Principles of Mathematics 12

April 2004 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		Sub-Organizers
1. Problem Solving	A	Problem Solving and Cross Topic Problems
2. Patterns and Relations	B	Geometric Sequences and Series
	C/D	Logarithms and Exponents
	C/D	Trigonometry
3. Shape and Space	E	Conics
	F	Transformations
4. Statistics and Probability	G	Combinatorics
	G	Probability
	G	Statistics

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	A	U	1.5	2	C3	21.	D	H	1.5	2	D3
2.	D	K	1.5	2	D5	22.	C	K	1.5	3	E1
3.	B	U	1.5	2	C4	23.	B	U	1.5	3	E2
4.	A	U	1.5	2	D6	24.	D	H	1.5	3	E2
5.	D	U	1.5	2	C3	25.	B	U	1.5	3	F5
6.	B	U	1.5	2	C5	26.	A	U	1.5	3	F2
7.	A	U	1.5	2	C7, C5	27.	A	U	1.5	3	F6
8.	B	U	1.5	2	C8	28.	C	H	1.5	3	F1, F3; A2
9.	A	H	1.5	2	C4; B1; A2	29.	B	U	1.5	4	G7, G8
10.	C	K	1.5	2	B1	30.	B	U	1.5	4	G6
11.	C	U	1.5	2	B1	31.	D	U	1.5	4	G8
12.	C	U	1.5	2	B1	32.	C	U	1.5	4	G11
13.	C	U	1.5	2	B1, B2	33.	C	U	1.5	4	G12
14.	B	H	1.5	2	B1; C2; A2	34.	B	H	1.5	4	G11
15.	D	K	1.5	2	D2	35.	A	U	1.5	4	G3
16.	B	U	1.5	2	C1	36.	A	H	1.5	4	G2
17.	D	U	1.5	2	D3	37.	C	U	1.5	4	G1
18.	D	U	1.5	2	D1	38.	C	U	1.5	4	G2
19.	D	U	1.5	2	D4	39.	A	K	1.5	4	G1
20.	C	U	1.5	2	C2	40.	B	H	1.5	4	G2

Multiple Choice = 60 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1.	1	U	5	2	C2
2.	2	U	4	2	E3; F1; A2
3.	3	U	5	3	F3
4a.	4	U	2	2	D7
4b.	5	U	2	2	D7
5.	6	U	4	4	G6
6a.	7	H	3	4	G12
6b.	8	H	1	4	G12
7.	9	H	4	2	C7

Written Response = 30 marks

Multiple Choice = 60 (40 questions)
Written Response = 30 (7 questions)
EXAMINATION TOTAL = 90 marks

LEGEND:

Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
CO = Curriculum Organizer
PLO = Prescribed Learning Outcome

1. Solve algebraically: $2\log_4 x - \log_4(x+3) = 1$ **(5 marks)**

solution

$$\log_4 x^2 - \log_4(x+3) = 1 \quad \leftarrow \textbf{1 mark}$$

$$\log_4 \frac{x^2}{x+3} = 1 \quad \leftarrow \textbf{1 mark}$$

$$4 = \frac{x^2}{x+3} \quad \leftarrow \textbf{1 mark}$$

$$4x + 12 = x^2$$

$$0 = x^2 - 4x - 12$$

$$0 = (x-6)(x+2)$$

$$\textbf{1 mark} \rightarrow \quad x = 6, \qquad x = -2$$

$$\Downarrow$$

reject $\leftarrow$ **1 mark**

$$\therefore\ x = 6$$

2. The circle with equation $x^2+6x+y^2+2y=0$ is translated 2 units to the right to form a new circle. Determine the equation of the new circle and change to standard form. **(4 marks)**

solution

$$x^2+6x+y^2+2y=0$$

$$\underbrace{(x-2)^2}_{\substack{\uparrow\\ \frac{1}{2}\ \textbf{mark}}}+6\underbrace{(x-2)}_{\substack{\uparrow\\ \frac{1}{2}\ \textbf{mark}}}+y^2+2y=0$$

