# Mathematics 12

June 1998 Provincial Examination

## ANSWER KEY / SCORING GUIDE

**Topics:**
1. Trigonometry
2. Quadratic Relations
3. Exponential and Logarithmic Functions
4. Polynomial Functions
5. Sequences and Series
6. Introduction to Calculus
7. Geometry
8. Problem Solving

**Part A: Multiple Choice**

| Q | K | C | T | ILO | Q | K | C | T | ILO |
|---|---|---|---|---|---|---|---|---|---|
| 1. | C | K | 2 | 18 | 26. | C | K | 4 | 38 |
| 2. | D | U | 2 | 14 | 27. | A | U | 4 | 40 |
| 3. | C | U | 2 | 17 | 28. | C | U | 4 | 40 |
| 4. | C | U | 2 | 18 | 29. | D | U | 4 | 41 |
| 5. | B | U | 2 | 15 | 30. | B | U | 4 | 37 |
| 6. | D | U | 2 | 12 | 31. | A | U | 4 | 39 |
| 7. | D | U | 2 | 22 | 32. | D | U | 4 | 43 |
| 8. | B | U | 2 | 16 | 33. | D | H | 4 | 35 |
| 9. | D | H | 2 | 19 | 34. | A | U | 5 | 46 |
| 10. | B | H | 2 | 20 | 35. | A | U | 5 | 46 |
| 11. | C | K | 1 | 01 | 36. | C | K | 5 | 47 |
| 12. | C | U | 1 | 05 | 37. | B | U | 5 | 46 |
| 13. | C | U | 1 | 03 | 38. | C | H | 5 | 45 |
| 14. | B | U | 1 | 02 | 39. | A | K | 6 | 57 |
| 15. | B | U | 1 | 04 | 40. | A | U | 6 | 56 |
| 16. | D | U | 1 | 08 | 41. | A | U | 6 | 50 |
| 17. | B | H | 1 | 06 | 42. | D | U | 6 | 60 |
| 18. | D | H | 1 | 08 | 43. | B | U | 6 | 53 |
| 19. | B | K | 3 | 29 | 44. | C | U | 6 | 52 |
| 20. | A | U | 3 | 31 | 45. | C | H | 6 | 53 |
| 21. | A | U | 3 | 33 | 46. | D | U | 7 | 63 |
| 22. | D | U | 3 | 25 | 47. | B | U | 7 | 63 |
| 23. | A | U | 3 | 30 | 48. | C | U | 8 | 64 |
| 24. | B | H | 3 | 32 | 49. | D | U | 8 | 64 |
| 25. | A | H | 3 | 31 | 50. | B | U | 8 | 64 |

**Multiple Choice = 50 marks**

**Part B: Written Response**

| Q | B | C | S | T | ILO |
|---|---|---|---|---|---|
| 1. | 1 | U | 3 | 3 | 32 |
| 2. | 2 | U | 2 | 1 | 08 |
| 3. | 3 | U | 3 | 5 | 48 |
| 4. | 4 | U | 3 | 6 | 58 |
| 5. | 5 | U | 3 | 2 | 21 |
| 6. | 6 | H | 4 | 7 | 63 |
| 7. | 7 | U | 2 | 8 | 64 |

**Written Response = 20 marks**

Multiple Choice = 50 (50 questions)
Written Response = 20 (7 questions)
**EXAMINATION TOTAL = 70 marks**

**LEGEND:**
**Q** = Question Number  **K** = Keyed Response  **C** = Cognitive Level
**B** = Score Box Number  **S** = Score  **T** = Topic
**ILO** = Intended Learning Outcome

## PART B: WRITTEN RESPONSE

**Value: 20 marks** **Suggested Time: 45 minutes**

**INSTRUCTIONS:** Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

**Full marks will NOT be given for the final answer only.**

1. Solve for $x$: $\log(3x-5)+\log(2x-1)=1$ **(3 marks)**

**SOLUTION:**

$$\log(3x-5)+\log(2x-1)=1$$

$$\log(3x-5)(2x-1)=1 \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$(3x-5)(2x-1)=10 \quad \leftarrow \textbf{1 mark}$$

$$6x^2-13x+5=10$$

$$6x^2-13x-5=0 \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$(3x+1)(2x-5)=0$$

