JUNE 1994 MATHEMATICS 12 PROVINCIAL EXAMINATION
ANSWER KEY / SCORING GUIDE

ITEM CLASSIFICATION

TOPICS:
1. Trigonometry
2. Quadratic Relations
3. Exponential and Logarithmic Functions
4. Polynomial Functions
5. Sequences and Series
6. Introduction to Calculus
7. Geometry
8. Problem Solving

PART A: MULTIPLE-CHOICE

Q	C	T	K	S	ILO	Q	C	T	K	S	ILO
1.	K	2	A	1	12.13	26.	K	4	B	1	12.38
2.	U	2	C	1	12.14	27.	U	4	D	1	12.35
3.	U	2	A	1	12.12	28.	U	4	C	1	12.41
4.	U	2	D	1	12.16	29.	U	4	B	1	12.40
5.	U	2	D	1	12.17	30.	U	4	D	1	12.37
6.	U	2	A	1	12.21	31.	U	4	A	1	12.39
7.	item deleted			1	12.20	32.	U	4	B	1	12.43
8.	U	2	A	1	12.15	33.	H	4	D	1	12.34
9.	H	2	A	1	12.17	34.	K	5	B	1	12.46
10.	H	2	C	1	12.23	35.	U	5	C	1	12.46
11.	K	1	B	1	12.02	36.	U	5	C	1	12.47
12.	K	1	A	1	12.07	37.	U	5	C	1	12.46
13.	U	1	D	1	12.02	38.	H	5	B	1	12.45
14.	U	1	A	1	12.03	39.	K	6	A	1	12.57
15.	U	1	C	1	12.03	40.	U	6	C	1	12.50
16.	H	1	D	1	12.04	41.	U	6	B	1	12.57
17.	U	1	A	1	12.07	42.	K	6	A	1	12.58
18.	H	1	A	1	12.02	43.	U	6	D	1	12.56
19.	K	3	D	1	12.28	44.	U	6	C	1	12.61
20.	U	3	C	1	12.32	45.	H	6	C	1	12.53
21.	U	3	B	1	12.30	46.	H	7	D	1	12.63
22.	U	3	B	1	12.24	47.	U	7	B	1	12.63
23.	U	3	D	1	12.29	48.	U	8	B	1	12.64
24.	U	3	D	1	12.31	49.	U	8	C	1	12.64
25.	H	3	B	1	12.31	50.	H	8	D	1	12.64

PART B: WRITTEN-RESPONSE

Q	B	C	T	S	ILO	Q	B	C	T	S	ILO
1.	1	U	2	3	12.18	5.	5	H	7	4	12.63
2.	2	U	1	2	12.06	6.	6	U	5	3	12.48
3.	3	U	8	2	12.64	7.	7	U	6	3	12.62
4.	4	U	3	3	12.32						

KEY:
Q = Question | **B** = Score box number | **C** = Cognitive level
T = Topic | **S** = Score | **ILO** = Intended Learning Outcome
K = Keyed response

1. Write in standard form: $9x^2+y^2-54x+4y+49=0$ **(3 marks)**

Solution:

$$9x^2-54x+y^2+4y=-49$$

$$\underbrace{9}_{\frac{1}{2}\text{ mark}}\left(x^2-6x+\underline{9}\right)+\left(y^2+4y+\underline{4}\right)=-49+\overbrace{\underline{81}+\underline{4}}^{\frac{1}{2}\text{ mark each}}$$

