

Principles of Mathematics 12

August 2003 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		Sub-Organizers
1. Problem Solving	A	Problem Solving and Cross Topic Problems
2. Patterns and Relations	B	Geometric Sequences and Series
	C/D	Logarithms and Exponents
	C/D	Trigonometry
3. Shape and Space	E	Conics
	F	Transformations
4. Statistics and Probability	G	Combinatorics
	G	Probability
	G	Statistics

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	C	U	1.5	2	C3	23.	B	H	1.5	2	C2
2.	D	K	1.5	2	D6	24.	D	K	1.5	3	E2
3.	A	U	1.5	2	C4	25.	D	H	1.5	3	E2
4.	B	U	1.5	2	D6	26.	C	U	1.5	3	E3
5.	D	U	1.5	2	C8	27.	A	U	1.5	3	E2
6.	B	U	1.5	2	D6	28.	D	K	1.5	3	F1
7.	B	U	1.5	2	C5	29.	D	K	1.5	3	F2
8.	B	H	1.5	2	D7	30.	A	U	1.5	3	F3
9.	D	H	1.5	2	D5	31.	A	U	1.5	3	F6
10.	D	H	1.5	2	C5	32.	D	U	1.5	3	F6
11.	B	U	1.5	2	B1	33.	B	H	1.5	3	F6
12.	B	U	1.5	2	B1	34.	A	H	1.5	4	G5
13.	C	U	1.5	2	B1	35.	B	U	1.5	4	G8
14.	B	U	1.5	2	B3	36.	C	U	1.5	4	G7
15.	A	H	1.5	2	B1, A3	37.	C	U	1.5	4	G11
16.	D	K	1.5	2	D2	38.	C	U	1.5	4	G11
17.	A	U	1.5	2	D3	39.	B	H	1.5	4	G12
18.	A	U	1.5	2	C2	40.	C	U	1.5	4	G9
19.	B	U	1.5	2	C1	41.	C	U	1.5	4	G8
20.	D	U	1.5	2	C2	42.	C	K	1.5	4	G2
21.	D	U	1.5	2	D1	43.	B	U	1.5	4	G2
22.	B	H	1.5	2	D1	44.	C	H	1.5	4	G2

Multiple Choice = 66 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1a.	1	U	2	3	F4
1b.	2	U	2	3	F2, F5
2.	3	U	4	3	E2
3.	4	U	5	2	D1
4a.	5	U	2	4	G7
4b.	6	U	2	4	G6
5a.	7	U	2	4	G13
5b.	8	U	2	4	G13
6a.	9	U	2	4	G3, G8
6b.	10	U	2	4	G3, G8
7a.	11	U	3	2	C6
7b.	12	U	1	2	C6
8.	13	H	5	2	C7

Written Response = 34 marks

Multiple Choice = 66 (44 questions)

Written Response = 34 (8 questions)

EXAMINATION TOTAL = 100 marks

LEGEND:

Q = Question Number

B = Score Box Number

PLO = Prescribed Learning Outcome

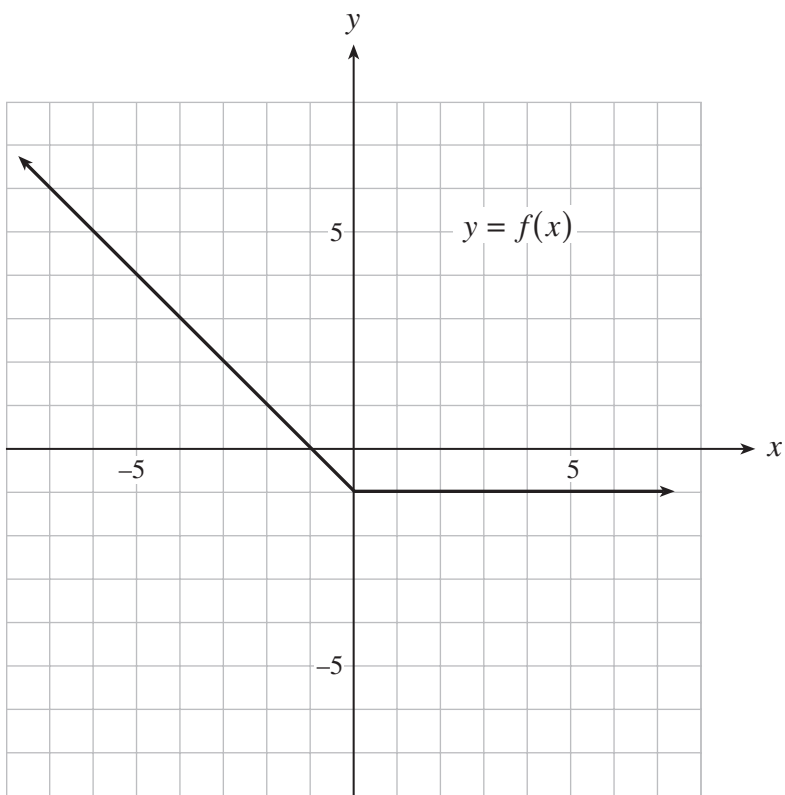
K = Keyed Response

S = Score

C = Cognitive Level

CO = Curriculum Organizer

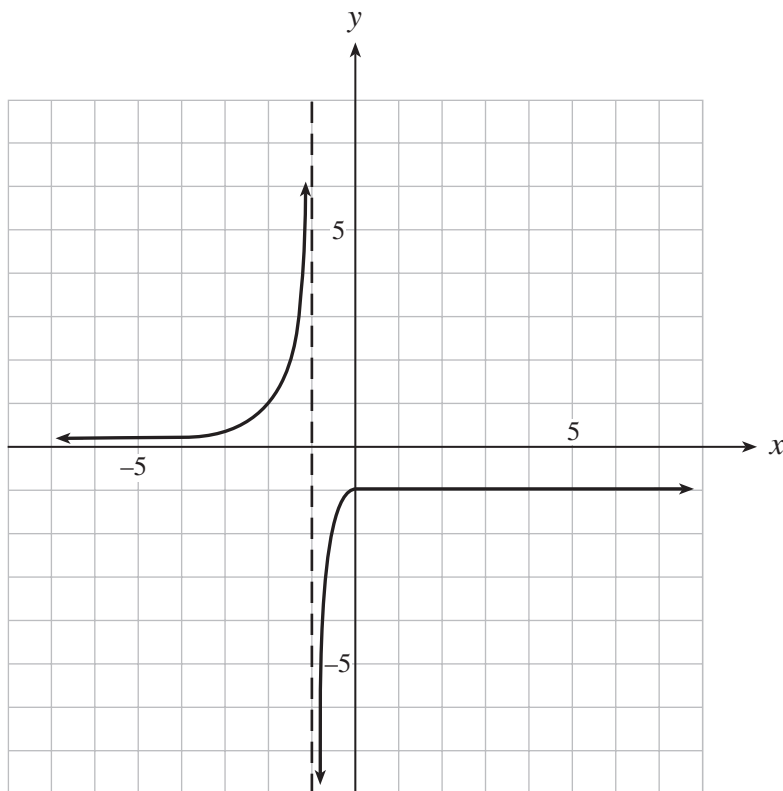
1. The graph of $y = f(x)$ is shown below.



a) On the grid provided, sketch the graph of $y = \frac{1}{f(x)}$.

(2 marks)

 solution



$\frac{1}{2}$ mark for $(-2, 1)$ and shape

$\frac{1}{2}$ mark for $(0, -1)$ and shape to left

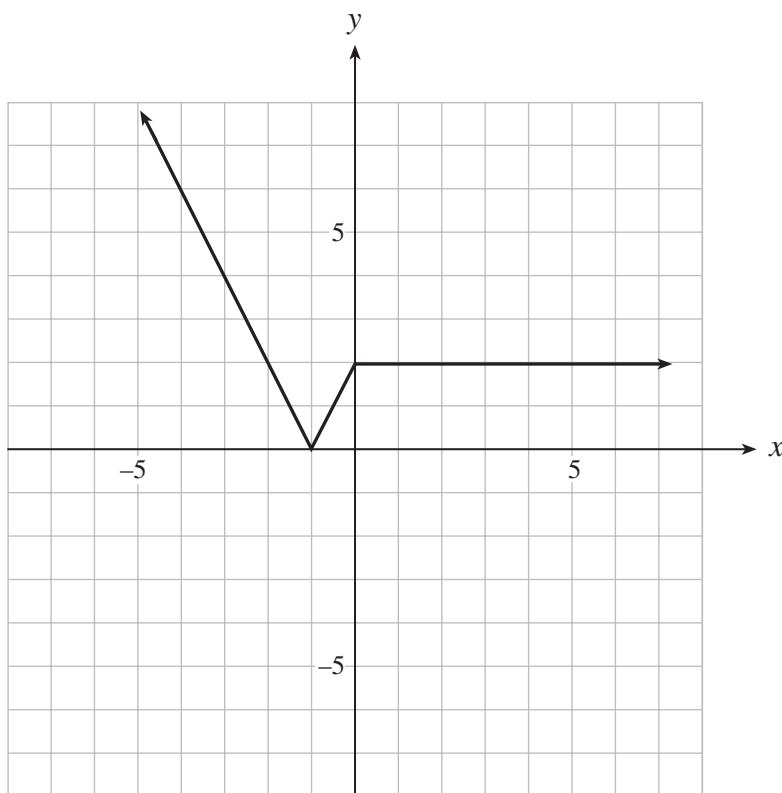
$\frac{1}{2}$ mark for horizontal portion of graph

