Mathematics 12

April 1996 Provincial Examination

ANSWER KEY / SCORING GUIDE

Topics: 1. Trigonometry
2.Quadratic Relations
3.Exponential and Logarithmic Functions
4.Polynomial Functions
5.Sequences and Series
6.Introduction to Calculus
7.Geometry
8.Problem Solving

Part A: Multiple Choice

Q	C	T	K	S	ILO	Q	C	T	K	S	ILO
1.	K	2	B	1	12.17	26.	K	4	D	1	12.38
2.	K	2	D	1	12.13	27.	U	4	D	1	12.35
3.	U	2	A	1	12.14	28.	U	4	B	1	12.40
4.	U	2	D	1	12.17	29.	U	4	D	1	12.40
5.	U	2	B	1	12.11	30.	U	4	A	1	12.43
6.	U	2	A	1	12.18	31.	U	4	C	1	12.40
7.	U	2	B	1	12.21	32.	U	4	A	1	12.42
8.	U	2	A	1	12.22	33.	H	4	A	1	12.36
9.	H	2	C	1	12.20	34.	K	5	D	1	12.46
10.	H	2	C	1	12.17, 12.64	35.	U	5	D	1	12.46
11.	K	1	B	1	12.01	36.	U	5	C	1	12.47
12.	U	1	B	1	12.02	37.	U	5	C	1	12.46
13.	U	1	C	1	12.05	38.	H	5	B	1	12.46
14.	U	1	A	1	12.06	39.	K	6	C	1	12.57
15.	U	1	B	1	12.07	40.	U	6	B	1	12.50
16.	U	1	B	1	12.09	41.	U	6	B	1	12.53
17.	U	1	C	1	12.09	42.	U	6	D	1	12.61
18.	H	1	A	1	12.06	43.	U	6	D	1	12.51
19.	K	3	B	1	12.29	44.	U	6	C	1	12.60
20.	U	3	A	1	12.26	45.	H	6	A	1	12.55
21.	U	3	D	1	12.24	46.	U	7	B	1	12.63
22.	U	3	B	1	12.31	47.	U	7	B	1	12.63
23.	U	3	B	1	12.32	48.	U	8	C	1	12.64
24.	H	3	A	1	12.30	49.	U	8	A	1	12.64
25.	H	3	D	1	12.31, 12.32	50.	H	8	A	1	12.64

Part B: Written Response

Q	B	C	T	S	ILO	Q	B	C	T	S	ILO
1.	1	U	3	3	12.32	5a.	5	U	3	2	12.58
2.	2	U	1	2	12.08	5b.	6	U	1	1	12.58
3.	3	U	2	3	12.15	6.	7	H	7	4	12.63
4.	4	U	5	3	12.46	7.	8	H	8	2	12.64

Multiple Choice = 50 (50 questions)
Written Response = 20 (7 questions)
Total = 70 marks

LEGEND:

Q = Question Number **C** = Cognitive Level **T** = Topic
K = Keyed Response **S** = Score **ILO** = Intended Learning Outcome
B = Score Box Number

PART B: WRITTEN RESPONSE

1. Solve: $\log_4(x-3)+\log_4(2x-4)=1$ **(3 marks)**

Solution:

$\log_4(x-3)(2x-4)=1$ $\leftarrow$ **½ mark**

$(x-3)(2x-4)=4$ $\leftarrow$ **1 mark**

$2x^2-10x+12=4$

$2x^2-10x+8=0$ $\leftarrow$ **½ mark**

$x^2-5x+4=0$

$(x-4)(x-1)=0$ $\leftarrow$ **½ mark**

$x=1 \qquad x=4$

$\underbrace{\qquad\qquad\qquad\qquad}$

$\uparrow$

½ mark for both answers

Deduct **½ mark** if no rejection

2. Prove the following identity: **(2 marks)**

$$\sin\theta + \cos\theta \cot\theta = \csc\theta$$

Solution:

	Left side	Right side
	$\sin\theta + \cos\theta \cot\theta$	$\csc\theta$
$\frac{1}{2}$ **mark** $\rightarrow$	$\sin\theta + \cos\theta \dfrac{\cos\theta}{\sin\theta}$	
$\frac{1}{2}$ **mark** $\rightarrow$	$\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta}$	
$\frac{1}{2}$ **mark** $\rightarrow$	$\dfrac{1}{\sin\theta}$	
$\frac{1}{2}$ **mark** $\rightarrow$	$\csc\theta$	

3. A point $P(x, y)$ moves in such a way that it is always the same distance from $A(-5, 2)$ as it is from $B(2, -4)$. Determine the equation of this locus. Write the answer in a simplified form.
(3 marks)