$\cancel{x=-\frac{1}{3}}$ $\qquad x=\frac{5}{2}$

reject

$\uparrow$ $\qquad\qquad \uparrow$

$\frac{1}{2}$ **mark** $\qquad \frac{1}{2}$ **mark**

2. Prove the identity: **(2 marks)**

$$\frac{\csc\theta}{\tan\theta+\cot\theta}=\cos\theta$$

**SOLUTION:**

| Left Side | Right Side |
|---|---|
| $\dfrac{\csc\theta}{\tan\theta+\cot\theta}$ | $\cos\theta$ |
| $\frac{1}{2}$ **mark** $\rightarrow$ $\dfrac{\frac{1}{\sin\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}$ | |
| $\frac{1}{2}$ **mark** $\rightarrow$ $\dfrac{\sin\theta\cos\theta\left(\frac{1}{\sin\theta}\right)}{\sin\theta\cos\theta\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)}$ | |
| $\frac{1}{2}$ **mark** $\rightarrow$ $\dfrac{\cos\theta}{\sin^2\theta+\cos^2\theta}$ | |
| $\frac{1}{2}$ **mark** $\rightarrow$ $\cos\theta$ | |

LS = RS

2. Prove the identity: **(2 marks)**

$$\frac{\csc\theta}{\tan\theta+\cot\theta}=\cos\theta$$

**ALTERNATE SOLUTION #1:**

| Left Side | Right Side |
|---|---|
| $\dfrac{\csc\theta}{\tan\theta+\cot\theta}$ | $\cos\theta$ |
| $\frac{1}{2}$ **mark** $\rightarrow$ $\dfrac{\frac{1}{\sin\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}$ | |
| $\frac{1}{2}$ **mark** $\rightarrow$ $\dfrac{\frac{1}{\sin\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}}$ | |
| $\frac{1}{2}$ **mark** $\rightarrow$ $\dfrac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta\cos\theta}}$ | |
| $\dfrac{1}{\sin\theta}\cdot\dfrac{\sin\theta\cos\theta}{1}$ | |
| $\frac{1}{2}$ **mark** $\rightarrow$ $\cos\theta$ | |

LS = RS

2. Prove the identity: **(2 marks)**

$$\frac{\csc\theta}{\tan\theta+\cot\theta}=\cos\theta$$

**ALTERNATE SOLUTION #2:**

| Left Side | Right Side | |
|---|---|---|
| $\dfrac{\csc\theta}{\tan\theta+\cot\theta}$ | $\cos\theta$ | |
| | $\dfrac{\frac{1}{\sin\theta}}{\tan\theta+\frac{1}{\tan\theta}}$ | $\leftarrow \frac{1}{2}$ **mark** |
| | $\dfrac{\frac{1}{\sin\theta}}{\frac{\tan^2\theta+1}{\tan\theta}}$ | $\leftarrow \frac{1}{2}$ **mark** |
| | $\dfrac{\frac{1}{\sin\theta}}{\frac{\sec^2\theta}{\tan\theta}}$ | $\leftarrow \frac{1}{2}$ **mark** |
| | $\dfrac{1}{\sin\theta}\cdot\dfrac{\tan\theta}{\sec^2\theta} = \dfrac{1}{\sin\theta}\cdot\dfrac{\sin\theta}{\cos\theta}\cdot\cos^2\theta$ | |
| | $\cos\theta$ | $\leftarrow \frac{1}{2}$ **mark** |

LS = RS

3. The 780 students of a high school form a pyramid shape by sitting in rows on the bleachers. The top row has one student, and each row below has one more student than the previous row. How many rows are required to seat all 780 students? **(3 marks)**

**SOLUTION:**

$a = 1$

$d = 1$ $\leftarrow \frac{1}{2}$ **mark**

$S_n = 780$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$780 = \frac{n}{2}\left[2 + (n-1)1\right]$ $\leftarrow$ **1 mark**