$\frac{1}{2}$ mark for asymptotic behaviour of graph

b) On the grid provided, sketch the graph of $y = 2|f(x)|$.

(2 marks)

 solution



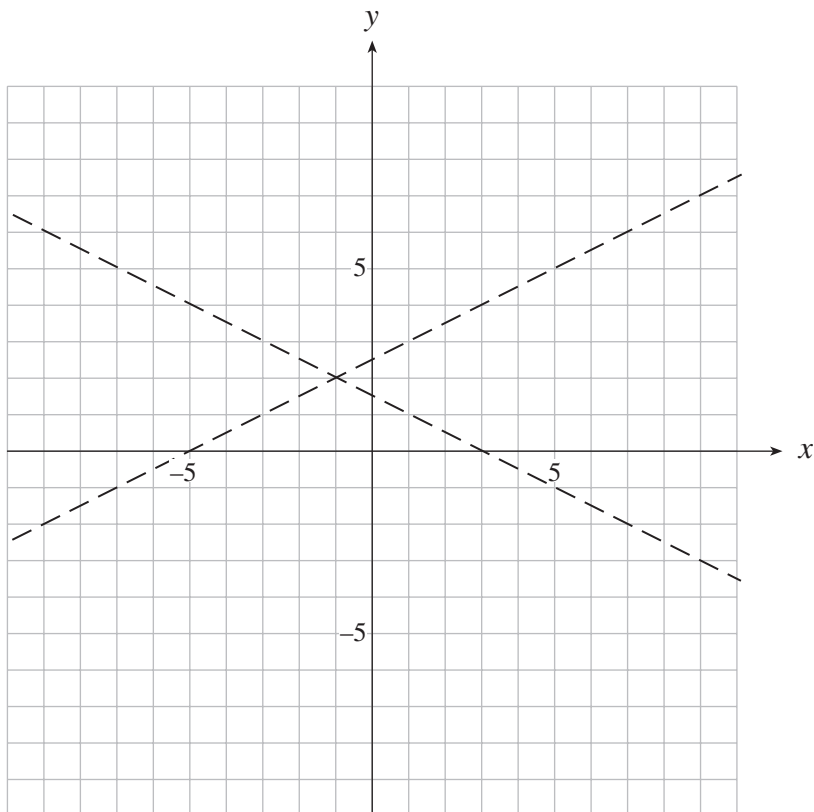
1 mark for absolute value

1 mark for vertical expansion

Note: Arrowheads on solution are not necessary.

2. The asymptotes of a hyperbola are shown below. Determine an equation of the hyperbola if the transverse axis is horizontal and has a length of 8. **(4 marks)**

(Note: The grid is provided for rough work only.)



π solution

$$\begin{array}{ccc}
 \frac{1}{2} \text{ mark} & & \frac{1}{2} \text{ mark} \\
 \downarrow & & \downarrow \\
 \frac{(x+1)^2}{16} - \frac{(y-2)^2}{4} = 1 & \leftarrow & \mathbf{1 \text{ mark}} \text{ for equation format} \\
 \uparrow & & \uparrow \\
 \mathbf{1 \text{ mark}} & & \mathbf{1 \text{ mark}}
 \end{array}$$

Note: A detailed graph of the hyperbola is not required.
 The equation does not have to be in standard form to receive full marks.

