Mathematics 12

April 1997 Provincial Examination

ANSWER KEY / SCORING GUIDE

Topics:
1. Trigonometry
2. Quadratic Relations
3. Exponential and Logarithmic Functions
4. Polynomial Functions
5. Sequences and Series
6. Introduction to Calculus
7. Geometry
8. Problem Solving

Part A: Multiple Choice

Q	C	T	K	S	ILO	Q	C	T	K	S	ILO
1.	K	2	C	1	13	26.	U	4	B	1	37
2.	K	2	C	1	17	27.	U	4	B	1	42
3.	U	2	B	1	12	28.	U	4	C	1	38
4.	U	2	D	1	16	29.	U	4	D	1	35
5.	U	2	C	1	21	30.	U	4	D	1	41
6.	U	2	B	1	17	31.	U	4	B	1	39
7.	U	2	C	1	20	32.	H	4	D	1	43
8.	U	2	A	1	18	33.	K	5	A	1	45
9.	H	2	B	1	15	34.	U	5	C	1	46
10.	H	2	A	1	17	35.	U	5	B	1	46
11.	K	1	C	1	05	36.	U	5	B	1	45
12.	K	1	A	1	02	37.	U	5	A	1	46
13.	U	1	D	1	03	38.	H	5	A	1	47
14.	U	1	C	1	01	39.	K	6	A	1	56
15.	U	1	B	1	07	40.	U	6	D	1	57
16.	H	1	C	1	06	41.	U	6	C	1	50
17.	H	1	B	1	03, 32	42.	U	6	C	1	53
18.	K	3	D	1	28	43.	U	6	C	1	61
19.	U	3	C	1	32	44.	U	6	A	1	62
20.	U	3	B	1	31	45.	H	6	D	1	51
21.	U	3	B	1	31	46.	H	7	A	1	63
22.	U	3	B	1	24	47.	H	7	C	1	63
23.	H	3	A	1	31	48.	U	8	D	1	64
24.	H	3	A	1	29	49.	H	8	A	1	64
25.	K	4	D	1	36	50.	H	8	D	1	64

Part B: Written Response

Q	B	C	T	S	ILO	Q	B	C	T	S	ILO
1.	1	U	2	3	22	5a.	5	U	3	1	30
2.	2	U	1	3	08	5b.	6	U	3	2	30
3.	3	U	5	2	46	6.	7	U	8	2	64
4.	4	U	6	3	58	7.	8	H	7	4	63

Multiple Choice = 50 (50 questions)
Written Response = 20 (7 questions)
Total = 70 marks

LEGEND:

Q = Question Number **C** = Cognitive Level **T** = Topic
K = Keyed Response **S** = Score **ILO** = Intended Learning Outcome
B = Score Box Number

PART B: WRITTEN RESPONSE

Value: 20 marks **Suggested Time: 45 minutes**

INSTRUCTIONS: Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided.

Full marks will NOT be given for the final answer only.

1. Determine all ordered pairs that satisfy the following system. **(3 marks)**

$$x^2 + y^2 = 5$$
$$y = 2x - 4$$

Solution:

$x^2 + (2x-4)^2 = 5$ ← **½ mark**

$x^2 + (4x^2 - 16x + 16) = 5$

$5x^2 - 16x + 11 = 0$ ← **½ mark**

$(5x-11)(x-1) = 0$ ← **½ mark**

} ← **1 mark for correct *x* - values from quadratic formula using CALCULATOR**

½ mark → $x = \frac{11}{5}$ or $x = 1$ **for BOTH *x* - values**

½ mark → $y = \frac{2}{5}$ or $y = -2$ **for BOTH *y* - values**

$\left(\frac{11}{5}, \frac{2}{5}\right)$ $(1, -2)$

or

$(2.2, .4)$ $(1, -2)$

} ← **½ mark for ORDERED PAIRS**

2. Prove:

$$\frac{\sin 2\theta}{2 - 2\cos^2 \theta} = \cot \theta$$ **(3 marks)**

Solution:

