

**Mathematics 12**  
 January 1996 Provincial Examination  
**ANSWER KEY / SCORING GUIDE**

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- Topics:**
1. Trigonometry
  2. Quadratic Relations
  3. Exponential and Logarithmic Functions
  4. Polynomial Functions
  5. Sequences and Series
  6. Introduction to Calculus
  7. Geometry
  8. Problem Solving

**Part A: Multiple-choice**

<b>Q</b>	<b>C</b>	<b>T</b>	<b>K</b>	<b>S</b>	<b>ILO</b>	<b>Q</b>	<b>C</b>	<b>T</b>	<b>K</b>	<b>S</b>	<b>ILO</b>
1.	K	2	D	1	12.17	26.	K	4	D	1	12.38
2.	U	2	C	1	12.13	27.	U	4	B	1	12.41
3.	K	2	B	1	12.20	28.	U	4	D	1	12.37
4.	U	2	C	1	12.12	29.	U	4	C	1	12.40
5.	U	2	A	1	12.18	30.	U	4	B	1	12.40
6.	U	2	C	1	12.19	31.	U	4	C	1	12.35
7.	U	2	A	1	12.15	32.	H	4	B	1	12.39, 12.35
8.	U	2	D	1	12.17	33.	K	5	C	1	12.46
9.	H	2	C	1	12.20	34.	U	5	A	1	12.46
10.	H	2	A	1	12.15	35.	U	5	D	1	12.45
11.	K	1	C	1	12.05	36.	U	5	B	1	12.47, 12.46
12.	U	1	D	1	12.01	37.	U	5	C	1	12.46
13.	U	1	B	1	12.02	38.	H	5	A	1	12.45, 12.64
14.	U	1	C	1	12.07	39.	K	6	B	1	12.62
15.	U	1	B	1	12.02	40.	U	6	D	1	12.57
16.	U	1	B	1	12.07	41.	U	6	B	1	12.51
17.	H	1	A	1	12.05, 12.64	42.	U	6	A	1	12.50
18.	H	1	D	1	12.07, 12.46	43.	U	6	D	1	12.52
19.	K	3	A	1	12.28	44.	U	6	D	1	12.62
20.	U	3	A	1	12.26	45.	H	6	C	1	12.58
21.	U	3	A	1	12.32	46.	U	7	B	1	12.63
22.	U	3	A	1	12.32	47.	U	7	B	1	12.63
23.	U	3	B	1	12.31	48.	U	8	C	1	12.64
24.	H	3	A	1	12.24	49.	U	8	B	1	12.64
25.	H	3	D	1	12.32	50.	U	8	D	1	12.64

**Part B: Written-response**

<b>Q</b>	<b>B</b>	<b>C</b>	<b>T</b>	<b>S</b>	<b>ILO</b>	<b>Q</b>	<b>B</b>	<b>C</b>	<b>T</b>	<b>S</b>	<b>ILO</b>
1.	1	U	5	2	12.46	4b.	5	U	6	1	12.60, 12.43
2.	2	U	2	3	12.21	5.	6	U	3	3	12.33
3.	3	U	1	3	12.03	6.	7	H	8	2	12.64
4a.	4	U	6	2	12.60	7.	8	H	7	4	12.63

Multiple-choice = 50 (50 questions)

Written-response = 20 (7 questions)

**Total = 70 marks**

**LEGEND:**

**Q** = Question Number

**C** = Cognitive Level

**T** = Topic

**K** = Keyed Response

**S** = Score

**ILO** = Intended Learning Outcome

**B** = Score Box Number

**PART B: WRITTEN-RESPONSE**

1. Determine the sum of the arithmetic series  $7 + 2 + (-3) + (-8) + \dots + (-213)$ . **(2 marks)**

**Solution:**

$$t_n = a + (n-1)d$$

$$-213 = 7 + (n-1)(-5) \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$-220 = -5(n-1)$$

$$45 = n \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$S_n = \frac{n}{2}(a+l)$$

$$= \frac{45}{2}(7-213) \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$= -4\,635 \quad \leftarrow \frac{1}{2} \text{ mark}$$

