**



$$RE = \sqrt{6^2 - 2^2} = \sqrt{32}$$

$$\frac{AR}{\sqrt{32}} = \frac{\sqrt{32}}{2}$$

$$AR = \frac{(\sqrt{32})(\sqrt{32})}{2}$$

$$AR = 16$$

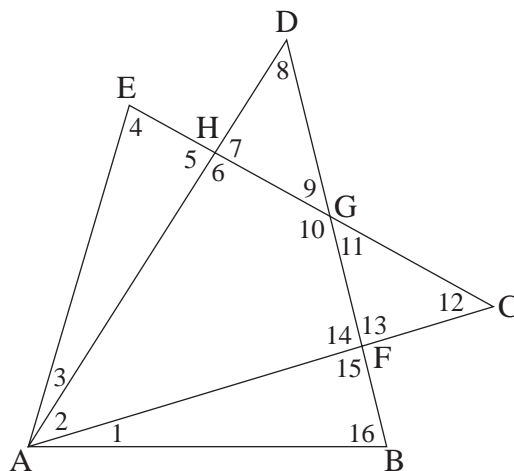


7. Complete the proof.

(4 marks)

Given: •  $\angle 8 = \angle 12$   
 •  $\angle 4 = \angle 16$

Prove: •  $\angle 3 = \angle 1$



**Solution:**

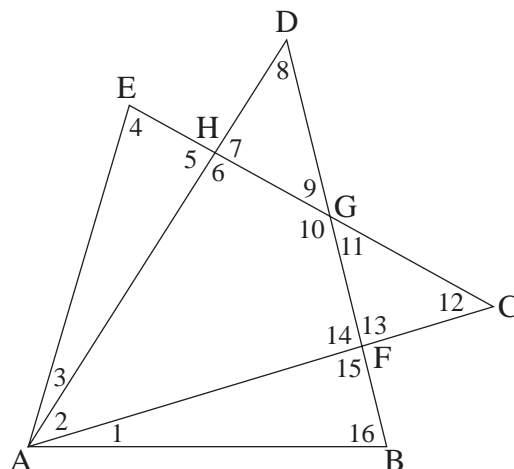
Proof	
Statement	Reason
$\angle 8 = \angle 12$	given
$\angle 4 = \angle 16$	given
$\angle EAC = \angle BAD$	3rd $\angle$ s of $\Delta$ s are =
$\angle 3 + \angle 2 = \angle 1 + \angle 2$	substitution
$\angle 3 = \angle 1$	equation property of subtraction

7. Complete the proof.

(4 marks)

Given: •  $\angle 8 = \angle 12$   
 •  $\angle 4 = \angle 16$

Prove: •  $\angle 3 = \angle 1$



**Alternate Solution:**

Proof	
Statement	Reason
$\angle 8 = \angle 12$	given
$\angle 9 = \angle 11$	vertically opposite $\angle$ s are =
$\angle 7 = \angle 13$	3rd $\angle$ s of $\Delta$ s are =
$\angle 5 = \angle 7, \angle 15 = \angle 13$	vertically opposite $\angle$ s are =
$\angle 5 = \angle 15$	both = to = angles
$\angle 4 = \angle 16$	given
$\angle 3 = \angle 1$	3rd $\angle$ s of $\Delta$ s are =

**END OF KEY**