$780 = \frac{n}{2}(1+n)$

$1\,560 = n + n^2$

$n + n^2 - 1\,560 = 0$ $\leftarrow \frac{1}{2}$ **mark**

$(n+40)(n-39) = 0$

$n = -40$ $\quad$ $n = 39$
reject
$\uparrow$
$\frac{1}{2}$ **mark**

$\therefore 39$ rows are required $\leftarrow \frac{1}{2}$ **mark**

**ALTERNATE SOLUTION:**

$1 + 2 + 3 + \ldots + n = 780$

$a = 1$

$\ell = n$ $\leftarrow \frac{1}{2}$ **mark**

$S_n = \frac{n}{2}(a + \ell) = 780$

$\frac{n}{2}(1+n) = 780$ $\leftarrow$ **1 mark**

$n + n^2 = 1\,560$

$n + n^2 - 1\,560 = 0$ $\leftarrow \frac{1}{2}$ **mark**

$(n+40)(n-39) = 0$

$n = -40$ $\quad$ $n = 39$
reject
$\uparrow$
$\frac{1}{2}$ **mark**

$\therefore 39$ rows are required $\leftarrow \frac{1}{2}$ **mark**

4. Determine all values of $x$ such that the function $f(x) = x^4 - 18x^2 + 8$ is **decreasing**. **(3 marks)**

**SOLUTION:**

$$f(x) = x^4 - 18x^2 + 8$$

$$f'(x) = 4x^3 - 36x \qquad \leftarrow \tfrac{1}{2} \text{ **mark**}$$

$$4x^3 - 36x = 0 \qquad \leftarrow \tfrac{1}{2} \text{ **mark**}$$

$$4x(x+3)(x-3) = 0$$

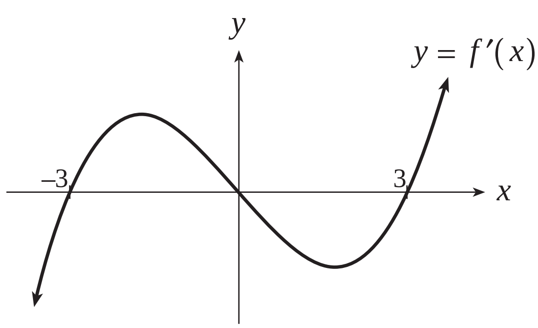
critical points:

$x = 0$ , $x = -3$ , $x = 3$ $\leftarrow \frac{1}{2}$ **mark**

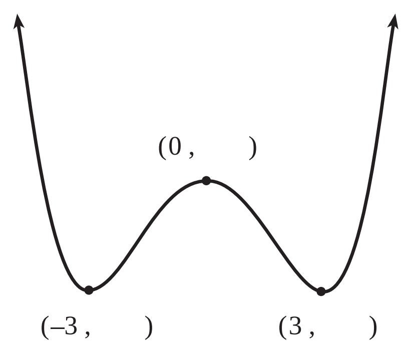
$f'(x) = 4x(x+3)(x-3) < 0$ $\leftarrow \frac{1}{2}$ **mark**

