

Mathematics 12

January 1998 Provincial Examination

ANSWER KEY / SCORING GUIDE

- Topics:**
1. Trigonometry
 2. Quadratic Relations
 3. Exponential and Logarithmic Functions
 4. Polynomial Functions
 5. Sequences and Series
 6. Introduction to Calculus
 7. Geometry
 8. Problem Solving

Part A: Multiple Choice

Q	K	C	T	ILO		Q	K	C	T	ILO
1.	B	K	2	17		26.	A	K	4	40
2.	D	U	2	17		27.	C	U	4	38
3.	A	U	2	12		28.	B	U	4	37
4.	C	U	2	17		29.	D	U	4	41
5.	D	U	2	22		30.	B	U	4	35
6.	A	U	2	21		31.	A	U	4	40
7.	C	U	2	18		32.	D	U	4	43
8.	B	U	2	17		33.	A	H	4	39
9.	B	H	2	15		34.	D	K	5	46
10.	DELETED					35.	D	U	5	46
11.	B	K	1	01		36.	B	U	5	47
12.	B	K	1	05		37.	C	U	5	46
13.	C	U	1	02		38.	C	H	5	45
14.	C	U	1	03		39.	C	K	6	57
15.	A	U	1	06		40.	C	K	6	50
16.	B	U	1	08		41.	B	U	6	51
17.	B	U	1	09		42.	D	K	6	56
18.	D	H	1	08		43.	C	U	6	61
19.	A	U	3	31		44.	B	U	6	62
20.	A	U	3	31		45.	D	H	6	53
21.	D	U	3	30		46.	D	H	7	63
22.	C	U	3	32		47.	A	H	7	63
23.	B	U	3	24		48.	B	U	8	64
24.	A	H	3	31		49.	D	U	8	64
25.	A	H	3	30		50.	B	H	8	64

Multiple Choice = 50 marks

Part B: Written Response

Q	B	C	S	T	ILO
1.	1	U	3	5	46
2.	2	U	2	1	08
3.	3	U	3	6	60
4a.	4	U	2	3	26
4b.	5	U	1	3	32
5.	6	U	3	2	19
6.	7	H	4	7	63
7.	8	H	2	8	64

Written Response = 20 marks

Multiple Choice = 50 (50 questions)

Written Response = 20 (7 questions)

EXAMINATION TOTAL = 70 marks

LEGEND:

Q = Question Number

B = Score Box Number

ILO = Intended Learning Outcome

K = Keyed Response

S = Score

C = Cognitive Level

T = Topic

PART B: WRITTEN RESPONSE

Value: 20 marks

Suggested Time: 45 minutes

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

Full marks will NOT be given for the final answer only.

1. Find the sum of the first 30 terms of the geometric series $10 + 12 + 14.4 + \dots$
(Accurate to at least 2 decimal places.)

(3 marks)

SOLUTION:

$$r = \frac{12}{10} = 1.2 \quad \leftarrow \mathbf{1 \text{ mark}}$$

$$\text{use } S_n = \frac{a(1-r^n)}{1-r}$$

where $a = 10$, $n = 30$

$$S_{30} = \frac{10(1-1.2^{30})}{1-1.2} \quad \leftarrow \mathbf{1 \text{ mark}}$$

$$\approx 11\,818.816$$

$$= 11\,818.82 \quad \leftarrow \mathbf{1 \text{ mark}}$$

2. Prove the identity.

(2 marks)

$$\frac{1 - \cos \theta}{\sin^2 \theta} = \frac{1}{1 + \cos \theta}$$

SOLUTION:

$$\frac{1 - \cos \theta}{\sin^2 \theta} = \frac{1}{1 + \cos \theta}$$

Left Side	Right Side
$\frac{1 - \cos \theta}{\sin^2 \theta}$	$\frac{1}{1 + \cos \theta}$
$\frac{1}{2}$ mark $\rightarrow = \frac{1 - \cos \theta}{1 - \cos^2 \theta}$	
1 mark $\rightarrow = \frac{1 - \cos \theta}{(1 + \cos \theta)(1 - \cos \theta)}$	
$\frac{1}{2}$ mark $\rightarrow = \frac{1}{1 + \cos \theta}$	