Solution:

$d_1 = d_2$ ← **½ mark** (for equating distance)

½ mark → $\sqrt{(x+5)^2+(y-2)^2} = \sqrt{(x-2)^2+(y+4)^2}$ ← **½ mark**

½ mark → $x^2+10x+25+y^2-4y+4 = x^2-4x+4+y^2+8y+16$ ← **½ mark**

$$\begin{aligned} -12y &= -14x-9 \\ 14x-12y+9 &= 0 \\ 14x-12y &= -9 \\ y &= \frac{7}{6}x+\frac{3}{4} \end{aligned}$$

← **½ mark**

3. A point $P(x, y)$ moves in such a way that it is always the same distance from $A(-5, 2)$ as it is from $B(2, -4)$. Determine the equation of this locus. Write the answer in a simplified form.

(3 marks)

Alternate Solution:

The locus is the perpendicular bisector of segment AB

Concept of midpoint $\leftarrow$ **½ mark**

$$\text{midpoint of AB} = \left(\frac{-5+2}{2}\ ,\ \frac{2-4}{2}\right)$$

$$= \left(-\frac{3}{2}\ ,\ -1\right)$$

$\uparrow$

½ mark

$$\text{slope of AB} = \frac{2+4}{-5-2}$$

$$= -\frac{6}{7} \qquad \leftarrow \textbf{½ mark}$$

$$\therefore\ m_{\perp} = \frac{7}{6} \qquad \leftarrow \textbf{½ mark}$$

$\therefore$ equation of locus $\quad y+1 = \frac{7}{6}\left(x+\frac{3}{2}\right)$

or $\quad y = \frac{7}{6}x + \frac{3}{4}$

or $\quad 14x - 12y = -9$

$\leftarrow$ **1 mark**

4. In a geometric series, the first term is 640 and the fourth term is 1080. Find the sum of the first 20 terms of this series. (Accurate to the nearest whole number.) **(3 marks)**

Solution:

$a = 640$
$n = 4$
$t_4 = 1080$
} ← **½ mark**

$t_n = ar^{n-1}$

$1080 = 640r^3$ ← **½ mark** for substitution in t_n formula

$\frac{1080}{640} = r^3$

$1.190550789 = r$ **or** $\frac{3}{4}\sqrt[3]{4}$ ← **½ mark**

$S_n = \frac{a(1-r^n)}{1-r}$

$= \frac{640(1-1.190550789^{20})}{1-1.190550789}$ ← **1 mark**

$= 106\,574.236$
$\therefore \quad S_n \approx 106\,574$
} ← **½ mark** for either one

5. a) Determine the x values of the critical points of $f(x) = x^4 - 8x^2$. **(2 marks)**

Solution:

$f'(x) = 4x^3 - 16x$ $\leftarrow$ **$\frac{1}{2}$ mark**

$0 = 4x^3 - 16x$ $\leftarrow$ **$\frac{1}{2}$ mark**

$0 = x^3 - 4x$

$0 = x\left(x^2 - 4\right)$

$0 = x(x-2)(x+2)$ $\leftarrow$ **$\frac{1}{2}$ mark**

$x = 0$ or $x - 2 = 0$ or $x + 2 = 0$

$x = 0$ $\quad x = 2$ $\quad x = -2$ $\leftarrow$ **$\frac{1}{2}$ mark**

(deduct for missing any value)

b) For what values of x is $f(x) = x^4 - 8x^2$ **decreasing**? **(1 mark)**

Solution:

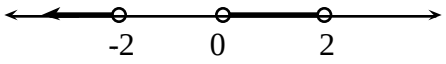
$f(x)$ is decreasing when $f'(x) < 0$ ← **½ mark**

$4x^3 - 16x < 0$

$4x\left(x^2 - 4\right) < 0$

$4x(x+2)(x-2) < 0$

$f(x)$ is decreasing for:



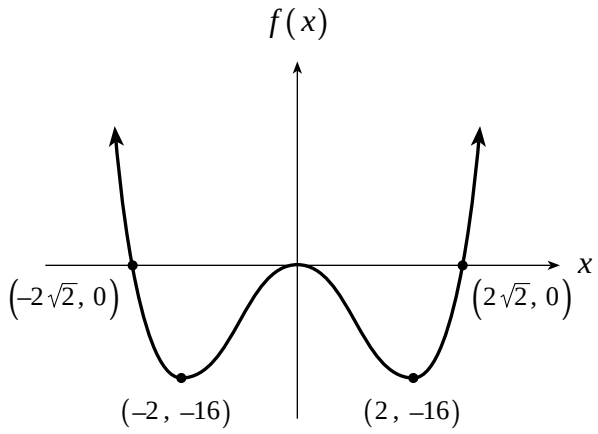
← **½ mark**

$x < -2$ or $0 < x < 2$

b) For what values of x is $f(x) = x^4 - 8x^2$ **decreasing**? **(1 mark)**

Alternate Solution:

Analysis of original 4th degree polynomial



$\frac{1}{2}$ **mark** for knowing that solution is where $f(2)$ is decreasing.