**OR** **Alternate Graphing Solution:**

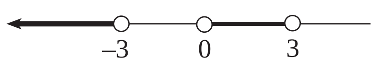
Graph of $f'(x)$



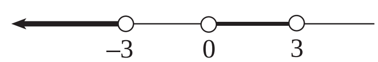

$f(x) = x^4 - 18x^2 + 8$




$x < -3$ **or** $0 < x < 3$ $\leftarrow$ **1 mark**




$x < -3$ **or** $0 < x < 3$ $\leftarrow$ **1 mark**

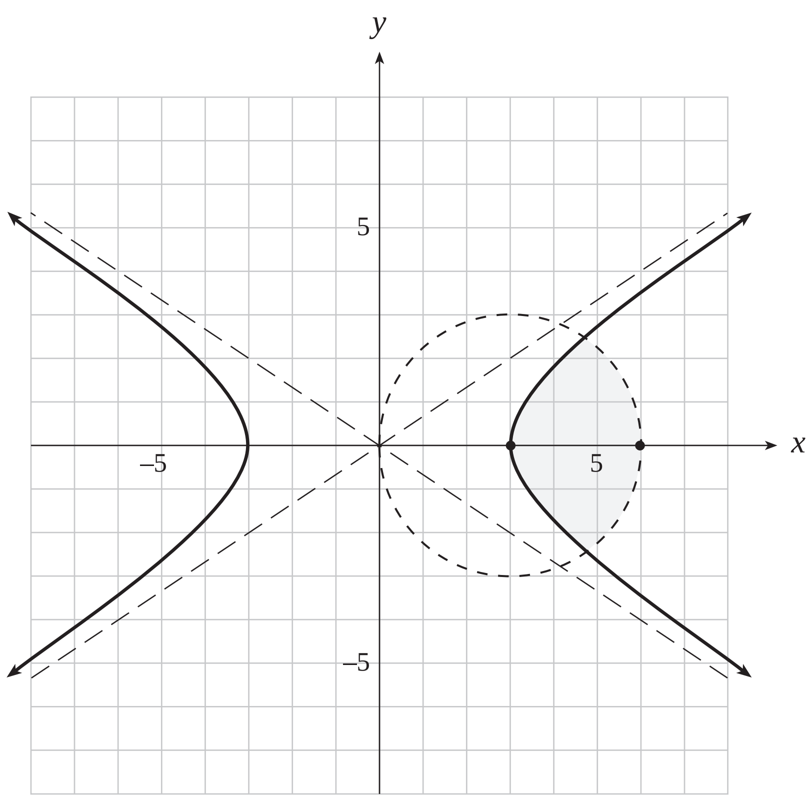


5. Graph the solution of the following system of inequalities on the grid provided. **(3 marks)**

$$\frac{x^2}{9} - \frac{y^2}{4} \geq 1$$

$$(x-3)^2 + y^2 < 9$$

**SOLUTION:**




**1 mark for graph of hyperbola**

**1 mark for graph of circle**

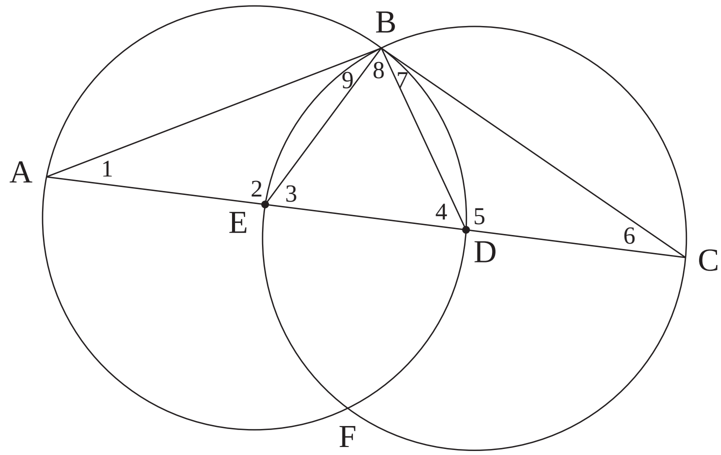
**$\frac{1}{2}$ mark for correct shading of circle**

