

**Principles of Mathematics 12**  
 June 2004 Provincial Examination  
**ANSWER KEY / SCORING GUIDE**

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**CURRICULUM:**

<b>Organizers</b>		<b>Sub-Organizers</b>
1. Problem Solving	A	Problem Solving and Cross Topic Problems
2. Patterns and Relations	B	Geometric Sequences and Series
	C/D	Logarithms and Exponents
	C/D	Trigonometry
3. Shape and Space	E	Conics
	F	Transformations
4. Statistics and Probability	G	Combinatorics
	G	Probability
	G	Statistics

**Part A: Multiple Choice**

<b>Q</b>	<b>K</b>	<b>C</b>	<b>S</b>	<b>CO</b>	<b>PLO</b>	<b>Q</b>	<b>K</b>	<b>C</b>	<b>S</b>	<b>CO</b>	<b>PLO</b>
1.	A	U	1.5	2	C4	21.	D	H	1.5	2	D1
2.	C	U	1.5	2	C3	22.	A	K	1.5	3	E2
3.	B	U	1.5	2	C5	23.	A	U	1.5	3	E3
4.	C	K	1.5	2	D6, F4	24.	C	H	1.5	3	E2
5.	D	U	1.5	2	C7	25.	A	K	1.5	3	F1
6.	D	U	1.5	2	C6	26.	A	U	1.5	3	F3
7.	B	U	1.5	2	D5	27.	D	U	1.5	3	F6
8.	A	U	1.5	2	C8	28.	C	H	1.5	3	F6
9.	B	H	1.5	2	D6	29.	D	K	1.5	4	G6
10.	A	U	1.5	2	B1	30.	A	U	1.5	4	G8
11.	D	U	1.5	2	B1	31.	C	H	1.5	4	G4
12.	B	K	1.5	2	B3	32.	C	U	1.5	4	G7
13.	D	U	1.5	2	B1	33.	D	H	1.5	4	G7
14.	D	H	1.5	2	B3	34.	B	U	1.5	4	G11
15.	B	K	1.5	2	D3	35.	B	U	1.5	4	G13
16.	C	K	1.5	2	C2	36.	B	U	1.5	4	G8
17.	D	U	1.5	2	C1	37.	B	H	1.5	4	G12
18.	C	U	1.5	2	C2	38.	A	U	1.5	4	G2
19.	C	U	1.5	2	C2	39.	C	U	1.5	4	G1
20.	A	H	1.5	2	D1	40.	B	H	1.5	4	G2

**Multiple Choice = 60 marks**

## Part B: Written Response

<b>Q</b>	<b>B</b>	<b>C</b>	<b>S</b>	<b>CO</b>	<b>PLO</b>
1a.	1	U	2	3	F6
1b.	2	U	3	3	F4
2.	3	U	4	3	E2
3.	4	U	5	2	D1, D3
4.	5	U	4	4	G12
5a.	6	U	2	4	G8, G12
5b.	7	U	2	4	G8, G3
6.	8	U	4	2	D7
7.	9	H	4	2	C8

**Written Response = 30 marks**

Multiple Choice = 60 (40 questions)

Written Response = 30 (7 questions)

