

Principles of Mathematics 12

January 2003 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		Sub-Organizers
1. Problem Solving	A	Problem Solving and Cross Topic Problems
2. Patterns and Relations	B	Geometric Sequences and Series
	C/D	Logarithms and Exponents
	C/D	Trigonometry
3. Shape and Space	E	Conics
	F	Transformations
4. Statistics and Probability	G	Combinatorics
	G	Probability
	G	Statistics

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	D	U	1.5	2	C3	23.	C	H	1.5	2	D3, A2
2.	C	U	1.5	2	D6	24.	D	K	1.5	3	E2
3.	C	U	1.5	2	C5, A9	25.	D	K	1.5	3	E2
4.	D	U	1.5	2	C4	26.	B	U	1.5	3	E2
5.	D	U	1.5	2	D6	27.	B	H	1.5	3	E2
6.	C	U	1.5	2	C4, C5	28.	A	K	1.5	3	F1
7.	B	U	1.5	2	C7	29.	A	K	1.5	3	F3
8.	A	U	1.5	2	C8	30.	B	U	1.5	3	F5
9.	B	U	1.5	2	D7	31.	B	U	1.5	3	F5
10.	D	H	1.5	2	C6	32.	C	U	1.5	3	F1, F2, F3
11.	B	U	1.5	2	B1	33.	B	H	1.5	3	F3, F1
12.	C	K	1.5	2	B1	34.	C	U	1.5	4	G4
13.	B	U	1.5	2	B1	35.	D	U	1.5	4	G6
14.	C	U	1.5	2	B1	36.	C	H	1.5	4	G8
15.	B	H	1.5	2	B1	37.	A	K	1.5	4	G9
16.	D	U	1.5	2	D4	38.	D	U	1.5	4	G11
17.	D	U	1.5	2	C1	39.	C	U	1.5	4	G8, G13
18.	C	U	1.5	2	D1	40.	C	H	1.5	4	G11
19.	D	U	1.5	2	D1	41.	D	K	1.5	4	G2
20.	A	U	1.5	2	D2	42.	A	U	1.5	4	G1
21.	A	U	1.5	2	D4	43.	A	U	1.5	4	G2
22.	A	H	1.5	2	D1	44.	A	H	1.5	4	G2

Multiple Choice = 66 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1a.	1	U	2	3	F2, F4
1b.	2	U	2	3	F2, F4
2.	3	U	4	3	E3
3.	4	U	5	2	C2
4a.	5	U	2	4	G5, G7
4b.	6	U	2	4	G5, G7
5a.	7	U	2	4	G12
5b.	8	U	2	4	G12
6a.	9	U	2	4	G2
6b.	10	U	2	4	G2
7.	11	U	4	2	C6
8.	12	H	5	2	C8

Written Response = 34 marks

Multiple Choice = 66 (44 questions)

Written Response = 34 (8 questions)

EXAMINATION TOTAL = 100 marks

LEGEND:

Q = Question Number

B = Score Box Number

PLO = Prescribed Learning Outcome

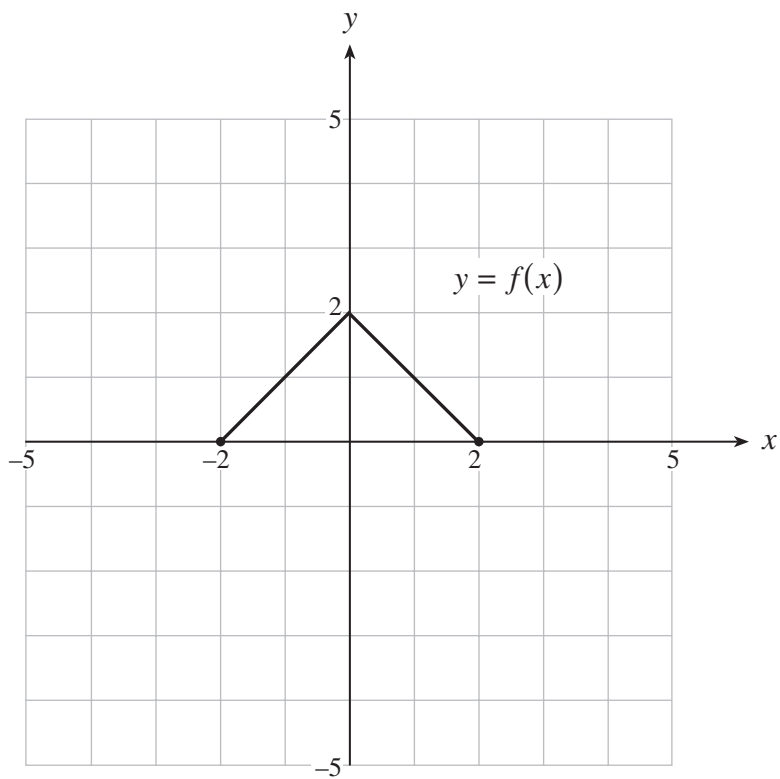
K = Keyed Response

S = Score

C = Cognitive Level

CO = Curriculum Organizer

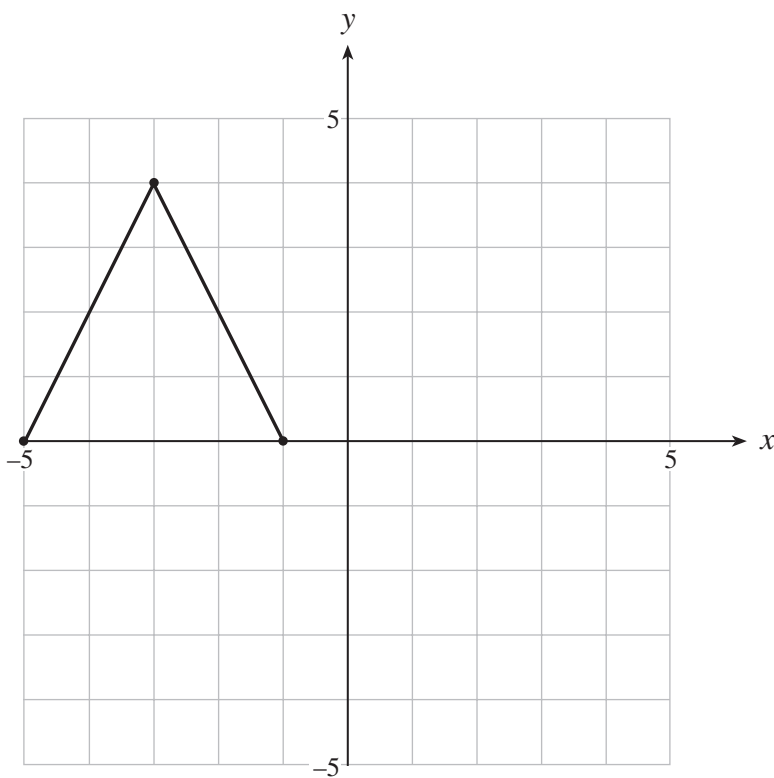
1. The graph of $y = f(x)$ is shown below.



a) On the grid provided, sketch the graph of $y = 2f(x + 3)$.

(2 marks)

 solution



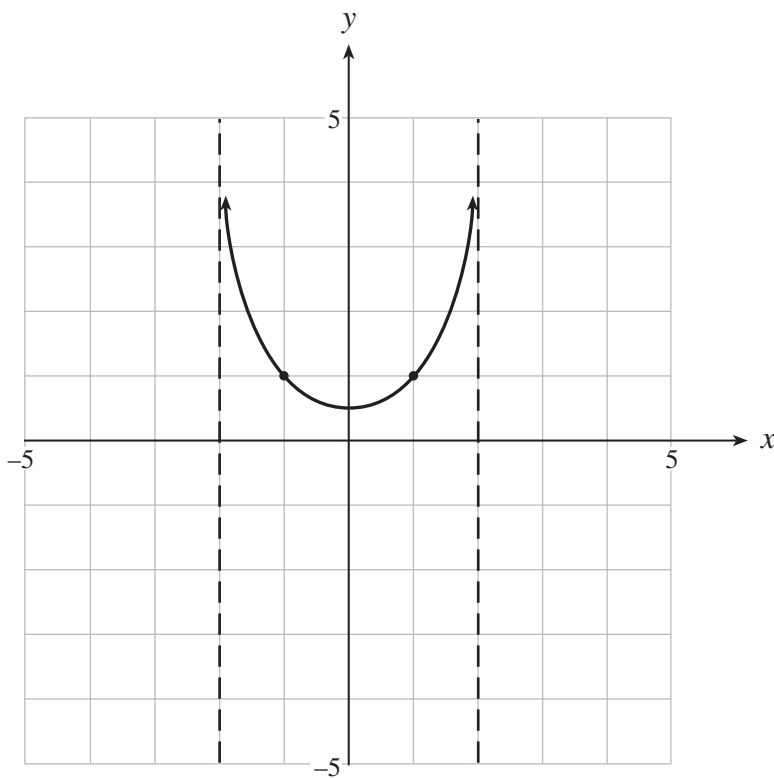
1 mark for horizontal translation

1 mark for vertical expansion

b) On the grid provided, sketch the graph of $y = \frac{1}{f(x)}$.