**$\frac{1}{2}$ mark for correct shading of hyperbola**

6. Complete the proof. **(4 marks)**

Given: 2 circles with centres E and D
BE = BD
A, E, D, C are collinear

Prove: $\Delta ABC$ is isosceles




**SOLUTION:**

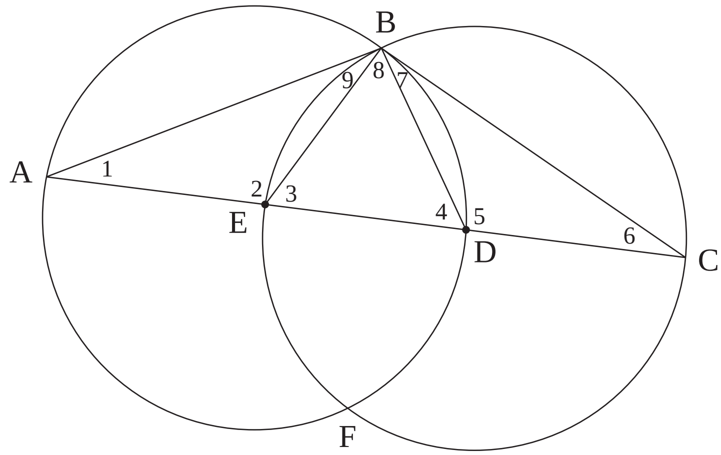
PROOF

| | Statement | Reason |
|---|---|---|
| | $BE = BD$ | given |
| **$\frac{1}{2}$ mark** $\rightarrow$ | $\angle 3 = \angle 4$ | $\angle$s opposite = sides are = |
| **1 mark** $\rightarrow$ | $\angle 2 = \angle 5$ | supplements of = $\angle$s are = |
| **$\frac{1}{2}$ mark** $\rightarrow$ | $AE = ED$ | = radii |
| | $CD = ED$ | = radii |
| **$\frac{1}{2}$ mark** $\rightarrow$ | $AE = CD$ | both = ED, substitution |
| **$\frac{1}{2}$ mark** $\rightarrow$ | $\Delta AEB \cong \Delta CDB$ | SAS |
| **$\frac{1}{2}$ mark** $\rightarrow$ | $AB = CB$ or $\angle 1 = \angle 6$ | CPCTC |
| **$\frac{1}{2}$ mark** $\rightarrow$ | $\Delta ABC$ is isosceles | definition of isosceles |

6. Complete the proof. **(4 marks)**

Given: 2 circles with centres E and D
BE = BD
A, E, D, C are collinear

Prove: $\Delta ABC$ is isosceles




**ALTERNATE SOLUTION #1:**

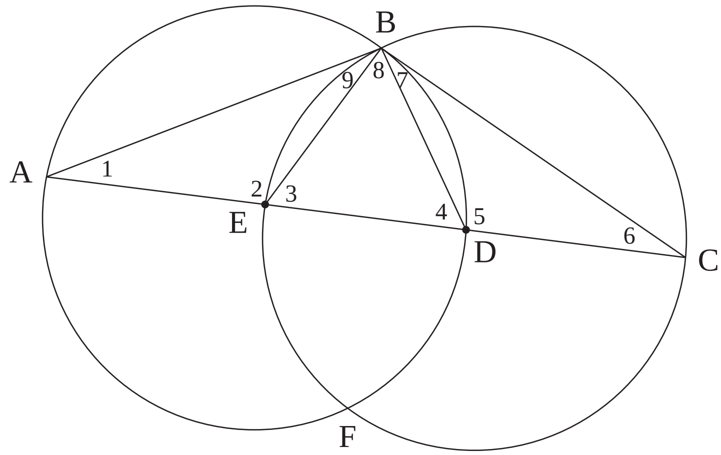
PROOF

| | Statement | Reason |
|---|---|---|
| | BE = BD | given |
| **½ mark** → | $\angle ABD = 90°$, $\angle CBE = 90°$ | inscribed ∠ on diameter |
| **½ mark** → | AE = ED | = radii |
| | ED = DC | = radii |
| **1 mark** → | AE + ED = ED + DC | equation property of addition |
| **½ mark** → | AD = EC | substitution |
| **½ mark** → | $\Delta ABD \cong \Delta CBE$ | HL |
| **½ mark** → | AB = CB | CPCTC |
| **½ mark** → | $\Delta ABC$ is isosceles | definition of isosceles |