**EXAMINATION TOTAL = 90 marks**

### **LEGEND:**

**Q** = Question Number

**B** = Score Box Number

**PLO** = Prescribed Learning Outcome

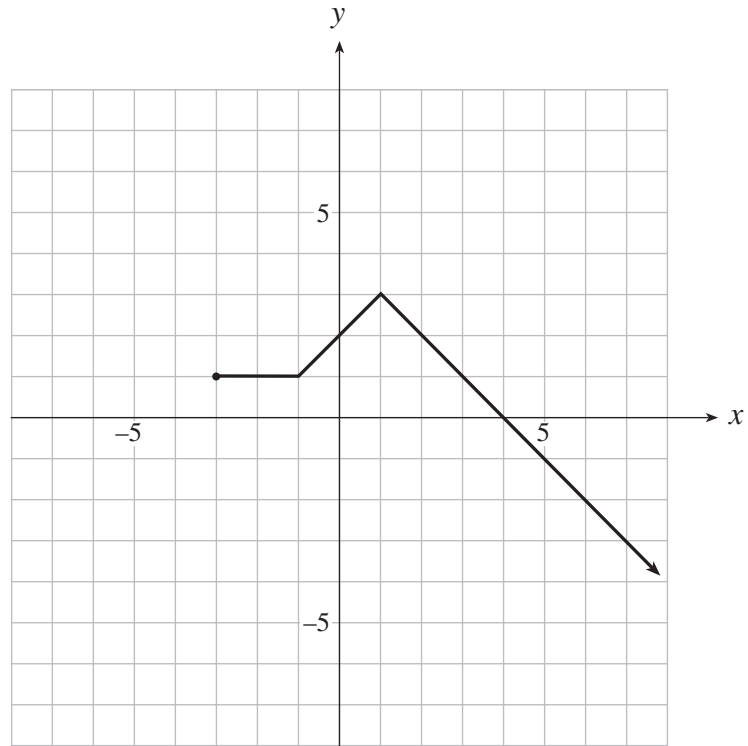
**K** = Keyed Response

**S** = Score

**C** = Cognitive Level

**CO** = Curriculum Organizer

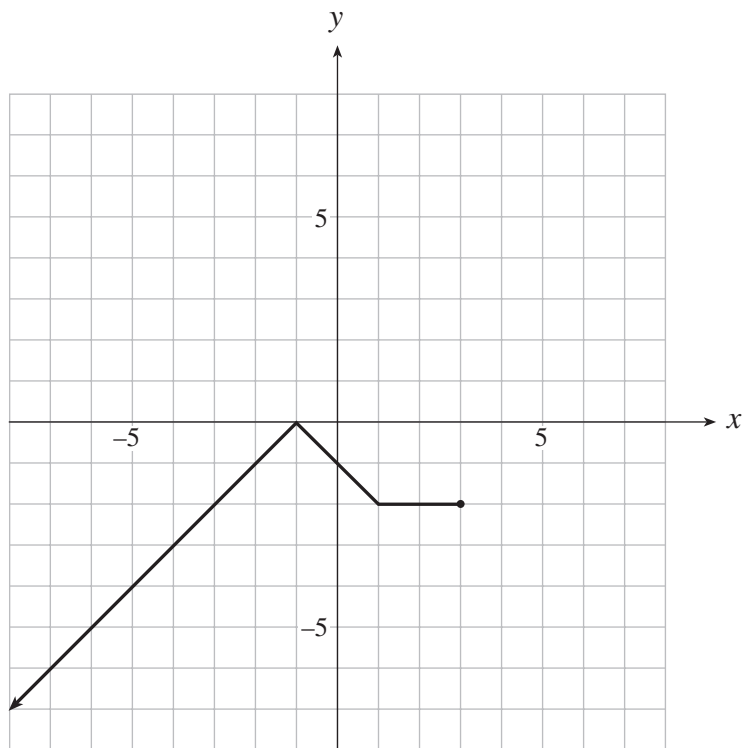
1. The graph of  $y = f(x)$  is shown below.



a) On the grid provided, sketch the graph of  $y = f(-x) - 3$ .

(2 marks)