3. Malcolm bought a new car for \$24 000. Every year it will depreciate in value by 8%. How long will it take for the car to be worth \$16 000? **(5 marks)**

(Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.)

solution

$$\frac{1}{2} \text{ mark} \quad 1 \text{ mark} \quad \frac{1}{2} \text{ mark}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$24\,000(1 - .08)^t = 16\,000$$

$$0.92^t = \frac{16\,000}{24\,000} = \frac{2}{3} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\log 0.92^t = \log \frac{2}{3} \quad \leftarrow 1 \text{ mark}$$

$$t \log 0.92 = \log \frac{2}{3} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$t = \frac{\log \frac{2}{3}}{\log 0.92} \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$t = 4.86 \text{ years} \quad \leftarrow \frac{1}{2} \text{ mark}$$

4. a) How many groups of 3 chairs can be chosen from 7 chairs if the chairs are all different colours?
(2 marks)

 solution

1 mark 1 mark

↓ OR ↓

$${}^7C_3 = \frac{7!}{3!4!}$$

$$= 35 \quad \leftarrow \text{1 mark}$$

b) How many different ways can 7 chairs be arranged in a row if 2 of the chairs are blue, 3 are yellow, 1 is red and 1 is green? (Assume that all of the chairs are identical except for colour.)
(2 marks)

 solution

$\frac{1}{2}$ mark

↓

$$\frac{7!}{2!3!} = 420 \quad \leftarrow \frac{1}{2} \text{ mark}$$

↑ ↑

$\frac{1}{2}$ mark each

5. A hand of five cards is dealt from a standard deck of 52 cards.

a) What is the probability that the hand contains exactly 1 club?

(2 marks)

 solution

$$P(1 \text{ club}) = \frac{\overbrace{({}_{13}C_1)}^{\frac{1}{2} \text{ mark}} \overbrace{({}_{39}C_4)}^{\frac{1}{2} \text{ mark}}}{\underset{\uparrow \frac{1}{2} \text{ mark}}{52}C_5} \approx 0.4114195678 \approx 0.41 \leftarrow \frac{1}{2} \text{ mark}$$

b) What is the probability that the hand contains at most 1 club?

(2 marks)

 solution

$$P(\text{at most 1 club}) = P(0 \text{ clubs}) + P(1 \text{ club})$$

$$\begin{aligned} & \frac{1}{2} \text{ mark} \quad \frac{1}{2} \text{ mark} \quad \leftarrow \left(\text{this } \frac{1}{2} \text{ mark is for } \textit{adding} \right. \\ & \quad \downarrow \quad \downarrow \quad \left. \text{the answer from part a) } \right) \\ & = \frac{{}_{39}C_5}{{}_{52}C_5} + \frac{({}_{13}C_1)({}_{39}C_4)}{{}_{52}C_5} \\ \frac{1}{2} \text{ mark} \rightarrow & \approx 0.6329531813 \\ & \approx 0.63 \leftarrow \frac{1}{2} \text{ mark} \end{aligned}$$

6. In a large city in BC the probability that a car has air conditioning is 0.72. If 200 cars are randomly selected, determine the probability that between 130 and 132 cars inclusive have air conditioning by using the following methods.

- a) Use the binomial distribution to obtain this probability.
(Answer accurate to at least 4 decimal places.)

(2 marks)

solution

$$p = 0.72 \quad n = 200 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\begin{aligned} P(130 \leq X \leq 132) &= \text{binomcdf}(200, 0.72, 132) - \text{binomcdf}(200, 0.72, 129) \quad \leftarrow 1\frac{1}{2} \text{ marks} \\ &= 0.0244056415 \\ &\approx 0.0244 \end{aligned}$$

OR

$$p = 0.72 \quad n = 200 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\begin{aligned} P(130 \leq X \leq 132) &= \text{binompdf}(200, 0.72, 130) + \text{binompdf}(200, 0.72, 131) + \text{binompdf}(200, 0.72, 132) \quad \leftarrow 1\frac{1}{2} \text{ marks} \\ &= 0.0244056408 \\ &\approx 0.0244 \end{aligned}$$