LS = RS

2. Prove the identity.

(2 marks)

$$\frac{1 - \cos \theta}{\sin^2 \theta} = \frac{1}{1 + \cos \theta}$$

ALTERNATE SOLUTION:

$$\frac{1 - \cos \theta}{\sin^2 \theta} = \frac{1}{1 + \cos \theta}$$

Left Side	Right Side
$\frac{1 - \cos \theta}{\sin^2 \theta}$	$\frac{1}{1 + \cos \theta}$
	$\frac{1}{(1 + \cos \theta)} (1 - \cos \theta)$ ← 1 mark
	$\frac{1 - \cos \theta}{1 - \cos^2 \theta}$ ← $\frac{1}{2}$ mark
	$\frac{1 - \cos \theta}{\sin^2 \theta}$ ← $\frac{1}{2}$ mark

LS = RS

3. A particle moves along the x -axis such that its distance from the origin is given by $x(t) = 2t^2 + 60t$ where x is in centimetres and t is in seconds. When the particle's velocity is 72 cm/sec, determine its distance $x(t)$ from the origin. **(3 marks)**

SOLUTION:

$$v = x'(t) \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$x'(t) = 4t + 60 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$4t + 60 = 72 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$t = 3 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$x(3) = 2(3)^2 + 60(3) \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$= 198 \text{ cm} \quad \leftarrow \frac{1}{2} \text{ mark}$$

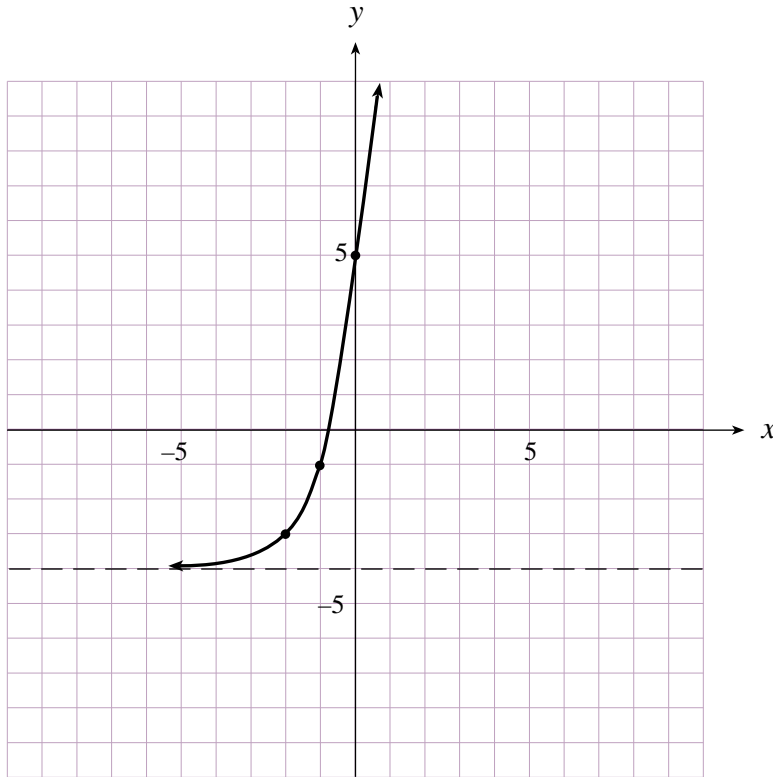
(Note: do not deduct for missing units.)

The following question has two parts: a) and b)

4. a) Graph the function $y = 3^{x+2} - 4$ on the grid provided. Indicate the asymptote with a dotted or broken line and clearly show at least three points on the curve. (2 marks)

SOLUTION:

$$y = 3^{x+2} - 4$$



asymptote: $\leftarrow \frac{1}{2}$ mark

shape: $\leftarrow 1$ mark

3 points: $\leftarrow \frac{1}{2}$ mark

Note: The asymptote need not be drawn if asymptotic behaviour is clearly indicated along $y = -4$.

b) Calculate the zero of this function. (Accurate to at least 2 decimal places.)