Left side	Right side
$\dfrac{\sin 2\theta}{2 - 2\cos^2 \theta}$	$\cot \theta$
½ mark → $= \dfrac{2\sin\theta\cos\theta}{2(1 - \cos^2\theta)}$	
1 mark →	
½ mark → $= \dfrac{2\sin\theta\cos\theta}{2\sin^2\theta}$	
½ mark → $= \dfrac{\cos\theta}{\sin\theta}$	
½ mark → $= \cot\theta$	

3. In a geometric sequence, $t_3 = 22\ 500$ and $t_6 = 38\ 880$. Find the first term. **(2 marks)**

Solution:

$t_3 = 22\ 500$ $\qquad$ $t_6 = 38\ 880$

$ar^2 = 22\ 500$ $\qquad$ $ar^5 = 38\ 880$ $\quad \leftarrow \frac{1}{2}$ **mark**

$a = \dfrac{22\ 500}{r^2}$ $\qquad$ $\dfrac{22\ 500}{r^2} r^5 = 38\ 880$ $\quad \leftarrow \frac{1}{2}$ **mark**

$r^3 = 1.728$

$r = 1.2$ $\quad \leftarrow \frac{1}{2}$ **mark**

$a = 15\ 625$ $\quad \leftarrow \frac{1}{2}$ **mark**

Alternate Solution:

$t_6 = ar^5 = 38\ 880$ $\quad \leftarrow \frac{1}{2}$ **mark**

$t_3 = ar^2 = 22\ 500$

$\dfrac{ar^5}{ar^2} = \dfrac{38\ 880}{22\ 500}$ $\quad \leftarrow \frac{1}{2}$ **mark**

$r^3 = 1.728$

$r = 1.2$ $\quad \leftarrow \frac{1}{2}$ **mark**

$a = 15\ 625$ $\quad \leftarrow \frac{1}{2}$ **mark**

4. Determine all values of x such that the function $f(x) = x^4 - 8x^2 - 9$ is increasing. **(3 marks)**

Solution:

$$f(x) = x^4 - 8x^2 - 9$$

$$f'(x) = 4x^3 - 16x \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$4x^3 - 16x = 0 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

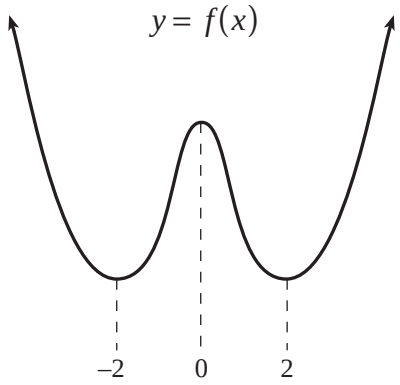
$$4x\left(x^2 - 4\right) = 0$$

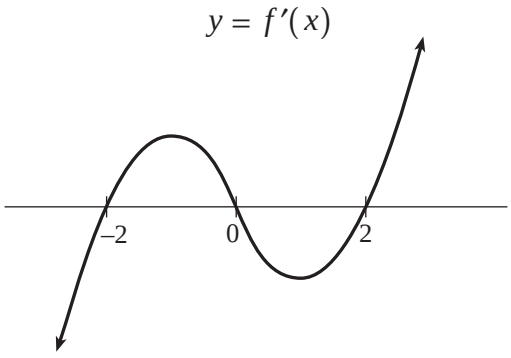
$$4x(x+2)(x-2) = 0$$

$\therefore$ critical points $x = 0$, $x = 2$, $x = -2$ $\leftarrow \tfrac{1}{2}$ **mark**

increasing $\Rightarrow$ $f'(x) > 0$ $\leftarrow \tfrac{1}{2}$ **mark for concept**

Consider the graph of a fourth degree function: **or** Consider the graph of the derived function:





$\therefore$ $f(x)$ is increasing on

$\leftarrow$ **1 mark** $\left(\tfrac{1}{2}\right.$ **mark for each interval**$\left.\right)$

or **or**

$-2 < x < 0$ or $x > 2$ $\leftarrow$ **1 mark** $\left(\tfrac{1}{2}\right.$ **mark for each interval**$\left.\right)$

(**Note:** Can state solution algebraically or graphically.)