6. Complete the proof. **(4 marks)**

Given: 2 circles with centres E and D
BE = BD
A, E, D, C are collinear

Prove: $\Delta$ABC is isosceles




**ALTERNATE SOLUTION #2:**

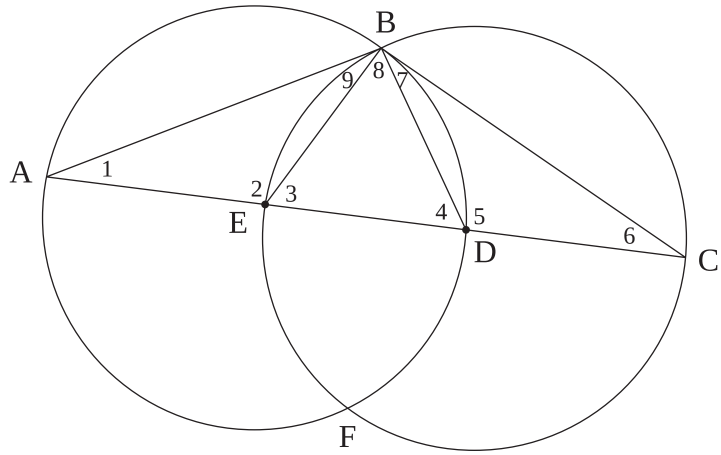
PROOF

| | Statement | Reason |
|---|---|---|
| **1 mark** $\rightarrow$ | $\angle ABD = 90°$, $\angle CBE = 90°$ | inscribed $\angle$ on diameter |
| **1 mark** $\rightarrow$ | AB is tangent to circle D at B | tangent $\perp$ radius |
| | CB is tangent to circle E at B | tangent $\perp$ radius |
| $\frac{1}{2}$ **mark** $\rightarrow$ | $\angle 1 = \angle 7$, $\angle 6 = \angle 9$ | $\angle$ between tangent and chord |
| | $\angle ABD = \angle CBE$ | both = 90°, substitution |
| $\frac{1}{2}$ **mark** $\rightarrow$ | $\angle ABD - \angle 8 = \angle CBE - \angle 8$ | equation property of subtraction |
| | $\angle 9 = \angle 7$ | substitution |
| $\frac{1}{2}$ **mark** $\rightarrow$ | $\angle 1 = \angle 6$ | both = to = $\angle$s (substitution) |
| $\frac{1}{2}$ **mark** $\rightarrow$ | $\Delta$ABC is isosceles | definition of isosceles |

6. Complete the proof. **(4 marks)**

Given: 2 circles with centres E and D
BE = BD
A, E, D, C are collinear

Prove: $\Delta ABC$ is isosceles




**ALTERNATE SOLUTION #3:**

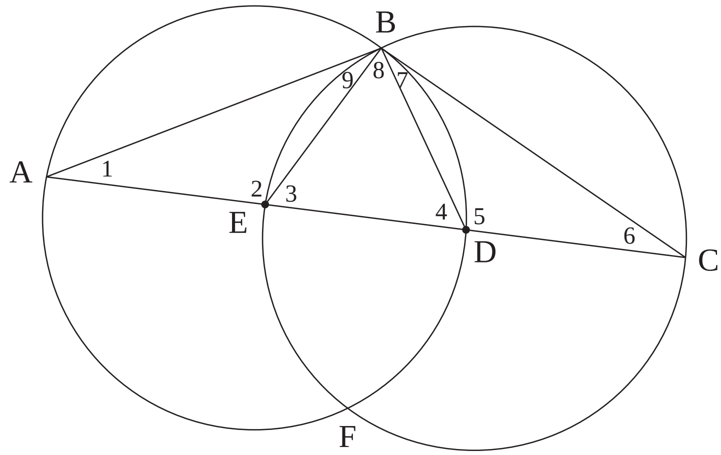
PROOF

| | Statement | Reason |
|---|---|---|
| **1 mark** → | $\angle ABD = 90°$, $\angle CBE = 90°$ | inscribed $\angle$ on diameter |
| | $\angle ABD = \angle CBE$ | both = $90°$, substitution |
| | BE = BD | given |
| **½ mark** → | $\angle 3 = \angle 4$ | $\angle$s opposite = sides are = |
| **2 marks** → | $\angle 1 = \angle 6$ | 3rd $\angle$s in $\Delta$s are = |
| **½ mark** → | $\Delta ABC$ is isosceles | definition of isosceles |

6. Complete the proof. **(4 marks)**

Given: 2 circles with centres E and D
BE = BD
A, E, D, C are collinear

Prove: $\Delta ABC$ is isosceles




**ALTERNATE SOLUTION #4:**

PROOF

| | Statement | Reason |
|---|---|---|
| | BE = BD | given |
| $\frac{1}{2}$ **mark** $\rightarrow$ | $\angle 3 = \angle 4$ | $\angle$s opposite = sides are = |
| $\frac{1}{2}$ **mark** $\rightarrow$ | AE = ED | = radii |
| | ED = DC | = radii |
| **1 mark** $\rightarrow$ | AE + ED = ED + DC | equation property of addition |
| $\frac{1}{2}$ **mark** $\rightarrow$ | AD = EC | substitution |
| $\frac{1}{2}$ **mark** $\rightarrow$ | $\Delta ABD \cong \Delta CBE$ | SAS |
| $\frac{1}{2}$ **mark** $\rightarrow$ | AB = BC | CPCTC |
| $\frac{1}{2}$ **mark** $\rightarrow$ | $\Delta ABC$ is isosceles | definition of isosceles |