OR

$$p = 0.72 \quad n = 200 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$P(x) = {}_n C_x p^x q^{n-x} \quad \Rightarrow$$

$$\begin{aligned} P(130 \leq X \leq 132) &= {}_{200} C_{130} (0.72)^{130} (0.28)^{70} + {}_{200} C_{131} (0.72)^{131} (0.28)^{69} + {}_{200} C_{132} (0.72)^{132} (0.28)^{68} \quad \leftarrow 1\frac{1}{2} \text{ marks} \\ &= 0.0244056708 \\ &\approx 0.0244 \end{aligned}$$

Note: Due to overflow errors, answers were not marked.

b) Use the normal approximation to the binomial distribution to obtain an estimate of this probability. (Answer accurate to at least 4 decimal places.) **(2 marks)**

solution

$$\mu = np = 200(0.72) = 144 \leftarrow \frac{1}{2} \text{ mark} \quad \sigma = \sqrt{npq} = \sqrt{200(0.72)(0.28)} = \sqrt{40.32} \approx 6.3498 \leftarrow \frac{1}{2} \text{ mark}$$

$$\begin{aligned} P(130 \leq X \leq 132) &= \text{normalcdf}(129.5, 132.5, 144, \sqrt{40.32}) \leftarrow \mathbf{1 \text{ mark}} \\ &= 0.0238648031 \\ &\approx 0.0239 \end{aligned}$$

alternate solution

If z-tables are used to find the normal approximation

$$\mu = np = 200(0.72) = 144 \leftarrow \frac{1}{2} \text{ mark} \quad \sigma = \sqrt{npq} = \sqrt{200(0.72)(0.28)} = \sqrt{40.32} \approx 6.3498 \leftarrow \frac{1}{2} \text{ mark}$$

$$\begin{array}{cc} \xrightarrow{\frac{1}{2} \text{ mark}} & \\ \downarrow & \downarrow \\ z_1 = \frac{129.5 - 144}{\sqrt{40.32}} = -2.283535358 \approx -2.28 & z_2 = \frac{132.5 - 144}{\sqrt{40.32}} = -1.811079767 \approx -1.81 \\ z_1 = \frac{129.5 - 144}{\sqrt{40.32}} = -2.283535358 \approx -2.28 & z_2 = \frac{132.5 - 144}{\sqrt{40.32}} = -1.811079767 \approx -1.81 \end{array}$$

$$\left. \begin{aligned} P(130 \leq X \leq 132) &\approx P(-2.28 < Z < -1.81) \\ &= 0.0352 - 0.0113 \\ &= 0.0239 \end{aligned} \right\} \leftarrow \frac{1}{2} \text{ mark}$$

7. a) Solve algebraically, giving exact values for x , where $0 \leq x < 2\pi$.

(3 marks)

$$2 \cos^2 x - \cos x - 1 = 0$$

 **solution**

$$2 \cos^2 x - \cos x - 1 = 0$$

$$(2 \cos x + 1)(\cos x - 1) = 0 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\cos x = -\frac{1}{2} \quad \text{or} \quad \cos x = 1$$

\uparrow \uparrow
 $\frac{1}{2}$ mark $\frac{1}{2}$ mark

$$x = \frac{2\pi}{3} \quad \text{or} \quad \frac{4\pi}{3} \quad x = 0$$

\uparrow \uparrow \uparrow
 $\frac{1}{2}$ mark $\frac{1}{2}$ mark $\frac{1}{2}$ mark

b) Give the general solution for this equation.

(Solve over the set of real numbers, giving exact value solutions.)

(1 mark)

 **solution**

$$\left. \begin{array}{l} x = \frac{2\pi}{3} + 2\pi n \\ x = \frac{4\pi}{3} + 2\pi n \end{array} \right\} \leftarrow \frac{1}{2} \text{ mark}$$

$$x = 2\pi n \quad \leftarrow \frac{1}{2} \text{ mark}$$

where $n \in I$

Note: $x = \frac{2\pi n}{3}$ would generate all solutions and receive **1 mark**

It is not necessary for students to state $n \in I$

8. Prove the identity:

(5 marks)

$$\frac{\cos \theta + \cot \theta}{1 + \sin \theta} = \cot \theta$$

 solution

LEFT SIDE	RIGHT SIDE
$\frac{\cos \theta + \cot \theta}{1 + \sin \theta} \quad \checkmark \frac{1}{2} \text{ mark}$	$\cot \theta$
$= \frac{\left(\cos \theta + \frac{\cos \theta}{\sin \theta} \right) \sin \theta}{(1 + \sin \theta) \sin \theta} \quad \left. \right\} \leftarrow 1 \text{ mark}$	
$1 \frac{1}{2} \text{ marks} \rightarrow = \frac{\cos \theta \sin \theta + \cos \theta}{(1 + \sin \theta) \sin \theta}$	
$1 \text{ mark} \rightarrow = \frac{\cos \theta (\sin \theta + 1)}{(1 + \sin \theta) \sin \theta}$	
$\frac{1}{2} \text{ mark} \rightarrow = \frac{\cos \theta}{\sin \theta}$	
$\frac{1}{2} \text{ mark} \rightarrow = \cot \theta$	
<p>LS = RS</p>	