 solution



**1 mark** reflection over y-axis

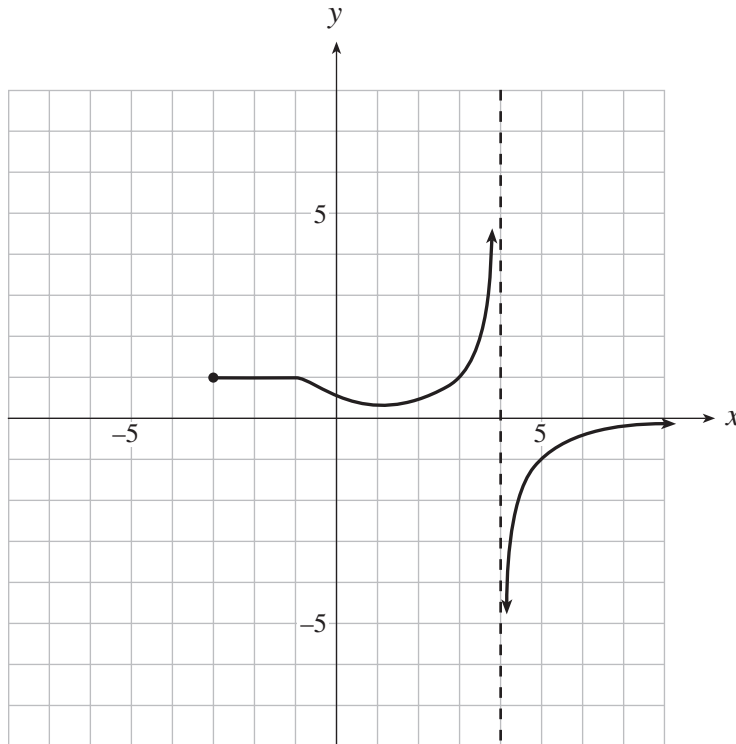
**1 mark** translation down 3

**Note:** Deduct  $\frac{1}{2}$  mark if graph does not stop on the right hand side.

b) On the grid provided, sketch the graph of  $y = \frac{1}{f(x)}$ .

(3 marks)

 solution



$\frac{1}{2}$  mark for vertical asymptote or asymptotic behaviour ( $x = 4$ )

$\frac{1}{2}$  mark for horizontal asymptote on right or asymptotic behaviour ( $y = 0$ )

$\frac{1}{2}$  mark for invariant point  $(5, -1)$

$\frac{1}{2}$  mark for invariant point  $(3, 1)$

$\frac{1}{2}$  mark for horizontal portion  $-3 \leq x \leq -1$

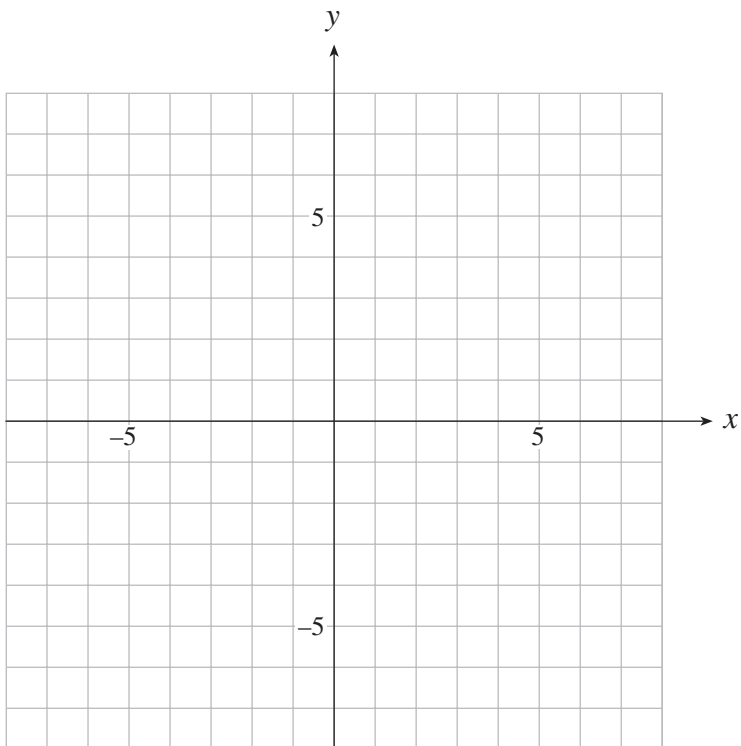
$\frac{1}{2}$  mark for shape from  $-1 \leq x \leq 3$