(1 mark)

SOLUTION:

$$0 = 3^{x+2} - 4 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$4 = 3^{x+2}$$

$$\log 4 = (x + 2)\log 3$$

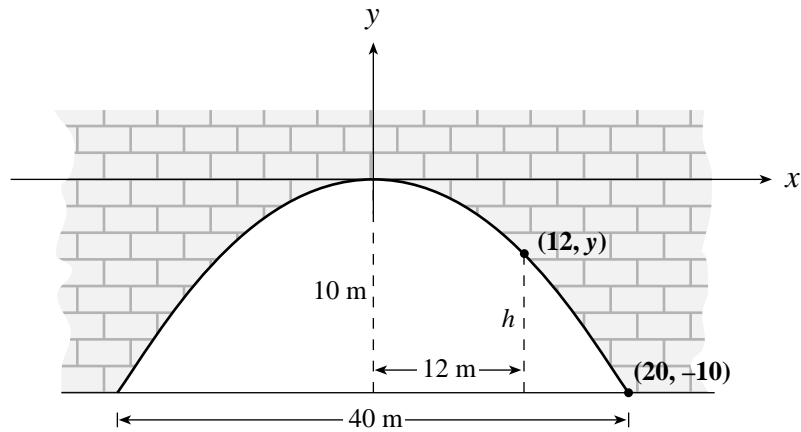
$$x + 2 = \frac{\log 4}{\log 3}$$

$$x = \frac{\log 4}{\log 3} - 2$$

$$x = -0.74 \quad \leftarrow \frac{1}{2} \text{ mark}$$

5. The arch of a bridge over a river has the shape of a parabola. The base is 40 m wide and the maximum height of the arch is 10 m. Find the height h of the arch at a distance of 12 m from the centre of the arch. (Accurate to at least 1 decimal place.) **(3 marks)**

SOLUTION:



$$y = ax^2 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$-10 = a(400) \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$-\frac{10}{400} = a$$

$$a = -\frac{1}{40} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$y = -\frac{1}{40}x^2 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\therefore y = -\frac{1}{40}(12)^2 \quad \leftarrow \frac{1}{2} \text{ mark}$$

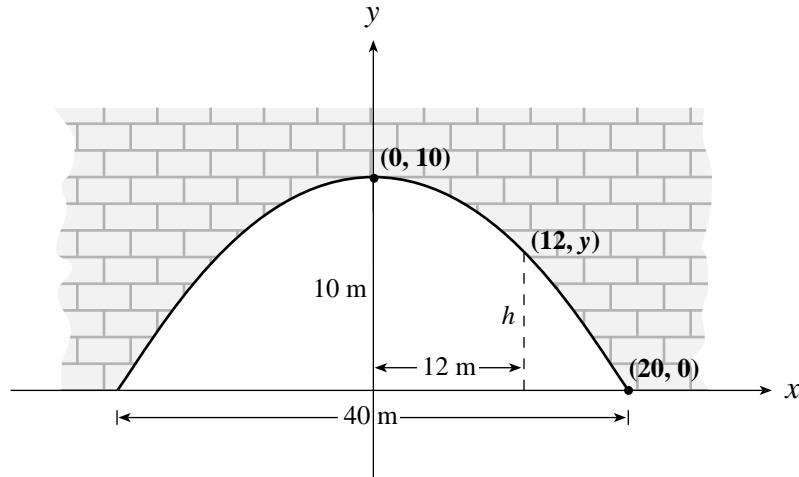
$$y = -3.6$$

$$h = 10 - 3.6$$

$$h = 6.4 \text{ m} \quad \leftarrow \frac{1}{2} \text{ mark}$$

5. The arch of a bridge over a river has the shape of a parabola. The base is 40 m wide and the maximum height of the arch is 10 m. Find the height h of the arch at a distance of 12 m from the centre of the arch. (Accurate to at least 1 decimal place.) **(3 marks)**

ALTERNATE SOLUTION:



$$y = ax^2 + 10 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$0 = 400a + 10 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$-\frac{1}{40} = a \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$y = -\frac{1}{40}x^2 + 10 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$y = -\frac{1}{40}(12)^2 + 10 \quad \leftarrow \frac{1}{2} \text{ mark}$$