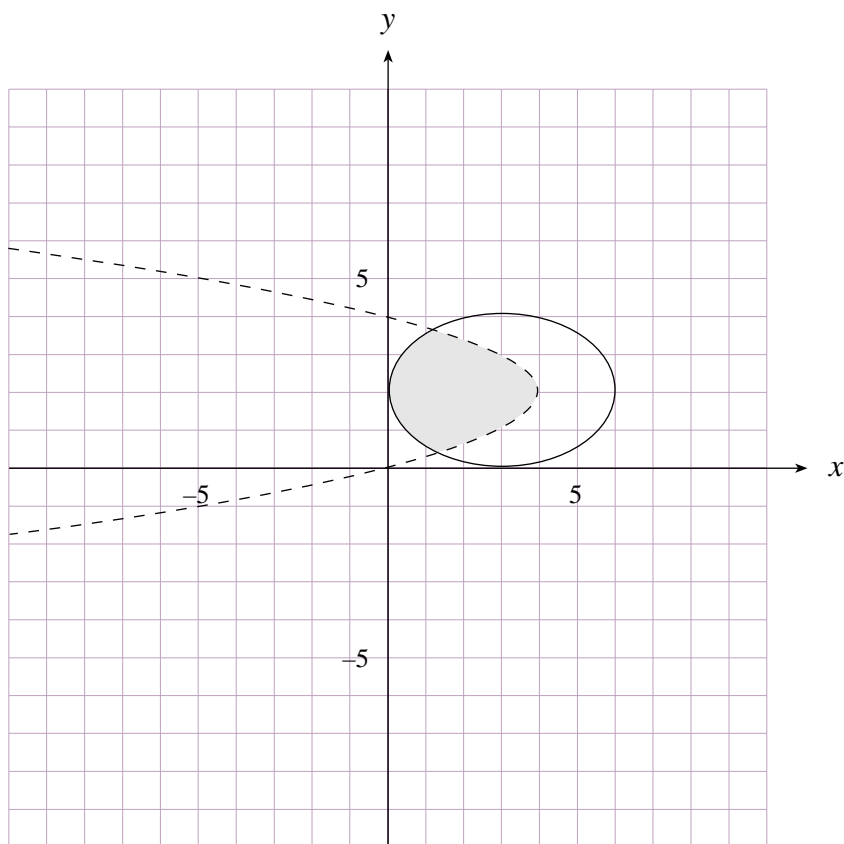
2. Graph the solution of the following system of inequalities on the grid provided.

(3 marks)

$$x < -(y-2)^2 + 4$$

$$\frac{(x-3)^2}{9} + \frac{(y-2)^2}{4} \leq 1$$

**Solution:**



**1 mark** for graph of parabola, vertex and 2 pts.

**1 mark** for graph of ellipse

$\frac{1}{2}$  **mark** for one dashed boundary line on parabola

$\frac{1}{2}$  **mark** for correct shaded region as solution set

3. Solve:  $2 \tan^2 x + 3 \tan x - 2 = 0$ ,  $0 \leq x < 2\pi$ . (Accurate to at least 2 decimal places.) (3 marks)

**Solution:**

$$2 \tan^2 x + 3 \tan x - 2 = 0$$

$$(2 \tan x - 1)(\tan x + 2) = 0$$

$$2 \tan x - 1 = 0 \text{ or } \tan x + 2 = 0$$

$$\frac{1}{2} \text{ mark} \rightarrow \tan x = \frac{1}{2} \text{ or } \tan x = -2 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$x = \underbrace{(0.46, 3.61, 2.03, 5.18)}$$

↑  
 $\frac{1}{2}$  mark for each

4. A particle moves along the  $x$ -axis so that its position at time  $t$  is  $x(t) = 2t^3 - 5t^2 - 4t + 3$ .  
( $x$  in cm and  $t$  in seconds.)

a) At what time(s) is the particle stationary?

**(2 marks)**

**Solution:**

Stationary when  $x'(t) = 0 \leftarrow \frac{1}{2}$  **mark** for concept

$$6t^2 - 10t - 4 = 0 \leftarrow \frac{1}{2}$$
 **mark** for derivative

$$t = -\frac{1}{3}s \quad \text{and} \quad t = 2s$$

$$\begin{array}{ccc} \uparrow & & \uparrow \\ \frac{1}{2} \text{ mark} & & \frac{1}{2} \text{ mark} \end{array}$$

b) At what time(s) is the particle moving to the left?