**Note:** Deduct  $\frac{1}{2}$  mark if graph does not stop at  $(-3, 1)$ .

**Note:** Arrowheads not necessary for full marks.

2. Determine the equation in standard form of the parabola with vertex  $(5, -2)$ , passing through the point  $(2, 0)$ , and having a horizontal axis of symmetry. **(4 marks)**

**Grid is provided for rough work only.**



## solution

$$x = a(y - k)^2 + h \quad \leftarrow \mathbf{1 \text{ mark}}$$

$$x = a(y + 2)^2 + 5 \quad \leftarrow \mathbf{1 \frac{1}{2} \text{ marks}}$$

$$2 = a(0 + 2)^2 + 5$$

$$a = -\frac{3}{4} \quad \leftarrow \mathbf{1 \frac{1}{2} \text{ marks}}$$

$$x = -\frac{3}{4}(y + 2)^2 + 5$$

**or**

$$x - 5 = -\frac{3}{4}(y + 2)^2$$

**or**

$$-\frac{4}{3}(x - 5) = (y + 2)^2$$

$\left. \begin{array}{l} x = -\frac{3}{4}(y + 2)^2 + 5 \\ \text{or} \\ x - 5 = -\frac{3}{4}(y + 2)^2 \\ \text{or} \\ -\frac{4}{3}(x - 5) = (y + 2)^2 \end{array} \right\} \leftarrow \mathbf{1 \text{ mark}}$

3. The population of a nest of ants can multiply threefold (triple) in 8 weeks. If the population is now 12 000, how many weeks will it take for the population to reach 300 000 ants?

(Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.) **(5 marks)**

### solution

$$300\,000 = 12\,000(3)^{\frac{t}{8}} \leftarrow \frac{1}{2} \text{ mark}$$

↑      ↑      ↑

$\frac{1}{2}$  mk     $\frac{1}{2}$  mk    $\frac{1}{2}$  mk

$$25 = 3^{\frac{t}{8}}$$

$$\log 25 = \log 3^{\frac{t}{8}} \quad \leftarrow 1 \text{ mark}$$

$$\log 25 = \frac{t}{8} \log 3 \quad \leftarrow 1 \text{ mark}$$

$$t = \frac{8 \log 25}{\log 3} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$t = 23.44 \text{ weeks} \quad \leftarrow \frac{1}{2} \text{ mark}$$

} 2 marks were given for showing how to solve an exponential equation using logs correctly.

### alternate solution

$$300\,000 = 12\,000(3)^{\frac{t}{8}} \leftarrow \frac{1}{2} \text{ mark}$$

↑      ↑      ↑

$\frac{1}{2}$  mk     $\frac{1}{2}$  mk    $\frac{1}{2}$  mk

$$25 = 3^{\frac{t}{8}}$$

$$\log_3 25 = \frac{t}{8} \quad \leftarrow 1 \frac{1}{2} \text{ marks}$$

$$8 \log_3 25 = t \quad \leftarrow \frac{1}{2} \text{ mark}$$

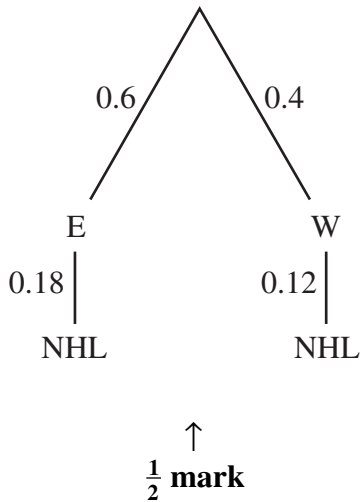
$$8 \frac{\log 25}{\log 3} = t \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$23.44 = t \quad \leftarrow \frac{1}{2} \text{ mark}$$



4. In the Canadian Junior Hockey League, 60% of the players are from Eastern Canada and 40% are from Western Canada. From this league, 18% of the Eastern players and 12% of the Western players go on to play in the NHL. If a randomly chosen NHL player who came from the Canadian Junior Hockey League is selected, what is the probability that he is from Western Canada? **(4 marks)**

