APRIL 1995 MATHEMATICS 12 PROVINCIAL EXAMINATION
ANSWER KEY / SCORING GUIDE

ITEM CLASSIFICATION

TOPICS

1. Trigonometry
2. Quadratic Relations
3. Exponential and Logarithmic Functions
4. Polynomial Functions
5. Sequences and Series
6. Introduction to Calculus
7. Geometry
8. Problem Solving

PART A: MULTIPLE-CHOICE

Q	C	T	K	S	ILO	Q	C	T	K	S	ILO
1.	K	2	C	1	12.15	26.	K	4	C	1	12.35
2.	U	2	D	1	12.14	27.	U	4	B	1	12.40
3.	U	2	A	1	12.13	28.	U	4	C	1	12.41
4.	U	2	A	1	12.17	29.	U	4	A	1	12.40
5.	U	2	A	1	12.11	30.	U	4	A	1	12.37
6.	U	2	D	1	12.21	31.	U	4	B	1	12.37
7.	U	2	A	1	12.15	32.	U	4	C	1	12.42
8.	U	2	A	1	12.22	33.	H	4	D	1	12.43/12.35
9.	H	2	B	1	12.17	34.	K	5	B	1	12.46
10.	H	2	C	1	12.20	35.	U	5	C	1	12.45
11.	K	1	C	1	12.01	36.	U	5	C	1	12.47
12.	U	1	B	1	12.02	37.	U	5	C	1	12.46
13.	U	1	C	1	12.05	38.	H	5	B	1	12.46
14.	U	1	B	1	12.07	39.	K	6	A	1	12.57
15.	U	1	A	1	12.03	40.	U	6	C	1	12.50
16.	U	1	B	1	12.06	41.	U	6	D	1	12.51
17.	U	1	D	1	12.03	42.	U	6	A	1	12.53
18.	H	1	A	1	12.05	43.	U	6	C	1	12.60
19.	K	3	D	1	12.28	44.	U	6	D	1	12.58
20.	U	3	D	1	12.29	45.	H	6	B	1	12.53
21.	U	3	C	1	12.32	46.	U	7	D	1	12.63
22.	U	3	C	1	12.26	47.	U	7	C	1	12.63
23.	U	3	D	1	12.24	48.	U	8	D	1	12.64
24.	H	3	B	1	12.32	49.	U	8	B	1	12.64
25.	H	3	B	1	12.30	50.	U	8	A	1	12.64

PART B: WRITTEN-RESPONSE

Q	B	C	T	S	ILO	Q	B	C	T	S	ILO
1.	1	U	3	3	12.32	4.	5	U	1	2	12.08
2a.	2	U	2	2	12.18	5.	6	U	5	3	12.46
2b.	3	U	2	1	12.18	6.	7	H	7	4	12.63
3.	4	U	6	3	12.56	7.	8	U	8	2	12.64

Multiple-choice = 50 (50 questions)
Written-response = 20 (7 questions)
Total = 70 marks

LEGEND:

Q = Question
K = Keyed response
B = Score box number
C = Cognitive level
S = Score
T = Topic
ILO = Intended Learning Outcome

PART B: WRITTEN-RESPONSE

1. Solve: $\log(x+11)+\log x=\log(x+1)+\log 6$ **(3 marks)**

Response:

$$\log(x+11)+\log x=\log(x+1)+\log 6$$

$$\log\left(x^2+11x\right)=\log(6x+6) \qquad \leftarrow \textbf{1 mark}$$

$$x^2+11x=6x+6 \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$x^2+5x-6=0$$

$$(x+6)(x-1)=0 \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$\underset{\substack{\uparrow\\ \text{reject}}}{x=-6} \qquad x=1 \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

reject $\leftarrow \tfrac{1}{2}$ **mark**

$$\therefore x=1$$

2. The equation of a hyperbola is $9x^2 - 4y^2 - 8y + 32 = 0$.

a) Change the equation to standard form. **(2 marks)**

Response:

$$9x^2 - 4y^2 - 8y + 32 = 0$$

$$9x^2 - 4\left(y^2 + 2y\right) = -32 \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}\ \text{(correctly factoring out '–4')}$$

$$9x^2 - 4\left(y^2 + 2y + 1\right) = -32 - 4 \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}\ \text{(completing the square and compensating correctly on R.S.)}$$

$$9x^2 - 4(y+1)^2 = -36 \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}\ \text{(factoring to perfect square)}$$