7. Given: Circle with centre O

PQ, PR, QR are tangents at A, B, C respectively

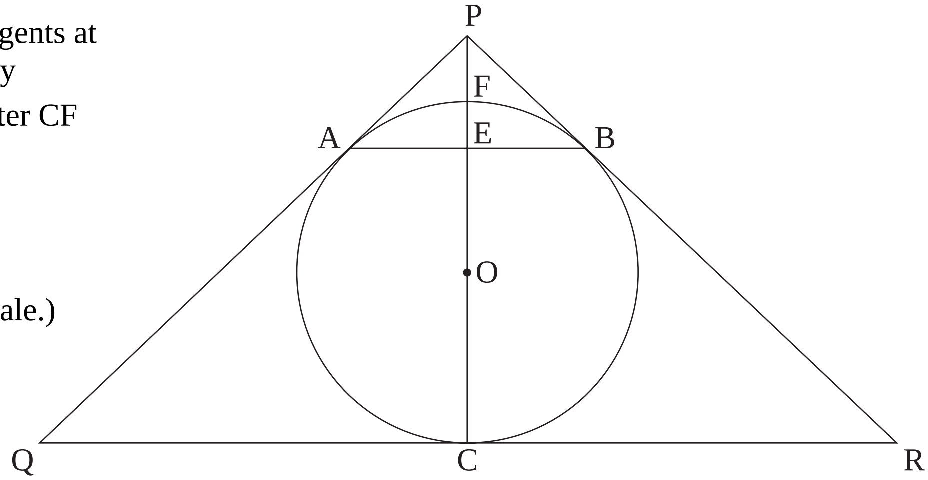
Chord AB $\perp$ diameter CF

$CF = 8$, $EF = 1$

**(2 marks)**

Determine the length of PE.

(Diagram is not drawn to scale.)




**SOLUTION:**

$OB = 4$

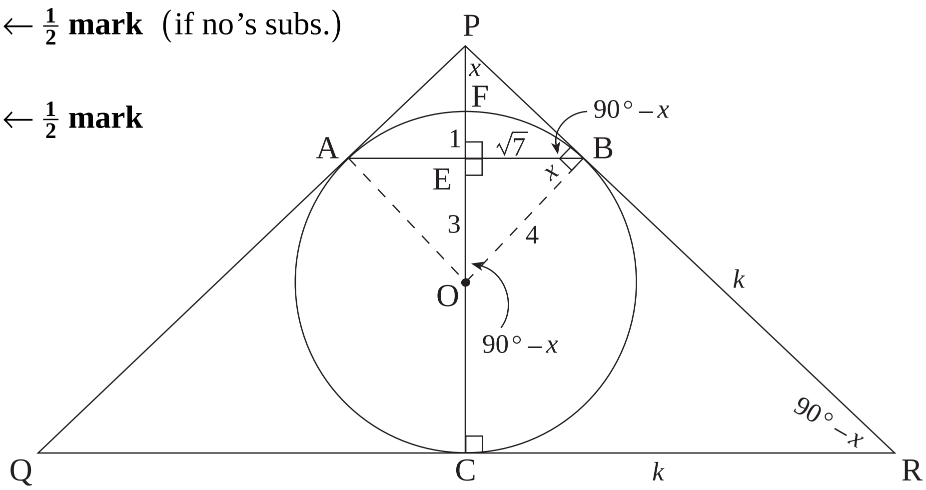
$EO = 3$

$\Rightarrow EB = \sqrt{7}$ (Pythagoras) $\leftarrow \frac{1}{2}$ **mark**

$\Delta PEB \sim \Delta BEO$ $\leftarrow \frac{1}{2}$ **mark** for any similar $\Delta$s leading to a solution (or ratio of sides)

$\frac{PE}{\sqrt{7}} = \frac{\sqrt{7}}{3}$ $\leftarrow \frac{1}{2}$ **mark** (if no's subs.)

$PE = \frac{7}{3}$ $\leftarrow \frac{1}{2}$ **mark**




**Using trig:**

$\angle x = 4.86°$ $\leftarrow \frac{1}{2}$ **mark**

tan or cos ratio $\leftarrow$ **1 mark**

$PE = 2.33$ $\leftarrow \frac{1}{2}$ **mark**

**Note:** only angle + $EB = \sqrt{1}$ $\leftarrow$ **1 mark**

**END OF KEY**