$$\left(x^2-4x+4\right)+(6x-12)+y^2+2y=0 \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$x^2+2x+y^2+2y-8=0 \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$\underbrace{\left(x^2+2x+1\right)+\left(y^2+2y+1\right)}_{\substack{\uparrow\\ \frac{1}{2}\ \textbf{mark}}}=\underbrace{8+1+1}_{\substack{\uparrow\\ \frac{1}{2}\ \textbf{mark}}}$$

$$\underbrace{(x+1)^2+(y+1)^2}_{\substack{\uparrow\\ \frac{1}{2}\ \textbf{mark}}}=\underbrace{10}_{\substack{\uparrow\\ \frac{1}{2}\ \textbf{mark}}}$$

alternate solution

$$x^2+6x+y^2+2y=0$$

$$x^2+6x+\underset{\substack{\uparrow\\ \frac{1}{2}\ \textbf{mark}}}{9}+y^2+2y+\underset{\substack{\uparrow\\ \frac{1}{2}\ \textbf{mark}}}{1}=9+1 \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$(x\underset{\substack{\uparrow\\ \frac{1}{2}\ \textbf{mark}}}{+}3)^2+(y\underset{\substack{\uparrow\\ \frac{1}{2}\ \textbf{mark}}}{+}1)^2=\underset{\substack{\uparrow\\ \frac{1}{2}\ \textbf{mark}}}{10}$$

translate 2 units right ⇒ $(x\underset{\substack{\uparrow\\ \textbf{1 mark}}}{+}1)^2+(y+1)^2=10$

3. Given the function $f(x)=\dfrac{3x}{x+1}$, determine the equation of the inverse function $f^{-1}(x)$.

(**Note:** Write the inverse function in the form $f^{-1}(x)=$ “ ”.) **(5 marks)**