 **solution**



$$P(W | \text{NHL}) = \frac{P(W \text{ and NHL})}{P(\text{NHL})} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$= \frac{(0.4)(0.12)}{(0.4)(0.12) + (0.6)(0.18)} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$\frac{1}{2}$  mark     $\frac{1}{2}$  mark     $\frac{1}{2}$  mark

$$= 0.3077$$

$$= 0.31 \quad \leftarrow 1 \text{ mark}$$

**Note:** Deduct  $\frac{1}{2}$  mark for rounding error.



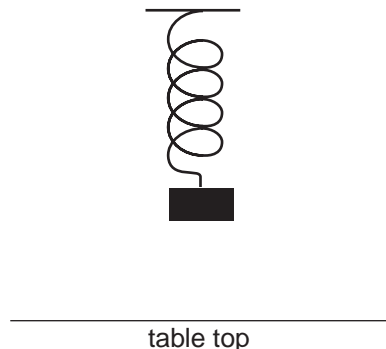
b) Use the normal approximation to the binomial distribution to obtain an estimate of this probability. (Answer accurate to at least 4 decimal places.) **(2 marks)**

 **solution**

$$\left. \begin{aligned} \mu &= np = 48(0.25) = 12 \\ \sigma &= \sqrt{npq} = \sqrt{48(.25)(.75)} = 3 \end{aligned} \right\} \leftarrow \frac{1}{2} \text{ mark}$$
$$\underbrace{\text{normalcdf}(9.5, 13.5, 12, 3)}_{\substack{\uparrow \\ \frac{1}{2} \text{ mark}}} = 0.4891 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$\frac{1}{2}$  mark continuity correction

6. A mass is supported by a spring so that it rests 50 cm above a table top, as shown in the diagram below. The mass is pulled down to a height of 20 cm above the table top and released at time  $t = 0$ . It takes 0.8 seconds for the mass to reach a maximum height of 80 cm above the table top. As the mass moves up and down, its height  $h$ , in cm, above the table top, is approximated by a sinusoidal function of the elapsed time  $t$ , in seconds, for a short period of time.



Determine an equation for a sinusoidal function that gives  $h$  as a function of  $t$ .

**(4 marks)**

**π solution**

$$h = -30 \cos \frac{2\pi}{1.6} t + 50$$

1 mk ↓
1 mk ↓
1 mark ↑

**1 mark for proper phase shift expression ( $t = t - 0$ )**

**OR**

$$h = 30 \sin \frac{2\pi}{1.6} (t - 0.4) + 50$$

**OR**

$$h = 30 \cos \frac{2\pi}{1.6} (t - 0.8) + 50$$

**OR**

$$h = -30 \sin \frac{2\pi}{1.6} (t - 1.2) + 50$$

**OR**

$$h = -30 \sin \frac{2\pi}{1.6} (t + 0.4) + 50$$

7. Prove the identity:

(4 marks)

$$\csc \theta \sin 2\theta - \sec \theta \cos 2\theta = \sec \theta$$

 solution

LEFT SIDE	RIGHT SIDE
<p style="text-align: center;">both for <math>\frac{1}{2}</math> mark</p> $\frac{1}{\sin \theta} \cdot 2 \sin \theta \cos \theta - \frac{1}{\cos \theta} (2 \cos^2 \theta - 1)$ <p style="text-align: center;"><math>\frac{1}{2}</math> mk      <math>\frac{1}{2}</math> mark</p> <p><math>1\frac{1}{2}</math> marks <math>\rightarrow</math> <math>2 \cos \theta - 2 \cos \theta + \frac{1}{\cos \theta}</math></p> <p><b>1 mark</b> <math>\rightarrow</math> <math>\frac{1}{\cos \theta}</math></p> <p style="text-align: center;"><math>\sec \theta</math></p>	<p style="text-align: center;"><math>\sec \theta</math></p> <p style="text-align: center;">LS = RS</p>

