Mathematics 12

January 1997 Provincial Examination

ANSWER KEY / SCORING GUIDE

Topics:
1. Trigonometry
2. Quadratic Relations
3. Exponential and Logarithmic Functions
4. Polynomial Functions
5. Sequences and Series
6. Introduction to Calculus
7. Geometry
8. Problem Solving

Part A: Multiple Choice

Q	C	T	K	S	ILO	Q	C	T	K	S	ILO
1.	K	2	D	1	15	26.	K	4	A	1	40
2.	U	2	C	1	14	27.	U	4	C	1	38
3.	K	2	D	1	17	28.	U	4	C	1	35
4.	U	2	D	1	12	29.	U	4	A	1	41
5.	U	2	D	1	16	30.	U	4	A	1	39
6.	U	2	B	1	21	31.	U	4	B	1	37
7.	U	2	D	1	18	32.	U	4	B	1	40
8.	U	2	B	1	22	33.	H	4	C	1	36
9.	H	2	C	1	17	34.	K	5	C	1	46
10.	H	2	B	1	19	35.	U	5	C	1	46
11.	K	1	C	1	01	36.	U	5	A	1	47
12.	U	1	A	1	05	37.	U	5	A	1	46
13.	U	1	D	1	02	38.	H	5	A	1	46
14.	U	1	A	1	08	39.	K	6	A	1	57
15.	U	1	D	1	09	40.	U	6	D	1	50
16.	U	1	A	1	06	41.	U	6	C	1	58
17.	U	1	A	1	08	42.	U	6	C	1	51
18.	H	1	B	1	03	43.	U	6	B	1	60
19.	K	3	D	1	28	44.	U	6	A	1	57
20.	U	3	D	1	31	45.	H	6	B	1	53
21.	U	3	B	1	26	46.	U	7	B	1	63
22.	U	3	B	1	31	47.	U	7	B	1	63
23.	U	3	C	1	32	48.	U	8	D	1	64
24.	H	3	D	1	31	49.	H	8	B	1	64
25.	H	3	C	1	24	50.	H	8	B	1	64

Part B: Written Response

Q	B	C	T	S	ILO
1.	1	U	1	2	03
2.	2	U	3	3	32
3.	3	U	5	3	46
4.	4	U	2	3	15
5.	5	U	8	2	64
6.	6	H	7	4	63
7.	7	U	6	3	62

Multiple Choice = 50 (50 questions)
Written Response = 20 (7 questions)
Total = 70 marks

LEGEND:
Q = Question Number **C** = Cognitive Level **T** = Topic
K = Keyed Response **S** = Score **ILO** = Intended Learning Outcome
B = Score Box Number

PART B: WRITTEN RESPONSE

Value: 20 marks **Suggested Time: 45 minutes**

INSTRUCTIONS:	Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question. Where required, place the final answer for each question in the space provided. **Full marks will NOT be given for the final answer only.**

1. Solve: $3\cos^2 x - 5\cos x - 2 = 0$, where $0 \le x < 2\pi$ (Accurate to at least 2 decimal places.)

(2 marks)

Solution:

$$3\cos^2 x - 5\cos x - 2 = 0$$

$$(3\cos x + 1)(\cos x - 2) = 0 \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\cos x = -\frac{1}{3} \qquad \cos = 2 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\text{ref}\ \angle = 1.23 \qquad \text{no solution}$$

$$\therefore\ x = 1.91\ ,\ 4.37$$

$$\uparrow \qquad \uparrow$$

$\tfrac{1}{2}$ **mark** $\tfrac{1}{2}$ **mark**

2. The population of a city is increasing at a rate of 6.5% each year. If the present population is 12 000, how long will it take for the population to reach 32 000? (Accurate to at least 1 decimal place.) **(3 marks)**

Solution:

$$12\ 000(1.065)^t = 32\ 000 \qquad \leftarrow \textbf{1 mark}$$

$$1.065^t = \frac{8}{3} \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$t \log 1.065 = \log \frac{8}{3} \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$t = \frac{\log \frac{8}{3}}{\log 1.065} \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$t = 15.6 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

Alternate Solution:

$$12\ 000(1.065)^t = 32\ 000 \qquad \leftarrow \textbf{1 mark}$$

$$1.065^t = \frac{8}{3} \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\log_{1.065} \frac{8}{3} = t \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\frac{\log \frac{8}{3}}{\log 1.065} = t \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$15.6 = t \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

