

Mathematics 12

August 1997 Provincial Examination

ANSWER KEY / SCORING GUIDE

- Topics:**
1. Trigonometry
 2. Quadratic Relations
 3. Exponential and Logarithmic Functions
 4. Polynomial Functions
 5. Sequences and Series
 6. Introduction to Calculus
 7. Geometry
 8. Problem Solving

Part A: Multiple Choice

Q	C	T	K	S	ILO	Q	C	T	K	S	ILO
1.	K	2	D	1	13	26.	K	4	B	1	38
2.	K	2	D	1	14	27.	U	4	A	1	35
3.	U	2	C	1	18	28.	U	4	C	1	40
4.	U	2	B	1	12	29.	U	4	A	1	37
5.	U	2	A	1	15	30.	U	4	D	1	41
6.	U	2	A	1	17	31.	H	4	D	1	43
7.	U	2	D	1	22	32.	K	5	C	1	45
8.	H	2	C	1	17, 64	33.	U	5	B	1	46
9.	U	2	D	1	19	34.	U	5	C	1	45
10.	H	2	B	1	16	35.	U	5	B	1	46
11.	K	1	D	1	02	36.	U	5	D	1	47
12.	U	1	C	1	01	37.	U	5	C	1	46
13.	U	1	D	1	05	38.	H	5	B	1	45, 03
14.	U	1	C	1	05	39.	K	6	B	1	57
15.	U	1	A	1	08	40.	U	6	B	1	51
16.	U	1	B	1	08	41.	U	6	B	1	50
17.	H	1	D	1	03	42.	U	6	C	1	60
18.	H	1	A	1	06	43.	U	6	A	1	59
19.	U	3	A	1	28	44.	U	6	C	1	52
20.	K	3	A	1	26	45.	H	6	D	1	59, 64
21.	U	3	B	1	31	46.	U	7	C	1	63
22.	U	3	D	1	25	47.	H	7	A	1	63
23.	U	3	C	1	33	48.	U	8	C	1	64
24.	H	3	D	1	31	49.	U	8	B	1	64
25.	H	3	C	1	30	50.	H	8	B	1	64

Part B: Written Response

Q	B	C	T	S	ILO	Q	B	C	T	S	ILO
1.	1	U	3	3	32	5.	5	U	4	2	39
2.	2	U	6	3	54	6.	6	H	7	4	63
3.	3	U	2	3	21	7.	7	U	8	2	64
4.	4	U	1	3	03						

Multiple Choice = 50 (50 questions)

Written Response = 20 (7 questions)

Total = 70 marks

LEGEND:

Q = Question Number

C = Cognitive Level

T = Topic

K = Keyed Response

S = Score

ILO = Intended Learning Outcome

B = Score Box Number

PART B: WRITTEN RESPONSE

Value: 20 marks

Suggested Time: 45 minutes

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

Full marks will NOT be given for the final answer only.

1. Solve: $\log(x + 4) = 1 - \log 2x$

(3 marks)

Solution:

$$\log(x + 4) + \log 2x = 1$$

$$\log[(2x)(x + 4)] = 1 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$2x^2 + 8x = 10^1 \quad \leftarrow \mathbf{1 \text{ mark}}$$

$$2x^2 + 8x - 10 = 0 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$x^2 + 4x - 5 = 0$$

$$(x + 5)(x - 1) = 0$$

$$x = -5 \quad x = 1$$

reject

↑

↑

$\frac{1}{2}$ mark $\frac{1}{2}$ mark

2. Determine an equation of the line tangent to the curve $y = x^3 - 3x^2 - 5x$ at the point where $x = -1$.

(3 marks)

Solution:

$$y' = m_t = 3x^2 - 6x - 5 \quad \left\{ \begin{array}{l} \frac{1}{2} \text{ mark for derivative} \\ \frac{1}{2} \text{ mark for concept derivative = slope of tangent} \end{array} \right.$$

$$y'(-1) = 3 + 6 - 5 = 4 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$y(-1) = -1 - 3 + 5 = 1 \quad \leftarrow \frac{1}{2} \text{ mark}$$