**Note:** Deduct  $\frac{1}{2}$  mark if LS and RS are not identical.

7. Prove the identity:

(4 marks)

$$\csc \theta \sin 2\theta - \sec \theta \cos 2\theta = \sec \theta$$

 **alternate solution 1**

LEFT SIDE	RIGHT SIDE
<p>both for <math>\frac{1}{2}</math> mark</p> $\frac{1}{\sin \theta} \cdot 2 \sin \theta \cos \theta - \frac{1}{\cos \theta} (\cos^2 \theta - \sin^2 \theta)$ <p style="text-align: center;"> <math>\frac{1}{2}</math> mark      <math>\frac{1}{2}</math> mark            ↓                    ↓         </p>	$\sec \theta$
<p><math>1\frac{1}{2}</math> marks →</p> $2 \cos \theta - \cos \theta + \frac{\sin^2 \theta}{\cos \theta}$	
$\cos \theta + \frac{\sin^2 \theta}{\cos \theta}$	
<p><math>\frac{1}{2}</math> mark →</p> $\frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta}$	
<p><math>\frac{1}{2}</math> mark →</p> $\frac{1}{\cos \theta}$	
$\sec \theta$	
	<p>LS = RS</p>

**Note:** Deduct  $\frac{1}{2}$  mark if LS and RS are not identical.

7. Prove the identity:

(4 marks)

$$\csc \theta \sin 2\theta - \sec \theta \cos 2\theta = \sec \theta$$

 alternate solution 2

LEFT SIDE	RIGHT SIDE
<p style="margin-left: 100px;">both for <math>\frac{1}{2}</math> mark</p> $\frac{1}{\sin \theta} \cdot 2 \sin \theta \cos \theta - \frac{1}{\cos \theta} (1 - 2 \sin^2 \theta)$ <p style="margin-left: 100px;"> <math>\frac{1}{2}</math> mark                      <math>\frac{1}{2}</math> mark  ↓                                      ↓ </p>	$\sec \theta$
$1\frac{1}{2}$ marks $\rightarrow$ $2 \cos \theta - \frac{1}{\cos \theta} + \frac{2 \sin^2 \theta}{\cos \theta}$	
$\frac{1}{2}$ mark $\rightarrow$ $\frac{2 \cos^2 \theta - 1 + 2 \sin^2 \theta}{\cos \theta}$	
	$\frac{2(\cos^2 \theta + \sin^2 \theta) - 1}{\cos \theta}$
	$\frac{2 - 1}{\cos \theta}$
$\frac{1}{2}$ mark $\rightarrow$ $\frac{1}{\cos \theta}$	
	$\sec \theta$
	LS = RS

**Note:** Deduct  $\frac{1}{2}$  mark if LS and RS are not identical.

7. Prove the identity:

(4 marks)

$$\csc \theta \sin 2\theta - \sec \theta \cos 2\theta = \sec \theta$$

 alternate solution 3

LEFT SIDE	RIGHT SIDE
$= \frac{\sin 2\theta}{\sin \theta} - \frac{\cos 2\theta}{\cos \theta}$ ← $\frac{1}{2}$ mark	$\sec \theta$
$= \frac{\sin 2\theta \cos \theta - \sin \theta \cos 2\theta}{\sin \theta \cos \theta}$ ← $\frac{1}{2}$ mark	
$= \frac{\sin(2\theta - \theta)}{\sin \theta \cos \theta}$ ← $1\frac{1}{2}$ marks	
$= \frac{\sin \theta}{\sin \theta \cos \theta}$ ← 1 mark	
$= \frac{1}{\cos \theta}$ ← $\frac{1}{2}$ mark	
$= \sec \theta$	
	LS = RS

**Note:** Deduct  $\frac{1}{2}$  mark if LS and RS are not identical.

END OF KEY