8. Prove the identity:

(5 marks)

$$\frac{\cos \theta + \cot \theta}{1 + \sin \theta} = \cot \theta$$

 alternate solution 1

LEFT SIDE	RIGHT SIDE
$\frac{\cos \theta + \cot \theta}{1 + \sin \theta}$ <p style="text-align: center;">↓ $\frac{1}{2}$ mark</p> $= \frac{\cos \theta + \frac{\cos \theta}{\sin \theta}}{1 + \sin \theta} \cdot \frac{(1 - \sin \theta)}{(1 - \sin \theta)} \left\} \leftarrow 1 \text{ mark}$	$\cot \theta$
$\frac{1}{2} \text{ mark} \rightarrow = \frac{\cos \theta - \sin \theta \cos \theta + \frac{\cos \theta}{\sin \theta} - \cos \theta}{1 - \sin^2 \theta}$	
$\frac{1}{2} \text{ mark} \rightarrow = \frac{-\sin \theta \cos \theta + \frac{\cos \theta}{\sin \theta}}{\cos^2 \theta}$	
$\frac{1}{2} \text{ mark} \rightarrow = \frac{\cos \theta \left(-\sin \theta + \frac{1}{\sin \theta} \right)}{\cos^2 \theta}$	
$\frac{1}{2} \text{ mark} \rightarrow = \left(\frac{-\sin \theta + \frac{1}{\sin \theta}}{\cos \theta} \right) \frac{\sin \theta}{\sin \theta}$	
$\frac{1}{2} \text{ mark} \rightarrow = \frac{-\sin^2 \theta + 1}{\cos \theta \sin \theta}$	
$\frac{1}{2} \text{ mark} \rightarrow = \frac{\cos^2 \theta}{\cos \theta \sin \theta}$	
$\frac{1}{2} \text{ mark} \rightarrow = \frac{\cos \theta}{\sin \theta}$	
<p>LS = RS</p>	

8. Prove the identity:

(5 marks)

$$\frac{\cos \theta + \cot \theta}{1 + \sin \theta} = \cot \theta$$

 alternate solution 2

	LEFT SIDE	RIGHT SIDE
	$\frac{\cos \theta + \cot \theta}{1 + \sin \theta}$	$\cot \theta$
3 marks →	$\left\{ \begin{aligned} &= \frac{\cos \theta + \frac{\cos \theta}{\sin \theta}}{1 + \sin \theta} \\ &= \frac{\sin \theta \cos \theta + \cos \theta}{\sin \theta (1 + \sin \theta)} \end{aligned} \right.$	
1 mark →	$= \frac{\cos \theta (1 + \sin \theta)}{\sin \theta} \cdot \frac{1}{1 + \sin \theta}$	
$\frac{1}{2}$ mark →	$= \frac{\cos \theta}{\sin \theta}$	
$\frac{1}{2}$ mark →	$= \cot \theta$	
		LS = RS

8. Prove the identity:

(5 marks)

$$\frac{\cos \theta + \cot \theta}{1 + \sin \theta} = \cot \theta$$

 **alternate solution 3**

LEFT SIDE	RIGHT SIDE
$\frac{\cos \theta + \cot \theta}{1 + \sin \theta}$	$\cot \theta$
3 marks $\rightarrow = \frac{\frac{\sin \theta}{\tan \theta} + \frac{1}{\tan \theta}}{1 + \sin \theta}$	
1 mark $\rightarrow = \frac{\sin \theta + 1}{\tan \theta} \cdot \frac{1}{1 + \sin \theta}$	
$\frac{1}{2}$ mark $\rightarrow = \frac{1}{\tan \theta}$	
$\frac{1}{2}$ mark $\rightarrow = \cot \theta$	
LS = RS	

END OF KEY