**(1 mark)**

**Solution:**

Moving left when  $x'(t) < 0 \leftarrow \frac{1}{2}$  **mark**

$$6t^2 - 10t - 4 < 0$$

$$-\frac{1}{3} < t < 2 \quad \leftarrow \frac{1}{2}$$
 **mark**

5. Suppose you invest \$10 at 10.2% per annum compounded annually. How many years would it take for your investment to grow to \$15 000? (Accurate to at least 1 decimal place.) **(3 marks)**

**Solution:**

$$15\,000 = 10(1.102)^n \quad \leftarrow \mathbf{1 \text{ mark}} \quad \left( \frac{1}{2} \text{ mark for formula concept, } \frac{1}{2} \text{ mark for values} \right)$$

$$1\,500 = 1.102^n \quad \leftarrow \frac{1}{2} \text{ mark}$$

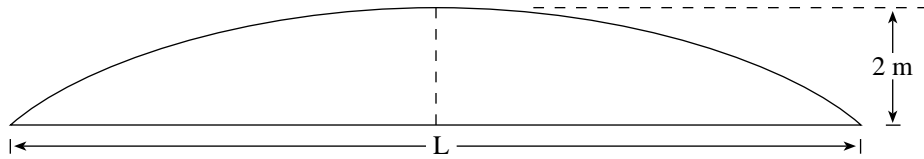
$$\log 1\,500 = n \log 1.102 \quad \text{or} \quad \log_{1.102} 1\,500 = n \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$n = \frac{\log 1\,500}{\log 1.102} \quad \left. \vphantom{n = \frac{\log 1\,500}{\log 1.102}} \right\} \leftarrow \frac{1}{2} \text{ mark}$$

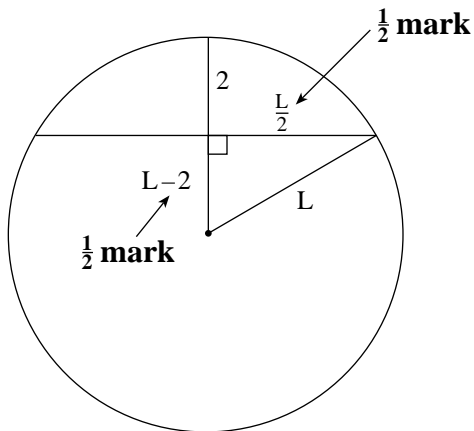
$$\begin{aligned} n &= 75.295 \\ \Rightarrow n &= 75.3 \text{ years} \end{aligned} \quad \left. \vphantom{\begin{aligned} n &= 75.295 \\ \Rightarrow n &= 75.3 \text{ years} \end{aligned}} \right\} \leftarrow \frac{1}{2} \text{ mark}$$

**Note:** If errors are made in either the initial formula or the ratio, etc., part marks could be earned by showing correct log applications thereafter.

6. A circular arch of a footbridge rises 2 m at the centre. If the horizontal length  $L$  of the footbridge is the same as the radius of the circle, calculate the value of  $L$ . (Accurate to at least 2 decimal places.) **(2 marks)**



**Solution:**



$$L^2 = (L - 2)^2 + \left(\frac{L}{2}\right)^2 \quad \left. \vphantom{L^2} \right\} \leftarrow \frac{1}{2} \text{ mark}$$