5. a) Determine the y-intercept of $y = \log_2(x+3)+1$. (Accurate to at least 2 decimal places.) **(1 mark)**

Solution:

$y = \log_2(x+3)+1$

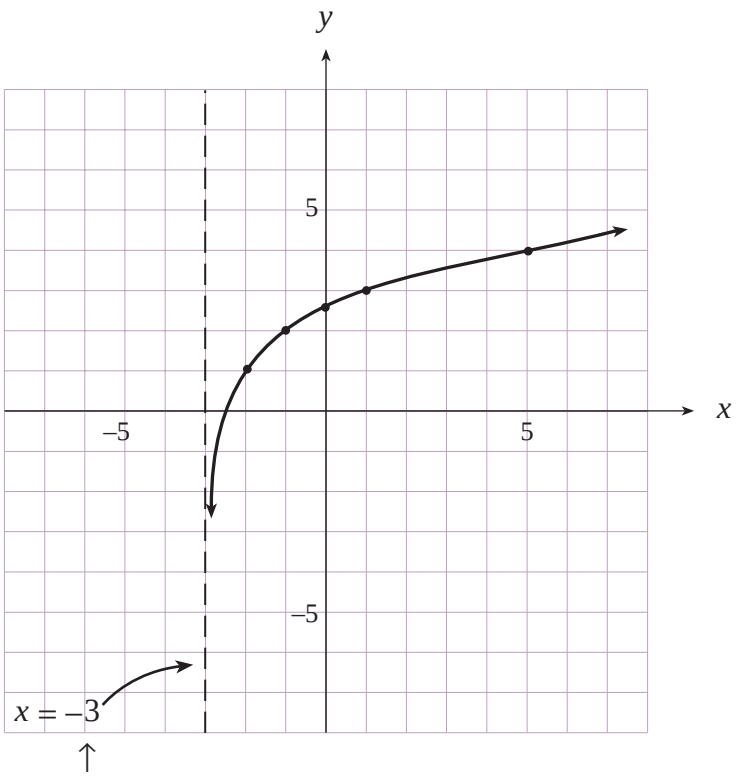
y-intercept $\Rightarrow x = 0$

$y = \log_2(0+3)+1$ $\left.\begin{matrix}\\ \\ \end{matrix}\right\} \leftarrow \frac{1}{2}$ **mark**

$y = 2.58$ $\leftarrow \frac{1}{2}$ **mark**

b) Graph $y = \log_2(x+3)+1$. Indicate the asymptote with a dotted or broken line and label it with its equation. **(2 marks)**

Solution:



x	y
–2.5	0
0	2.58
1	3
–1	2
5	4

$\frac{1}{2}$ **mark for correct labelling of given vertical asymptote**

$\frac{1}{2}$ **mark** for having asymptote in right place.

$\frac{1}{2}$ **mark** for correct shape of a log function.

$\frac{1}{2}$ **mark** for accuracy.

6. A river system currently has 4 million fish. Each year the population declines by 5%. After how many years will the population fall below 700 000 fish? (Accurate to the nearest year.)