solution

$$f(x)=\frac{3x}{x+1}$$

$$y=\frac{3x}{x+1}$$

$$x=\frac{3y}{y+1} \qquad \leftarrow \textbf{1 mark}$$

$$x(y+1) \text{ or } xy+x=3y \qquad \leftarrow \textbf{1 mark}$$

$$x=3y-xy \qquad \leftarrow \textbf{1 mark}$$

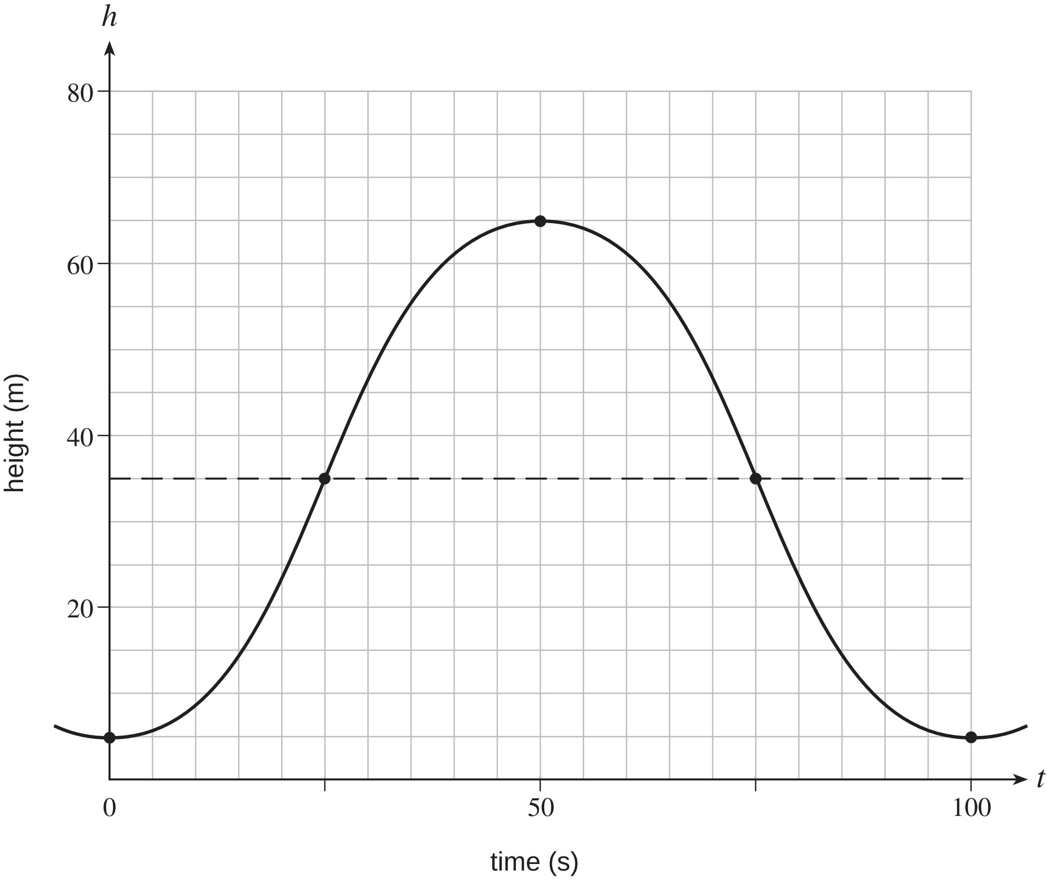
$$x=(3-x)y \qquad \leftarrow \textbf{1 mark}$$

$$\left.\begin{array}{c} \dfrac{x}{3-x}=y \\ \textbf{or} \\ f^{-1}(x)=\dfrac{x}{3-x} \\ \textbf{or} \\ f^{-1}(x)=\dfrac{-x}{x-3} \end{array}\right\} \leftarrow \textbf{1 mark}$$

4. A Ferris wheel with a radius of 30 m rotates once every 100 s. At time $t = 0$ s, passengers get on at the lowest point of its rotation which is 5 m above the ground.

   a) Using the grid below, graph how the height h of a passenger varies with respect to the elapsed time t during at least one rotation of the Ferris wheel. Clearly show at least 5 points on your graph and indicate the scale on the vertical axis. **(2 marks)**

solution



1 mark for general shape
1 mark for accuracy of 5 points

b) Determine a sine function that gives the passenger’s height h metres, above the ground as a function of time t seconds. **(2 marks)**

solution

$$h = \underset{\substack{\uparrow\\ \frac{1}{2}\,\mathbf{mk}}}{30}\sin \underset{\substack{\uparrow\\ \frac{1}{2}\,\mathbf{mk}}}{\frac{2\pi}{100}}\left(t - \underset{\substack{\uparrow\\ \frac{1}{2}\,\mathbf{mk}}}{25}\right) + \underset{\substack{\uparrow\\ \frac{1}{2}\,\mathbf{mk}}}{35}$$