$$\left.\begin{array}{l} \dfrac{x^2}{4} - \dfrac{(y+1)^2}{9} = -1 \\ \textbf{or} \\ \dfrac{(y+1)^2}{9} - \dfrac{x^2}{4} = 1 \end{array}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

b) Give the equations of the asymptotes of the hyperbola. **(1 mark)**

Response:

$$y + 1 = \frac{3}{2}x \qquad y + 1 = -\frac{3}{2}x$$

slopes: $\frac{3}{2}$ and $-\frac{3}{2}$ ← $\frac{1}{2}$ **mark**

y-intercept: -1 ← $\frac{1}{2}$ **mark**

3. Given $f(x) = x^2 - 3x$, use the **definition of the derivative** to show that $f'(x) = 2x - 3$.

(3 marks)

Response:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$= \lim_{h\to 0} \frac{(x+h)^2 - 3(x+h) - \left(x^2 - 3x\right)}{h} \qquad \leftarrow 1 \textbf{ mark}$$

$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h} \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$= \lim_{h\to 0} \frac{2xh + h^2 - 3h}{h} \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$= \lim_{h\to 0} \frac{h(2x + h - 3)}{h}$$

$$= \lim_{h\to 0} (2x + h - 3) \qquad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$= 2x - 3$$

$-\frac{1}{2}$ **mark** for confusing limit placement

4. Prove the identity. **(2 marks)**

$$\frac{\cos\theta + \sin\theta\tan\theta}{\sin\theta\sec\theta} = \csc\theta$$

Left side	Right side
$\dfrac{\cos\theta + \sin\theta\tan\theta}{\sin\theta\sec\theta}$	$\csc\theta$
$\frac{1}{2}$ **mark** → $\left\{ = \dfrac{\cos\theta + \sin\theta\left(\dfrac{\sin\theta}{\cos\theta}\right)}{\sin\theta\left(\dfrac{1}{\cos\theta}\right)} \right.$	$\left.\right\}$ **for substituting in both places**
$\frac{1}{2}$ **mark** → $\left\{ = \dfrac{\left(\cos\theta + \dfrac{\sin^2\theta}{\cos\theta}\right)\cos\theta}{\left(\dfrac{\sin\theta}{\cos\theta}\right)\cos\theta} \right.$	$\left.\right\}$ **for simplifying complex fraction**
$\frac{1}{2}$ **mark** → $\left\{ = \dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta} \right.$	
$\frac{1}{2}$ **mark** → $= \dfrac{1}{\sin\theta}$	
$= \csc\theta$	

LS = RS

5. The sum of the first 100 terms of an arithmetic series is $-11\ 200$. The sum of the first 200 terms is $17\ 600$. Determine the first term a and the common difference d of the series. **(3 marks)**

Response:

$-11\ 200 = \frac{100}{2}\left(2a+(100-1)d\right)$ ← **½ mark**

$-224 = 2a + 99d$ ←←←← ←←←←←←←

$17\ 600 = \frac{200}{2}\left(2a+(200-1)d\right)$ ← **½ mark** ↑ ↕ ↓ } ← **½ mark** **Attempt to simplify (both)**

$176 = 2a + 199d$ ←←←← ←←←←←←←

$$\begin{array}{r} 176 = 2a + 199d \\ -224 = 2a + 99d \\ \hline 400 = 100d \end{array}$$

} ← **½ mark** **Attempt to solve system**

$d = 4$ $a = -310$

↑ ↑

½ mark **½ mark**

½ mark correct formula

½ mark substitution into formula correctly

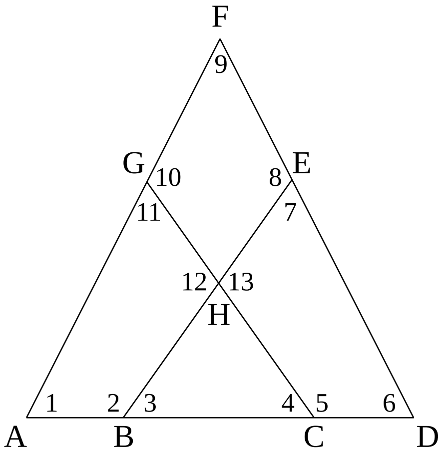
} ← **1 mark total**

If wrong formula, attempt to simplify <u>and</u> attempt to solve → Cap 1

6. Complete the proof. **(4 marks)**

Given: AF = DF
$\angle 2 = \angle 5$

Prove: $\angle 10 = \angle 8$



Response:

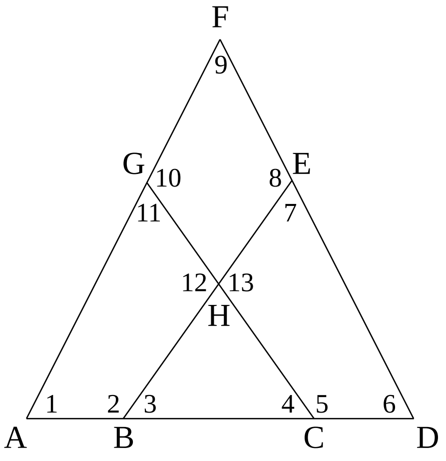
Proof

	Statement	Reason
2 marks →	AF = DF	given
	$\angle 1 = \angle 6$	$\angle$s opposite = sides are =
	$\angle 2 = \angle 5$	given
	$\angle 3 = \angle 4$	supplements of = $\angle$s are =
2 marks →	$\angle 7 = \angle 11$	3rd $\angle$s of Δs are =
	$\angle 10 = \angle 8$	supplements of = $\angle$s are =

6. Complete the proof. **(4 marks)**

Given: AF = DF
$\angle 2 = \angle 5$

Prove: $\angle 10 = \angle 8$



Alternate Response 1:

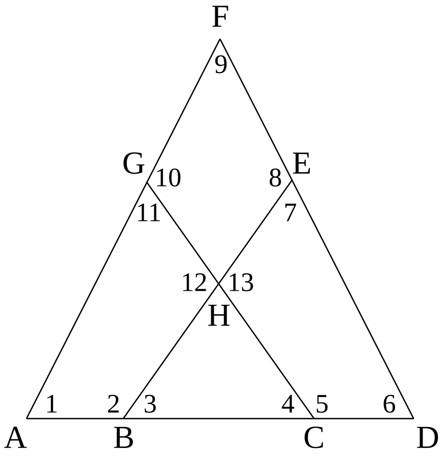
Proof

	Statement	Reason
2 marks →	AF = DF	given
	$\angle 1 = \angle 6$	$\angle$s opposite = sides are =
	$\angle 2 = \angle 5$	given
	$\angle 12 = \angle 13$	vertically opposite $\angle$s are =
2 marks →	$\angle 11 = \angle 7$	4th $\angle$s of quadrilaterals are =
	$\angle 10 = \angle 8$	supplements of = $\angle$s are =

6. Complete the proof. **(4 marks)**

Given: AF = DF
$\angle 2 = \angle 5$

Prove: $\angle 10 = \angle 8$



Alternate Response 2:

Proof

	Statement	Reason
2 marks → {	AF = DF	given
	$\angle 1 = \angle 6$	$\angle$s opposite = sides are =
	$\angle 2 = \angle 5$	given
	$\angle 9 = \angle 9$	same $\angle$
2 marks → {	$\angle 8 = \angle 10$	4th $\angle$s of quadrilaterals are =

7. Find a polynomial equation of lowest degree with integral coefficients such that one root of $f(x)=0$ is $\sqrt{2}+\sqrt{3}$. **(2 marks)**

Response:

$$x=\sqrt{2}+\sqrt{3} \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$x^2=\left(\sqrt{2}+\sqrt{3}\right)^2 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$x^2=2+2\sqrt{6}+3$$

$$x^2=5+2\sqrt{6}$$

$$x^2-5=2\sqrt{6}$$

$$\left(x^2-5\right)^2=\left(2\sqrt{6}\right)^2 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\left.\begin{aligned} x^4-10x^2+25&=24 \\ x^4-10x^2+1&=0 \end{aligned}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

7. Find a polynomial equation of lowest degree with integral coefficients such that one root of $f(x)=0$ is $\sqrt{2}+\sqrt{3}$. **(2 marks)**

Alternate Response:

$$x=\sqrt{2}+\sqrt{3} \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$x-\sqrt{2}=\sqrt{3}$$

$$\left(x-\sqrt{2}\right)^2=\left(\sqrt{3}\right)^2 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$x^2-2\sqrt{2}x+2=3$$

$$x^2-1=2\sqrt{2}x$$

$$\left(x^2-1\right)^2=\left(2\sqrt{2}x\right)^2 \qquad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\left.\begin{aligned} &x^4-2x^2+1=8x^2 \\ &x^4-10x^2+1=0 \end{aligned}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

END OF KEY