(2 marks)

Solution:

$$700\,000 = 4\,000\,000(1-0.05)^t \quad \leftarrow \tfrac{1}{2} \textbf{ mark for concept of decay}$$

$$\frac{700\,000}{4\,000\,000} = (0.95)^t$$

$$0.175 = (0.95)^t \quad \leftarrow \tfrac{1}{2} \textbf{ mark for } \mathbf{0.95}$$

$$\log 0.175 = \log 0.95^t$$

$$\log 0.175 = t \log 0.95 \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$\frac{\log 0.175}{\log 0.95} = t$$

$$34 = t \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

Alternate Solution:

$$700\,000 = 4\,000\,000(1-0.05)^t \quad \leftarrow \tfrac{1}{2} \textbf{ mark for concept of decay}$$

$$\frac{700\,000}{4\,000\,000} = (0.95)^t$$

$$0.175 = (0.95)^t \quad \leftarrow \tfrac{1}{2} \textbf{ mark for } \mathbf{0.95}$$

$$\log_{0.95} 0.175 = t \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

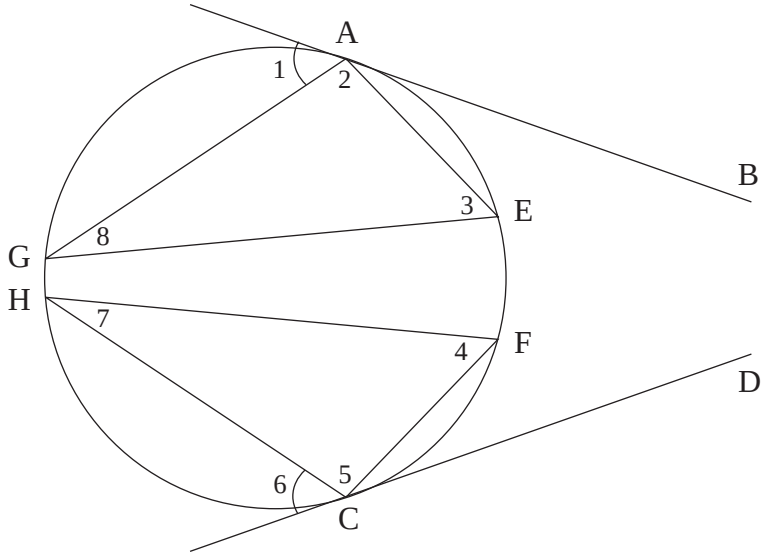
$$\frac{\log 0.175}{\log 0.95} = t$$

$$34 = t \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

7. Complete the proof. **(4 marks)**

Given: AB and CD are tangents
$AE = CF$
$\angle 1 = \angle 6$

Prove: $GE = HF$



Solution:

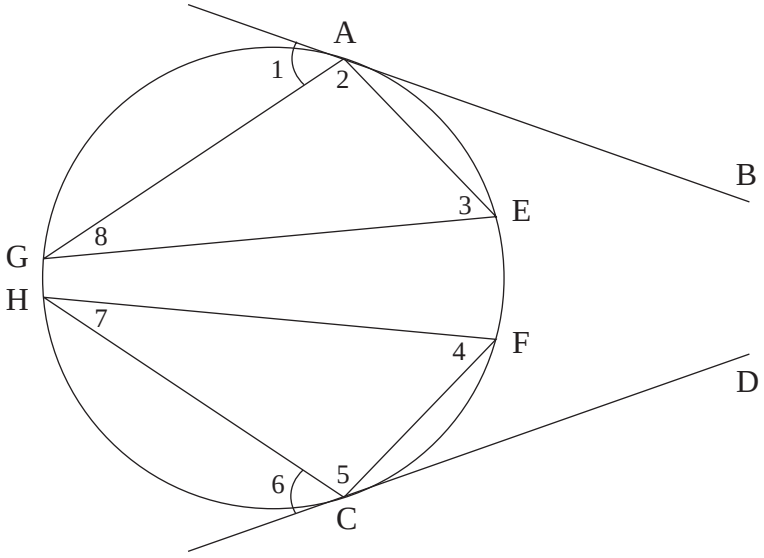
Proof

	Statement	Reason
	AB and CD are tangents	given
$\frac{1}{2}$ **mark** →	$\angle 1 = \angle 3$	$\angle$ between tangent and chord
$\frac{1}{2}$ **mark** →	$\angle 6 = \angle 4$	$\angle$ between tangent and chord
	$\angle 1 = \angle 6$	given
$\frac{1}{2}$ **mark** →	$\angle 3 = \angle 4$	both = to = angles / substitution
	$AE = CF$	given
$\frac{1}{2}$ **mark** →	$\angle 8 = \angle 7$	inscribed angles on = chords are =
$\frac{1}{2}$ **mark** →	$\angle 5 = \angle 2$	3rd $\angle$s of triangles are =
1 mark →	$\Delta AGE = \Delta CHF$	ASA
$\frac{1}{2}$ **mark** →	$GE = HF$	CPCTC

7. Complete the proof. **(4 marks)**

Given: AB and CD are tangents
AE = CF
$\angle 1 = \angle 6$

Prove: GE = HF



Alternate Solution #1:

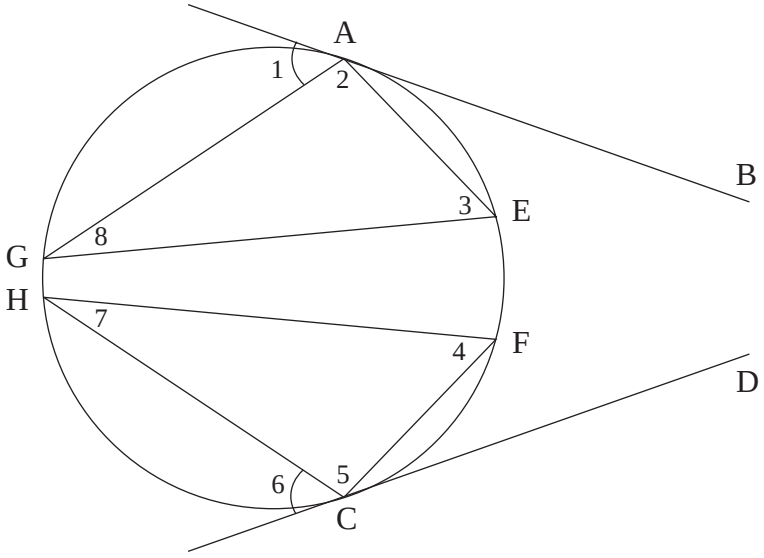
Proof

Statement		Reason
	AB and CD are tangents	given
$\frac{1}{2}$ **mark** →	$\angle 1 = \angle 3$	∠ between tangent and chord
$\frac{1}{2}$ **mark** →	$\angle 6 = \angle 4$	∠ between tangent and chord
	$\angle 1 = \angle 6$	given
$\frac{1}{2}$ **mark** →	$\angle 3 = \angle 4$	both = to = angles / substitution
	AE = CF	given
$\frac{1}{2}$ **mark** →	$\angle 8 = \angle 7$	inscribed angles on = chords are =
1 mark →	$\Delta AGE = \Delta CHF$	AAS
1 mark →	GE = HF	CPCTC

7. Complete the proof. **(4 marks)**

Given: AB and CD are tangents
AE = CF
$\angle 1 = \angle 6$

Prove: GE = HF



Alternate Solution #2:

Proof

Statement		Reason
	AB and CD are tangents	given
$\frac{1}{2}$ mark →	$\angle 1 = \angle 3$	$\angle$ between tangent and chord
$\frac{1}{2}$ mark →	$\angle 6 = \angle 4$	$\angle$ between tangent and chord
	$\angle 1 = \angle 6$	given
$\frac{1}{2}$ mark →	$\angle 3 = \angle 4$	both = to = angles / substitution
	AE = CF	given
$\frac{1}{2}$ mark →	$\angle 8 = \angle 7$	inscribed angles on = chords are =
$\frac{1}{2}$ mark →	$\Delta AGE = \Delta CHF$	3rd $\angle$s of triangles are =
$1\frac{1}{2}$ marks →	GE = HF	chords from = inscribed $\angle$s are =

END OF KEY