point is $\therefore (-1, 1)$

$$y - 1 = 4(x + 1) \quad \leftarrow 1 \text{ mark}$$

$$\text{or } y = 4x + 5$$

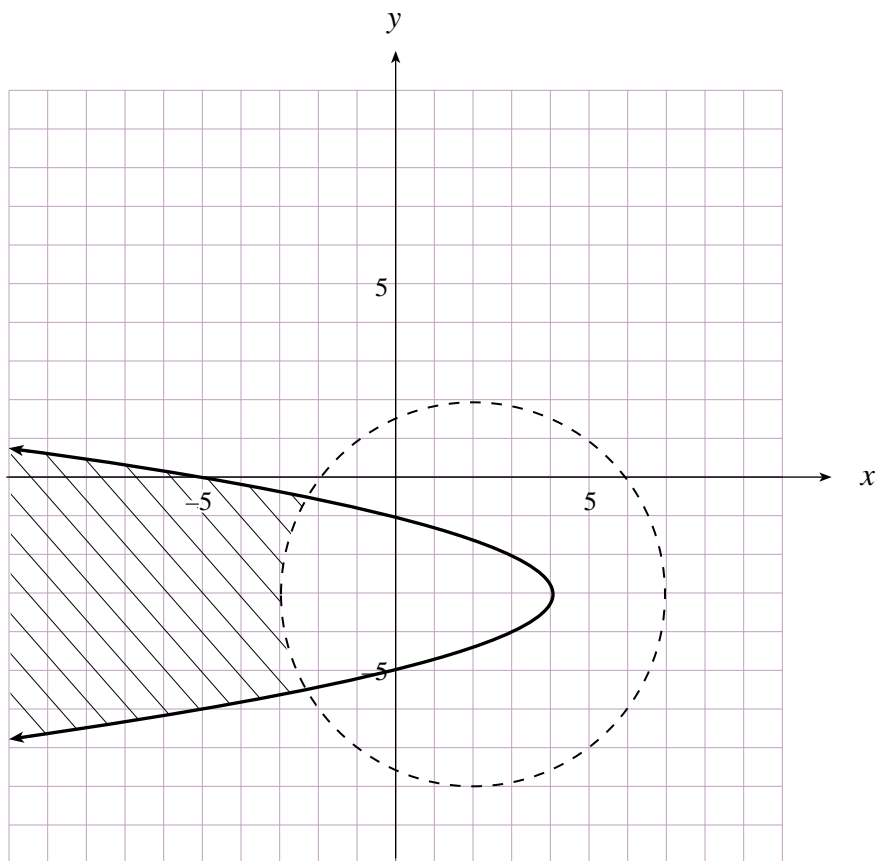
3. Graph the solution of the following system of inequalities on the grid provided.

(3 marks)

$$(x - 2)^2 + (y + 3)^2 > 25$$

$$x \leq -(y + 3)^2 + 4$$

Solution:



1 mark for graph of circle

1 mark for parabola

1 mark for correct region shaded

4. Solve for x : $\sec^2 x - \sec x - 6 = 0$, where $0 \leq x < 2\pi$. (Accurate to at least 2 decimal places.)
(3 marks)

Solution:

$$\sec^2 x - \sec x - 6 = 0$$

$$(\sec x - 3)(\sec x + 2) = 0$$

$$\frac{1}{2} \text{ mark} \rightarrow \sec x = 3$$

$$\cos x = \frac{1}{3}$$

$$\text{ref} = 1.23$$

$$x = 1.23, 5.05$$

↑ ↑

$\frac{1}{2}$ mark each

$$\sec x = -2 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\cos x = -\frac{1}{2}$$

$$\text{ref} = 1.05$$

$$x = 2.09, 4.19$$

↑ ↑

$\frac{1}{2}$ mark each

$$\left(\text{or } x = \frac{2\pi}{3}, \frac{4\pi}{3} \right)$$

5. A polynomial function of degree 3 has zeros 5, 3, -1, and passes through the point (2, -6). Determine an equation of this function. (Answer may be left in factored form.) **(2 marks)**

Solution:

$\frac{1}{2}$ mark for a $\frac{1}{2}$ mark for factors

$$p(x) = a(x-5)(x-3)(x+1)$$

$$-6 = a(2-5)(2-3)(2+1) \quad \leftarrow \frac{1}{2} \text{ mark for substituting point}$$

$$-6 = a(-3)(-1)(3)$$

$$-6 = 9a$$

$$-\frac{2}{3} = a$$

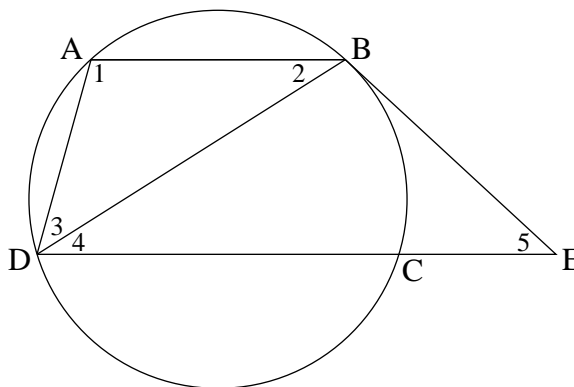
$$p(x) = -\frac{2}{3}(x-5)(x-3)(x+1) \quad \leftarrow \frac{1}{2} \text{ mark}$$

6. Complete the proof.

(4 marks)

Given: BE is a tangent
 D, C, E are collinear
 $\angle 3 = \angle 5$

Prove: $AB \parallel DE$



Solution:

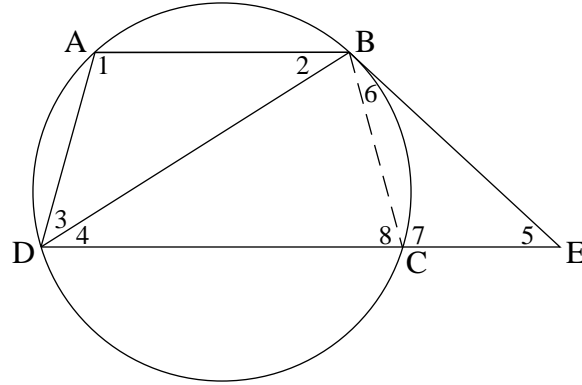
Statement	Proof	Reason
2 marks \rightarrow $\left\{ \begin{array}{l} \text{BE is a tangent} \\ \angle DBE = \angle 1 \end{array} \right.$		given \angle between tangent and chord
1 mark \rightarrow $\left\{ \begin{array}{l} \angle 3 = \angle 5 \\ \angle 2 = \angle 4 \end{array} \right.$		given 3rd \angle s of Δ s are =
1 mark \rightarrow $\left\{ \begin{array}{l} AB \parallel DE \end{array} \right.$		alternate interior \angle s are =

6. Complete the proof.

(4 marks)

Given: BE is a tangent
 D, C, E are collinear
 $\angle 3 = \angle 5$

Prove: $AB \parallel DE$



Alternate Solution:

	Statement	Proof	Reason
1 mark →	join BC		construction
	$\angle 3 = \angle 5$		\angle between tangent and chord
	$\angle 1 + \angle 8 = 180^\circ$		opposite \angle s of cyclic quadrilateral
1 mark →	$\angle 7 + \angle 8 = 180^\circ$		\angle s on a line
	$\angle 1 = \angle 7$		both supplementary to $\angle 8$
	$\angle 6 = \angle 2$		3rd \angle s of Δ s are =
2 marks →	BE is a tangent		given
	$\angle 6 = \angle 4$		\angle between tangent and chord
	$\angle 2 = \angle 4$		both = $\angle 6$ (substitution)
	$AB \parallel DE$		alternate interior \angle s are =

7. The line segment AB has endpoints A (13, 6), and B(1, 2). Find the coordinates of point P which divides the line segment AB in a ratio of 3:5, that is $\frac{PA}{PB} = \frac{3}{5}$. **(2 marks)**

Solution:

$$\left. \begin{array}{l} x\text{-value of P is } \frac{3}{8} \text{ of } x\text{-distance between AB} \\ y\text{-value of P is } \frac{3}{8} \text{ of } y\text{-distance between AB} \end{array} \right\} \leftarrow \frac{1}{2} \text{ mark}$$

$$\frac{3}{8}(12) = 4.5 \leftarrow \frac{1}{2} \text{ mark}$$

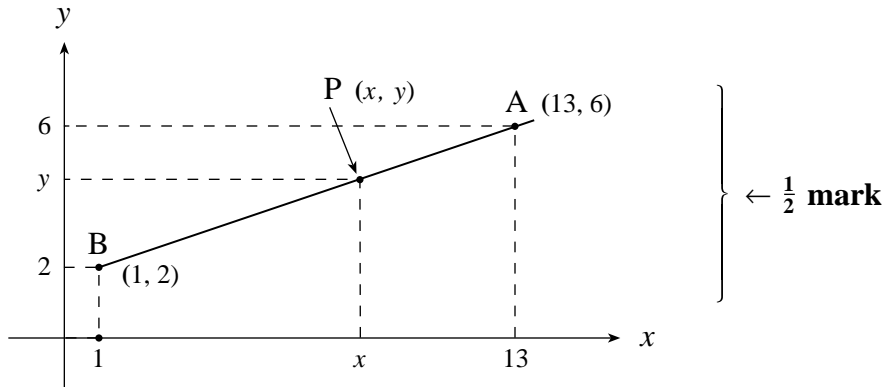
$$\frac{3}{8}(4) = 1.5 \leftarrow \frac{1}{2} \text{ mark}$$

$$(13 - 4.5, 6 - 1.5) = P(8.5, 4.5)$$

$$\begin{array}{c} \uparrow \\ \frac{1}{2} \text{ mark} \end{array}$$

7. The line segment AB has endpoints A (13, 6), and B(1, 2). Find the coordinates of point P which divides the line segment AB in a ratio of 3:5, that is $\frac{PA}{PB} = \frac{3}{5}$. **(2 marks)**

Alternate Solution:



$$\frac{13-x}{x-1} = \frac{3}{5} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\frac{6-y}{y-2} = \frac{3}{5} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$65 - 5x = 3x - 3$$

$$30 - 5y = 3y - 6$$

$$8x = 68$$

$$8y = 36$$

$$x = \frac{68}{8} = \frac{17}{2} = 8.5$$

$$y = \frac{36}{8} = \frac{9}{2} = 4.5$$

$$\therefore P = (8.5, 4.5) \quad \leftarrow \frac{1}{2} \text{ mark}$$

END OF KEY