alternate solution

$$h = -30\sin\frac{2\pi}{100}(t - 75) + 35$$

$$h = -30\sin\frac{2\pi}{100}(t + 25) + 35$$

Note: 1 mark for correct cosine function

5. Solve algebraically using factorial notation: ${}_nP_2 = 90$ **(4 marks)**

solution

$${}_nP_2 = 90$$

$$\frac{n!}{(n-2)!} = 90 \qquad \leftarrow \textbf{1 mark}$$

$$\frac{n(n-1)(n-2)!}{(n-2)!} = 90$$

$$n^2 - n = 90 \qquad \leftarrow \textbf{1 mark}$$

$$n^2 - n - 90 = 0$$

$$(n-10)(n+9) = 0$$

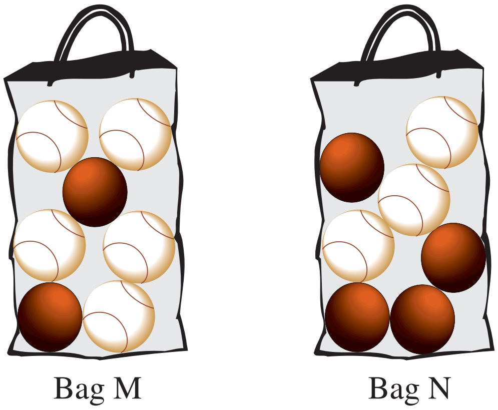
$$\textbf{1 mark} \rightarrow \quad n = 10 \qquad n = -9$$

$$\Downarrow$$

$$\text{reject} \qquad \leftarrow \textbf{1 mark}$$

$$\therefore\ n = 10$$

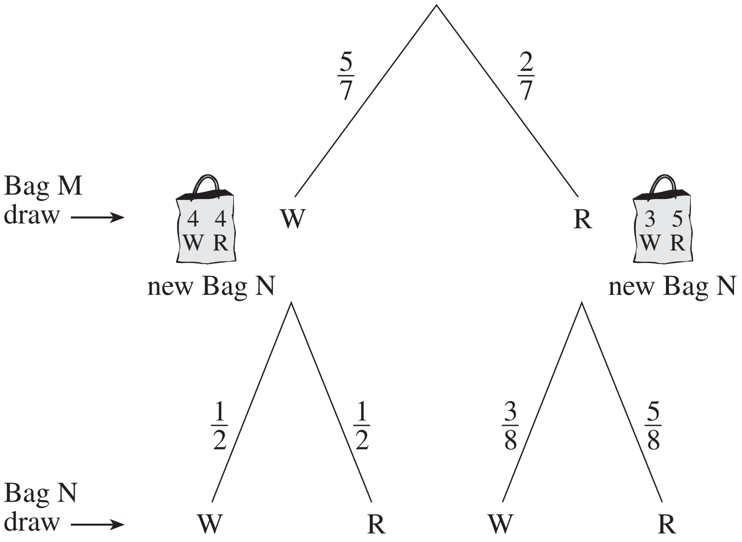
6. Bag M contains 5 white balls and 2 red balls. Bag N contains 3 white balls and 4 red balls.



a) A ball is randomly selected from Bag M and placed in Bag N. A ball is then randomly selected from Bag N. What is the probability that the ball selected from Bag N is white?

(3 marks)

solution



$$P\left(\mathrm{W}_{\text{bag N}}\right)=\underbrace{\left(\frac{5}{7}\right)\left(\frac{1}{2}\right)+\left(\frac{2}{7}\right)\left(\frac{3}{8}\right)}_{\substack{\uparrow\\ \textbf{3 marks}}}=\underbrace{\frac{13}{28}\ \text{ or }\ 0.46}_{\substack{\uparrow\\ \frac{1}{2}\ \textbf{mark}\\ \underline{\textbf{deducted}}\ \textbf{if wrong}}}$$

b) If a white ball is selected from Bag N, what is the probability that a red ball was transferred from Bag M to Bag N? **(1 mark)**

solution

$$
\begin{aligned}
P\left(\mathrm{R}_{\text{bag M}} \,\middle|\, \mathrm{W}_{\text{bag N}}\right) &= \frac{P\left(\mathrm{R}_{\text{bag M}} \text{ and } \mathrm{W}_{\text{bag N}}\right)}{P\left(\mathrm{W}_{\text{bag N}}\right)} \\
&= \frac{\frac{2}{7}\times\frac{3}{8}}{\left(\frac{5}{7}\right)\left(\frac{1}{2}\right) + \left(\frac{2}{7}\right)\left(\frac{3}{8}\right)} && \begin{array}{l} \leftarrow \tfrac{1}{2}\ \textbf{mark} \\ \leftarrow \tfrac{1}{2}\ \textbf{mark} \end{array} \\
&= \frac{3}{13} \quad \text{or} \quad 0.23 && \leftarrow \tfrac{1}{2}\ \textbf{mark}\ \underline{\textbf{deducted}}\ \textbf{if answer not}\ \mathbf{0 \le p \le 1}
\end{aligned}
$$