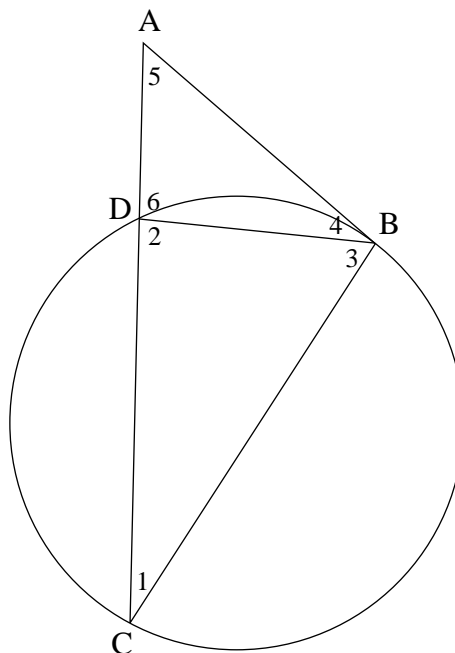
$$y = 6.4 \quad \leftarrow \frac{1}{2} \text{ mark}$$

6. Complete the proof.

(4 marks)

Given: AB is a tangent
 $\angle 3 = \angle 5$
 C, D, A are collinear

Prove: BC is a diameter



SOLUTION:

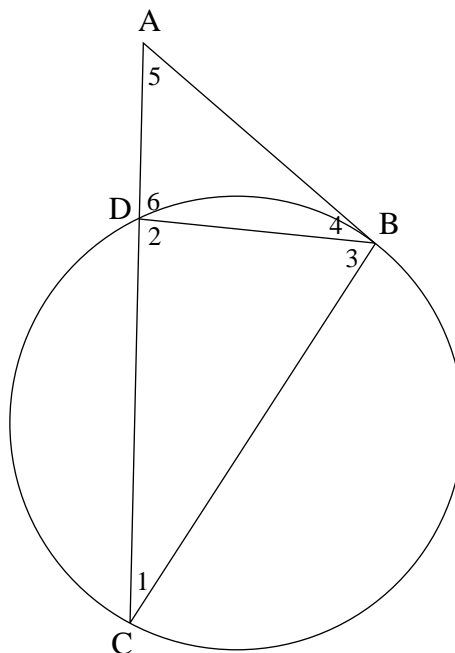
PROOF	
Statement	Reason
$\left. \begin{array}{l} \text{AB is a tangent} \\ \angle 4 = \angle 1 \\ \angle 3 = \angle 5 \\ \angle 2 = \angle 6 \end{array} \right\} \text{2 marks} \rightarrow$	<p>given</p> <p>\angle between tangent and chord</p> <p>given</p> <p>3rd \angles of Δs are =</p>
$\left. \begin{array}{l} \angle 2 + \angle 6 = 180^\circ \\ \angle 2 = 90^\circ \\ \text{BC is a diameter} \end{array} \right\} \text{2 marks} \rightarrow$	<p>\angles on a line</p> <p>$2 = \angle$s with sum of 180°</p> <p>inscribed \angle on BC = 90°</p>

6. Complete the proof.

(4 marks)

Given: AB is a tangent
 $\angle 3 = \angle 5$
 C, D, A are collinear

Prove: BC is a diameter



ALTERNATE SOLUTION:

PROOF	
Statement	Reason
$2 \text{ marks} \rightarrow \left\{ \begin{array}{l} AB \text{ is a tangent} \\ \angle 4 = \angle 1 \\ \angle 3 = \angle 5 \\ \angle 2 = \angle 6 \end{array} \right.$	given \angle between tangent and chord given 3rd \angle s of Δ s are =
$2 \text{ marks} \rightarrow \left\{ \begin{array}{l} \angle 2 = \angle 6 = 90^\circ \\ \angle 4 + \angle 5 = 90^\circ \\ \angle 4 + \angle 3 = 90^\circ \\ \angle ABC = 90^\circ \\ BC \text{ is a diameter} \end{array} \right.$	= supplementary \angle s complementary \angle s substitution substitution inscribed \angle on BC = 90°

7. A river system has a current population of 4 000 000 fish. In each year, an enhancement program produces a 7% growth in population followed by a 200 000 fish harvest (i.e. 200 000 fish are taken from the population). Determine the number of fish after the second harvest.

(2 marks)

SOLUTION:

$\frac{1}{2}$ mark $\frac{1}{2}$ mark
↓ ↓

$$P_1 = 4\,000\,000(1 + 0.07) - 200\,000$$
$$= 4\,080\,000$$

$\frac{1}{2}$ mark (recursion)
↓

$$P_2 = 4\,080\,000(1 + 0.07) - 200\,000$$
$$= 4\,165\,600$$

← $\frac{1}{2}$ mark

END OF KEY