↑ **½ mark for both** ↑

$$9(x-3)^2+(y+2)^2=36 \quad \leftarrow \textbf{½ mark}$$

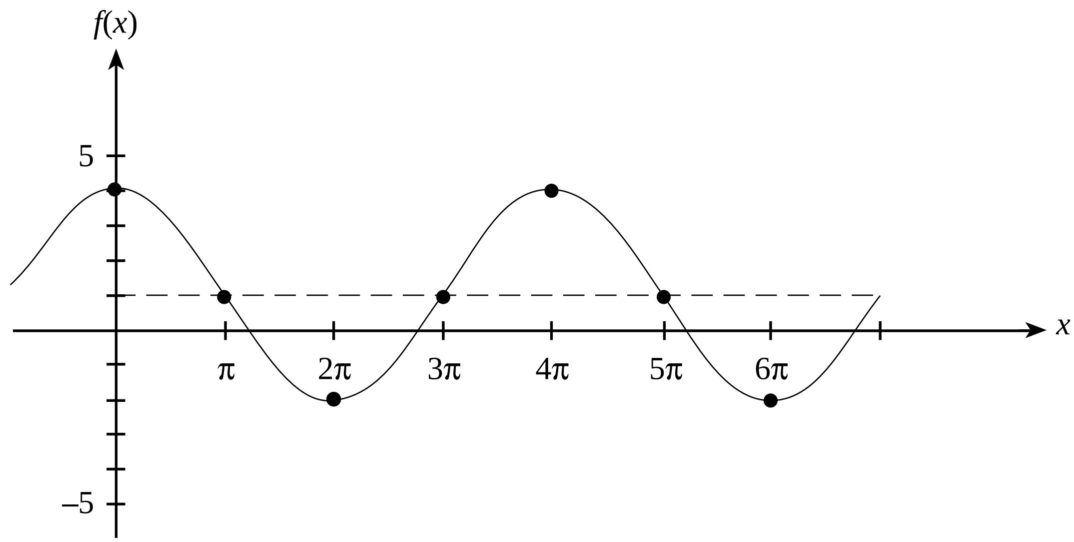
$$\frac{(x-3)^2}{4}+\frac{(y+2)^2}{36}=1 \quad \leftarrow \textbf{½ mark}$$

Marks capped at 2 if final line was **not** an equation of an ellipse.

Also acceptable for full marks: $\left(\frac{x-2}{2}\right)^2+\left(\frac{y+2}{6}\right)^2=1$

2. Graph at least one period of $f(x) = 3\cos\frac{1}{2}x + 1$ on the grid provided. **(2 marks)**

Solution:



$f(x) = 3\cos\frac{1}{2}x + 1$

Amplitude:	3	←	**½ mark**
Period:	$\frac{2\pi}{1/2} = 4\pi$	←	**½ mark** (had to be demonstrated on graph)
Vertical shift:	up one	←	**½ mark**
Shape:	cosine wave	←	**½ mark**

Notes: Shape ½ mark included some indication of concavity (saw-tooth not accepted) and had to pass through (π, 1) and (3π, 1).

If information about amplitude = 3, period = 4π, vertical displacement = 1 up **and** no graph shown, then received mark of ½.

3. Determine the value(s) of k for which the graph of the relation $(2+k)x^2 + (1-k^2)y^2 + x - 2y = 17$ represents a parabola. **(2 marks)**

Solution:

$2 + k = 0$ or $1 - k^2 = 0$ ← ½ **mark** for concept → $AB - H^2 = 0$ textbook justification

$k = -2$ or $k^2 = 1$

$k = \pm 1$

$k = -2,\quad 1,\quad -1$

↑ ↑ ↑

½ mark **½ mark** **½ mark**

4. Solve for x: $\log_5(2x+1) = 1 - \log_5(x+2)$ **(3 marks)**

Solution:

$\log_5(2x+1) + \log_5(x+2) = 1$ ← **½ mark**

$\log_5(2x+1)(x+2) = 1$ ← **½ mark**

$2x^2 + 5x + 2 = 5$ ← **1 mark**

$2x^2 + 5x - 3 = 0$

$(2x-1)(x+3) = 0$

$x = -3, \quad \frac{1}{2}$

reject ↑ ↑ **½ mark**

½ mark

$\therefore x = \frac{1}{2}$

AlternateSolution:

$\log_5(2x+1) = \log_5 5 - \log_5(x+2)$ ← **½ mark**

$\log_5(2x+1) = \log_5\left(\frac{5}{x+2}\right)$ ← **½ mark**

$2x + 1 = \frac{5}{x+2}$ ← **1 mark**

$(2x+1)(x+2) = 5$

$2x^2 + 5x - 3 = 0$

$(2x-1)(x+3) = 0$

$x = -3, \quad \frac{1}{2}$

reject ↑ ↑ **½ mark**

½ mark

$\therefore x = \frac{1}{2}$

5. Complete the proof. **(4 marks)**

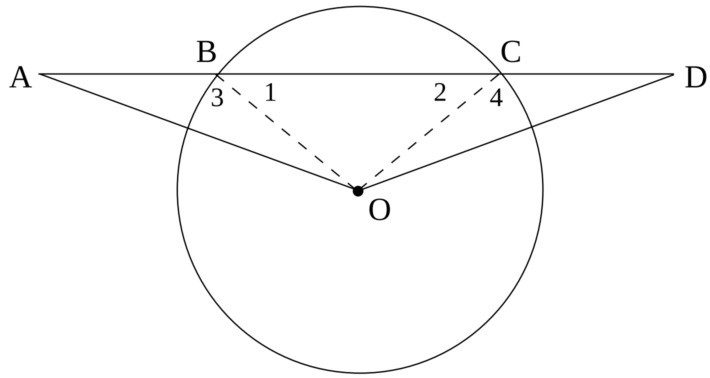
Given: O is the centre

A, B, C, D are collinear

AB = CD

Prove: OA = OD

Note: Students are encouraged to use numbers to label angles.



Solution:

Proof

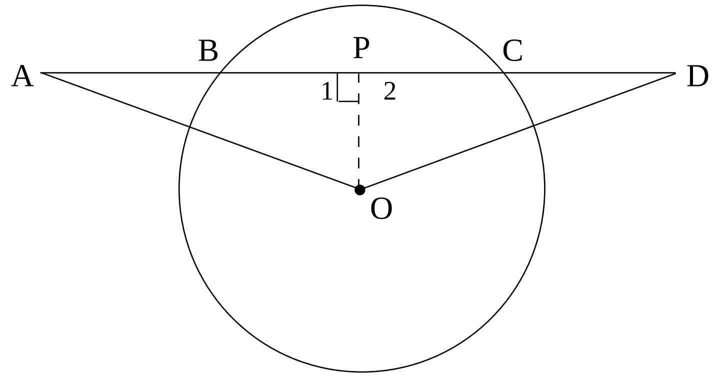
	Statement	Reason
2 marks	join OB and OC	
	O is the centre	given
	OB = OC	radii are =
	$\angle 1 = \angle 2$	$\angle$s opp = sides
	$\angle 3 = \angle 4$	supplements of = $\angle$s are =
	option: BC = BC (to *)	identity
	AB = CD	given
	AB + BC = CD + BC	addition property of equality
	AC = BD	substitution
2 marks	AB = CD	given
	*Δ OBA ≅ Δ OCD	SAS
	OA = OD	CPCTC

Subtract:

- ½ for missing construction
- 1 for major line missing
- ½ for given missing (AB = CD)
- ½ for wrong congruence / Δ symbol missing
- ½ for wrong correspondence
- ½ for wrong reason

* Cap at 2½ if $\angle 3 = \angle 4$ is missing but everything else is correct, **unless** using option indicated.