$\frac{1}{2}$ **mark** for correct solution.

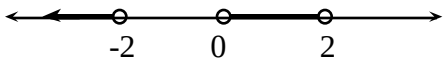
Decreasing $f(x)$ for

$x < -2$

$0 < x < 2$

OR

solution on number line

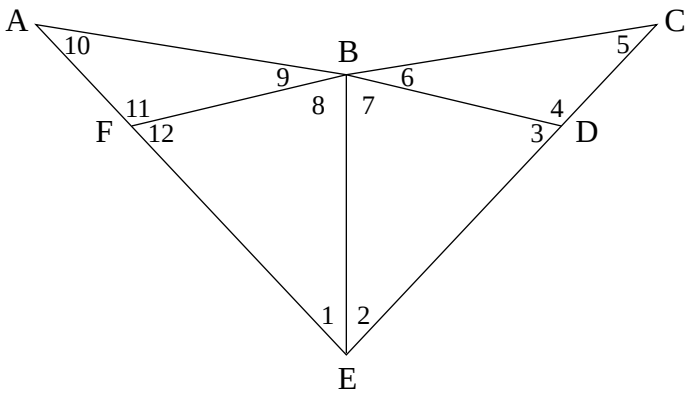


$x < -2$ or $0 < x < 2$

6. Complete the proof. **(4 marks)**

Given: $\angle 1 = \angle 2$
$EF = ED$

Prove: $\angle 11 = \angle 4$



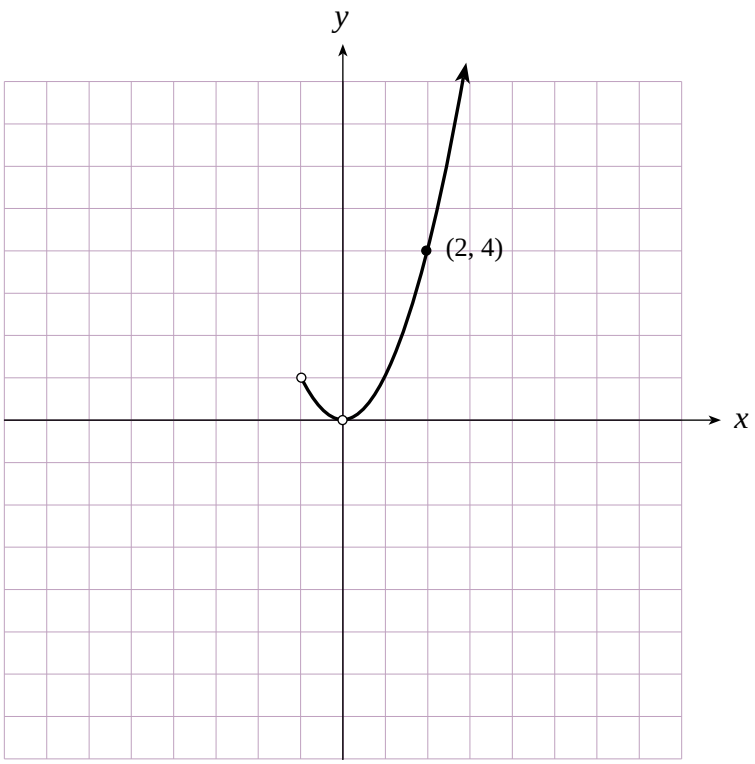
Solution:

Proof

Statement	Reason	
$\angle 1 = \angle 2$	given	
$EF = ED$	given	
$EB = EB$	same side	← **$\frac{1}{2}$ mark**
$\Delta FBE \cong \Delta DBE$	SAS	← **$1\frac{1}{2}$ marks**
$\angle 12 = \angle 3$	CPCTC	← **1 mark**
$\angle 11 = \angle 4$	supplements of $= \angle$s are $=$	← **1 mark**

7. Graph: $\log_{x+1} y = \log_{x+1} x^2$ **(2 marks)**

Solution:



$\frac{1}{2}$ **mark** for $y = x^2$ shape.

$\frac{1}{2}$ **mark** for right hand portion of parabola $(y = 0)$.

$\frac{1}{2}$ **mark** $\rightarrow x \neq -1$

$\frac{1}{2}$ **mark** $\rightarrow x \neq 0$

END OF KEY