3. Determine the sum of the arithmetic series $3+17+31+\ \dots\ +1\,151$.	**(3 marks)**

Solution:

$$t_n = a+(n-1)d \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$1\,151 = 3+(n-1)14 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$1\,148 = (n-1)14$$

$$82 = n-1$$

$$83 = n \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

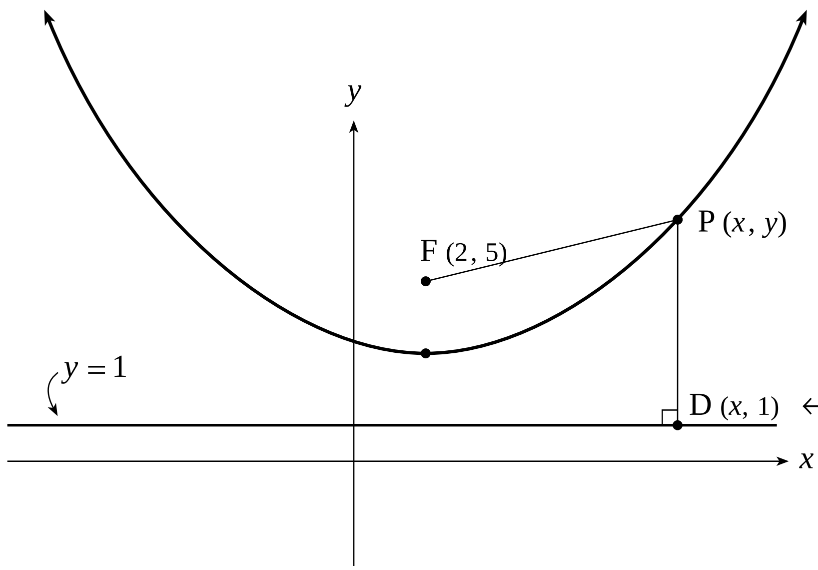
$$S_n = \frac{n}{2}(a+l)$$

$$= \frac{83}{2}(3+1\,151) \qquad \leftarrow \textbf{1 mark}$$

$$S_n = 47\,891 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

4. A point P moves such that it is always equidistant from the point $F(2, 5)$ and the line given by $y = 1$. Find an equation of this locus and write it in standard form. **(3 marks)**

Solution:



← $\frac{1}{2}$ **mark** for D $(x, 1)$

$$PF = PD$$

$\frac{1}{2}$ **mark** → $\sqrt{(x-2)^2 + (y-5)^2} = \sqrt{(x-x)^2 + (y-1)^2}$ ← $\frac{1}{2}$ **mark**

$$(x-2)^2 + y^2 - 10y + 25 = y^2 - 2y + 1$$ ← $\frac{1}{2}$ **mark**

$$(x-2)^2 + 24 = 8y$$ ← $\frac{1}{2}$ **mark**

$$y = \frac{1}{8}(x-2)^2 + 3$$

or

$$y - 3 = \frac{1}{8}(x-2)^2$$

← $\frac{1}{2}$ **mark**

4. A point P moves such that it is always equidistant from the point $F\left(2,\ 5\right)$ and the line given by $y=1$. Find an equation of this locus and write it in standard form. **(3 marks)**

Alternate Solution:

$y = \frac{1}{4p}(x-h)^2 + k$ ← **½ mark**

1 mark → $p = 2$ $\quad (h,\ k) = (2,\ 3)$ ← **½ mark**

$y = \frac{1}{8}(x-2)^2 + 3$ ← **1 mark**

5. A function is defined by the equation $f(t) = t^2 + 6t + 7$. Sketch the graph of $f(x) + f(y) = 0$.
(2 marks)

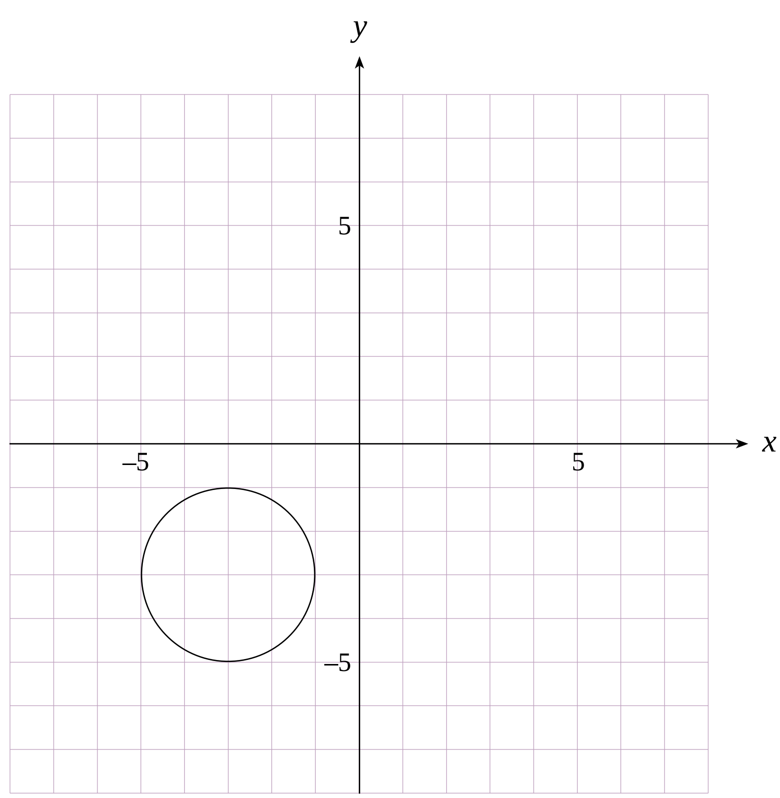
Solution:

$$f(x) + f(y) = 0$$

$$x^2 + 6x + 7 + y^2 + 6y + 7 = 0 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$x^2 + 6x + 9 + y^2 + 6y + 9 = -14 + 18 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$(x+3)^2 + (y+3)^2 = 4 \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

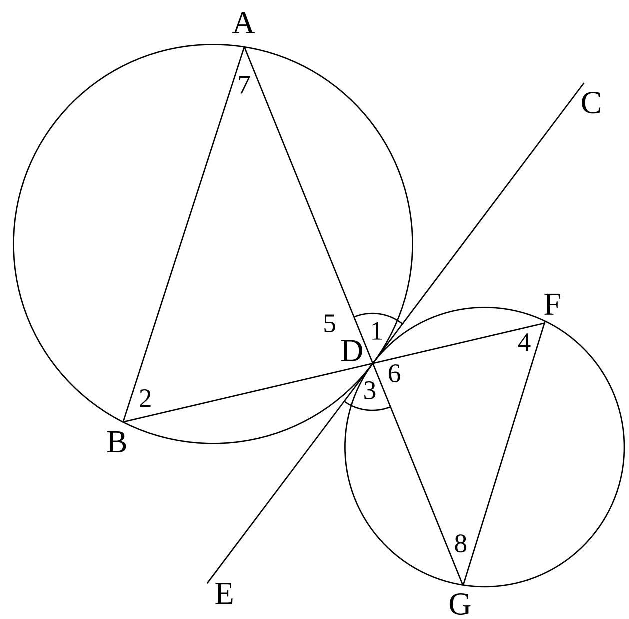


← $\tfrac{1}{2}$ **mark** for graph

6. Complete the proof. **(4 marks)**

Given: AG and BF intersect at D
CE is tangent to both circles at D

Prove: AB || FG



Solution:

Proof

	Statement	Reason	
	CE is tangent to both circles at D	given	
$\frac{1}{2}$ mark →	∠1 = ∠2	∠ between chord and tangent	← **$\frac{1}{2}$ mark**
$\frac{1}{2}$ mark →	∠3 = ∠4	∠ between chord and tangent	
$\frac{1}{2}$ mark →	∠1 = ∠3	vertically opposite ∠s are =	← **$\frac{1}{2}$ mark**
$\frac{1}{2}$ mark →	∠2 = ∠4	both = to = ∠s or substitution	← **$\frac{1}{2}$ mark**
	AB || FG	alternate interior ∠s are =	← **$\frac{1}{2}$ mark**

Note: Deduct $\frac{1}{2}$ mark if given is missing.

Deduct 1 mark if reason is wrong or missing.
or
Deduct $\frac{1}{2}$ mark if reason is poorly expressed.

7. Two numbers x and y differ by 50, where x is larger than y. If $R = x + y + xy$, determine the values of x and y such that R is a minimum. **(3 marks)**

Solution:

Let the numbers be x and y.

$$\left.\begin{aligned} x - y &= 50 \\ \therefore\ y &= x - 50 \end{aligned}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

Let $R = x + y + xy$

$$\left.\begin{aligned} R &= x + (x - 50) + x(x - 50) \\ R &= x^2 - 48x - 50 \end{aligned}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\frac{dR}{dx} = 2x - 48 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$2x - 48 = 0 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$x = 24 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$y = 24 - 50 = -26 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$x = 24$

$y = -26$

(**Note:** Only deduct $\frac{1}{2}$ **mark** if x and y are reversed.)

END OF KEY