AlternateSolution:



Proof

Marks	Statement	Reason
	join O to P, P on BC,	
	such that $OP \perp BC$	
	options:	
	∑construct mid - point P and use	
1½ marks	converse (chord $\perp$ bisector theorem)	
	∑construct $\perp$ bisector of BC	
	$\angle 1 = 90°$, $\angle 2 = 90°$	def. of $\perp$
	$\angle 1 = \angle 2$	both $= 90°$ (substitution)
	$BP = CP$	$\perp$ through centre is a bisector of the chord
	$AB = CD$	given
	$AB + BP = CD + CP$	equation property of addition (**1 mark**)
2½ marks	$AP = DP$	substitution (**1 mark**)
	$OP = OP$	same side (identity, common)
	$\Delta\ APO \cong \Delta\ DPO$	SAS
	$OA = OD$	CPCTC (Pythagoras without congruent Δs)

Subtract:
- 1 if construction not established correctly (cap at **3 marks**)
- 1 if major line missing
- ½ for incorrect congruence / Δ symbol missing
- ½ if identity missing

6. A college student was offered two different summer jobs. Job A would last 17 weeks and pay $225 per week, with weekly raises of $10. Job B would last 4 months and pay $1 100 per month, with monthly raises of 10% of the previous month's salary. How much more would the college student earn by accepting Job A? **(3 marks)**

Solution:

$$t_{17} = 225 + 16(10) = 385 \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$S_n = \frac{17}{2}(225 + 385) \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$= \$5\,185 \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

OR

$a = 225$, $n = 1$, $d = 10$ — all three required $\leftarrow \tfrac{1}{2}$ **mark**

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{17}{2}[450 + (16)10] \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$= \$5\,185 \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

½ **mark**
(for r = 1.1) ↓

$$S_n = \frac{1\,100\left[1 - (1.1)^4\right]}{1 - 1.1}$$

$$= \$5\,105.10 \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

Answer: $79.90 ← ½ **mark**

½ **mark**
(for r = 1.1) ↓

$$S_n = \frac{1\,100\left[1 - (1.1)^4\right]}{1 - 1.1}$$

$$= \$5\,105.10 \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

Answer: $79.90 ← ½ **mark**

7. Two numbers have the property that the sum of twice one number and three times the second number is 40. Find the two numbers such that their product is a maximum. **(3 marks)**

Solution:

let the numbers be x and y

$2x + 3y = 40$ $\leftarrow \frac{1}{2}$ **mark**

$$y = \frac{40 - 2x}{3}$$

product = P = xy $\leftarrow \frac{1}{2}$ **mark**

$P = x\left(\frac{40-2x}{3}\right) = \frac{40}{3}x - \frac{2}{3}x^2$ $\leftarrow \frac{1}{2}$ **mark**

$\frac{dP}{dx} = \frac{40}{3} - \frac{4}{3}x = 0$ $\leftarrow \frac{1}{2}$ **mark**

$$\frac{4}{3}x = \frac{40}{3}$$

$x = 10$ $\leftarrow \frac{1}{2}$ **mark**

$y = \frac{40 - 2(10)}{3} = \frac{20}{3}$ $\leftarrow \frac{1}{2}$ **mark**

Answer:

$10, \quad \frac{20}{3}$

AlternateSolution:

let the numbers be x and y

$2x + 3y = 40$ $\leftarrow \frac{1}{2}$ **mark**

$$y = \frac{40 - 2x}{3}$$

product = P = xy $\leftarrow \frac{1}{2}$ **mark**

$P = x\left(\frac{40-2x}{3}\right)$ $\leftarrow \frac{1}{2}$ **mark**

$$P = -\frac{2}{3}x^2 + \frac{40}{3}x$$

$$\frac{P}{-\frac{2}{3}} = x^2 - 20x + 100 - 100$$

$$\left.\begin{aligned} \frac{P}{-\frac{2}{3}} &= (x-10)^2 - 100 \\ \therefore P &= -\frac{2}{3}(x-10)^2 + \frac{200}{3} \end{aligned}\right\} \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

Answer:

$\therefore$ max at $x = 10$ $\leftarrow \frac{1}{2}$ **mark**

$\therefore$ $y = \frac{20}{3}$ $\leftarrow \frac{1}{2}$ **mark**

END OF KEY