7. Prove the identity: **(4 marks)**

$$\tan\theta\cos 2\theta + \tan\theta \;=\; \sin 2\theta$$

solution

LEFT SIDE	RIGHT SIDE
1 mark → $= \tan\theta\left(\cos 2\theta + 1\right)$	$\sin 2\theta$
$= \overset{\mathbf{\frac{1}{2}\ mark}}{\overset{\downarrow}{\frac{\sin\theta}{\cos\theta}}}\left(2\cos^2 \overset{\mathbf{1\ mark}}{\overset{\downarrow}{\theta - 1}} + 1\right)$	
½ mark → $= \frac{\sin\theta}{\cos\theta}\cdot 2\cos^2\theta$	
½ mark → $= 2\sin\theta\cos\theta$	
½ mark → $= \sin 2\theta$	

LS = RS

7. Prove the identity: **(4 marks)**

$$\tan\theta\cos 2\theta + \tan\theta \;=\; \sin 2\theta$$

alternate solution 1

LEFT SIDE	RIGHT SIDE
$= \dfrac{\sin\theta}{\cos\theta}\overset{\frac{1}{2}\textbf{ mark}}{\overset{\downarrow}{\left(\cos^2\theta-\sin^2\theta\right)}}+\overset{\frac{1}{2}\textbf{ mark}}{\overset{\downarrow}{\dfrac{\sin\theta}{\cos\theta}}}$	$\sin 2\theta$
$\textbf{1 mark} \rightarrow \quad = \dfrac{\sin\theta}{\cos\theta}\left(\cos^2\theta-\sin^2\theta+1\right)$	
$\frac{1}{2}\textbf{ mark} \rightarrow \quad = \dfrac{\sin\theta}{\cos\theta}\left(\cos^2\theta+\cos^2\theta\right)$	
$\frac{1}{2}\textbf{ mark} \rightarrow \quad = \dfrac{\sin\theta}{\cos\theta}\left(2\cos^2\theta\right)$	
$\frac{1}{2}\textbf{ mark} \rightarrow \quad = 2\sin\theta\cos\theta$	
$\frac{1}{2}\textbf{ mark} \rightarrow \quad = \sin 2\theta$	

LS = RS

7. Prove the identity: **(4 marks)**

$$\tan\theta\cos 2\theta + \tan\theta \;=\; \sin 2\theta$$

alternate solution 2

LEFT SIDE	RIGHT SIDE
$= \dfrac{\sin\theta}{\cos\theta}\overset{\frac{1}{2}\textbf{ mark}}{\overset{\downarrow}{\left(1-2\sin^2\theta\right)}} + \overset{\frac{1}{2}\textbf{ mark}}{\overset{\downarrow}{\dfrac{\sin\theta}{\cos\theta}}}$	$\sin 2\theta$
1 mark → $= \dfrac{\sin\theta}{\cos\theta}\left(1-2\sin^2\theta+1\right)$	
$\frac{1}{2}$ **mark** → $= \dfrac{\sin\theta}{\cos\theta}\left(2-2\sin^2\theta\right)$	
$= \dfrac{2\sin\theta}{\cos\theta}\left(1-\sin^2\theta\right)$	
$\frac{1}{2}$ **mark** → $= \dfrac{2\sin\theta}{\cos\theta}\left(\cos^2\theta\right)$	
$\frac{1}{2}$ **mark** → $= 2\sin\theta\cos\theta$	
$\frac{1}{2}$ **mark** → $= \sin 2\theta$	

LS = RS

END OF KEY