$$L^2 - 16L + 16 = 0$$

$$L = \frac{16 \pm \sqrt{192}}{2}$$

$$L = 14.93 \text{ or } 1.07$$

reject  $\downarrow$

$$14.93 \text{ m} \quad \leftarrow \frac{1}{2} \text{ mark}$$



7. Complete the proof.

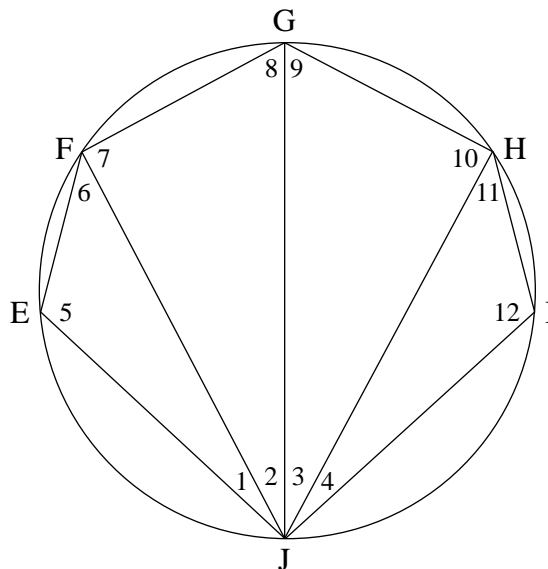
(4 marks)

Given: Circle with diameter GJ

$EF = IH$

$\angle 6 = \angle 11$

Prove:  $\angle 2 = \angle 3$



**Solution:**

Proof	
Statement	Reason
$1\frac{1}{2}$ marks $\rightarrow$ {	$EF = IH$ $\angle 1 = \angle 4$ $\angle 6 = \angle 11$ $\angle 5 = \angle 12$ $FJ = HJ$
1 mark $\rightarrow$ {	$\angle 8 = \angle 9$
1 mark $\rightarrow$ {	circle with diameter GJ $\angle 7 = 90^\circ, \angle 10 = 90^\circ$ $\angle 7 = \angle 10$
$\frac{1}{2}$ mark $\rightarrow$ {	$\angle 2 = \angle 3$
	given inscribed $\angle$ s on = chords are = given 3rd $\angle$ s of $\Delta$ s are = chords on = inscribed $\angle$ s are = inscribed $\angle$ s on = chords are = given inscribed $\angle$ on diameter = $90^\circ$ substitution (both = $90^\circ$ ) 3rd $\angle$ s of $\Delta$ s are =

7. Complete the proof.

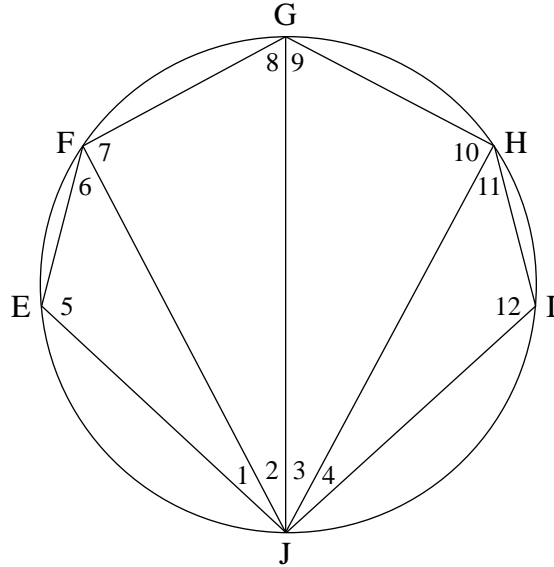
(4 marks)

Given: Circle with diameter GJ

$$EF = IH$$

$$\angle 6 = \angle 11$$

Prove:  $\angle 2 = \angle 3$



**Alternate Solution:**

Proof	
Statement	Reason
$1\frac{1}{2}$ marks $\rightarrow$ {	$EF = IH$ given $\angle 1 = \angle 4$ inscribed $\angle$ s on = chords are = $\angle 6 = \angle 11$ given $\triangle EFJ \cong \triangle IHJ$ AAS $FJ = HJ$ CPCTC
1 mark $\rightarrow$ {	$\angle 8 = \angle 9$ inscribed $\angle$ s on = chords are =
1 mark $\rightarrow$ {	circle with diameter GJ given $\angle 7 = 90^\circ, \angle 10 = 90^\circ$ inscribed $\angle$ on diameter = $90^\circ$ $\angle 7 = \angle 10$ substitution (both = $90^\circ$ )
$\frac{1}{2}$ mark $\rightarrow$ {	$\triangle GFJ \cong \triangle GHJ$ AAS $\angle 2 = \angle 3$ CPCTC

END OF KEY