(2 marks)

 solution



$\frac{1}{2}$ mark for invariant points

$\frac{1}{2}$ mark for asymptotes or asymptotic behaviour of the graph

$\frac{1}{2}$ mark for point $\left(0, \frac{1}{2}\right)$
(accept any y coordinate between 0 and 1)

$\frac{1}{2}$ mark for shape

2. Change to standard form.

(4 marks)

$$2y^2 + x + 12y + 23 = 0$$

 solution

$\frac{1}{2}$ mark each

↓ ↓

$$2(y^2 + 6y) + x = -23$$

$\frac{1}{2}$ mark

↓

$$2(y^2 + 6y + 9) + x = 18 - 23 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$1 \text{ mark} \rightarrow 2(y+3)^2 + x = -5$$

$$x = -2(y+3)^2 - 5 \quad \leftarrow 1 \text{ mark}$$

or

$$x + 5 = -2(y+3)^2$$

3. Solve algebraically: $2 \log(3 - x) = \log 4 + \log(6 - x)$

(5 marks)

 solution

$$2 \log(3 - x) = \log 4 + \log(6 - x)$$

1 mark

↓

1 mark

↓

$$\log(3 - x)^2 = \log(4(6 - x))$$

$$(3 - x)^2 = 4(6 - x) \quad \leftarrow \text{1 mark for antilog}$$

$$9 - 6x + x^2 = 24 - 4x$$

$$x^2 - 2x - 15 = 0 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$(x - 5)(x + 3) = 0$$

$$x = -3, 5 \quad \leftarrow \frac{1}{2} \text{ mark}$$

↓

reject \leftarrow 1 mark

$$\therefore x = -3$$

4. A toy box contains 4 different cars and 6 different trucks.

a) In how many ways can a collection of 5 toys be chosen if the collection must consist of 2 cars and 3 trucks? **(2 marks)**

 **solution**

$\frac{1}{2}$ mark

↓

$${}_4C_2 \times {}_6C_3 = 120$$

↑ ↑ ↑

$\frac{1}{2}$ mark each

b) In how many ways can a collection of 5 toys be chosen if the collection must consist of at least 3 cars? **(2 marks)**

 **solution**

$\frac{1}{2}$ mark

↓

$${}_4C_3 \times {}_6C_2 + {}_4C_4 \times {}_6C_1 = 66$$

↑ ↑ ↑

$\frac{1}{2}$ mark

$\frac{1}{2}$ mark

$\frac{1}{2}$ mark

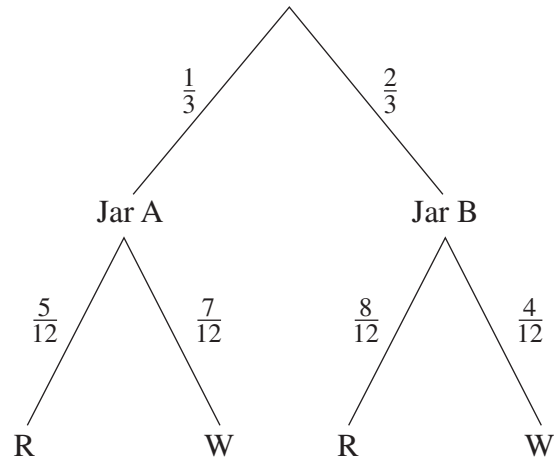
5. Jar A contains 5 red balls and 7 white balls. Jar B contains 8 red balls and 4 white balls. A fair die is rolled. If a 1 or 2 comes up, a ball is randomly selected from Jar A, otherwise, a ball is randomly selected from Jar B.

a) Find the probability that a white ball is selected.

(2 marks)

 solution

$$\begin{array}{c}
 \frac{1}{2} \text{ mark for both} \\
 \downarrow \quad \downarrow \\
 P(W) = \frac{1}{3} \times \frac{7}{12} + \frac{2}{3} \times \frac{4}{12} = \frac{15}{36} = \frac{5}{12} \\
 \uparrow \quad \uparrow \quad \uparrow \\
 \frac{1}{2} \text{ mark for both} \quad \frac{1}{2} \text{ mark} \\
 \underbrace{\hspace{10em}} \\
 \uparrow \\
 \frac{1}{2} \text{ mark for process}
 \end{array}$$



b) Given that the ball selected is white, find the probability that it came from Jar A.

(2 marks)

 solution

$$\begin{array}{c}
 \frac{1}{2} \text{ mark} \\
 \downarrow \\
 P(\text{Jar A} | W) = \frac{\frac{1}{3} \times \frac{7}{12}}{\frac{1}{3} \times \frac{7}{12} + \frac{2}{3} \times \frac{4}{12}} = \frac{\frac{7}{36}}{\frac{5}{12}} = \frac{7}{15} \\
 \uparrow \quad \uparrow \\
 \frac{1}{2} \text{ mark} \quad \frac{1}{2} \text{ mark} \\
 \underbrace{\hspace{10em}} \\
 \uparrow \\
 \frac{1}{2} \text{ mark for division} \\
 \text{(concept of conditional probability)}
 \end{array}$$

6. The life expectancy of a car tire produced by a particular plant is normally distributed with a mean of 50 000 km and a standard deviation of 3 000 km.

a) What percent of these tires lasts between 49 000 km and 52 000 km?

(2 marks)

 solution

$$\begin{array}{c}
 \frac{1}{2} \text{ mark} \quad \quad \quad \frac{1}{2} \text{ mark} \\
 \downarrow \quad \quad \quad \downarrow \\
 \underbrace{\hspace{10em}} \\
 \frac{1}{2} \text{ mark syntax} \rightarrow \text{normalcdf}(49\,000, 52\,000, 50\,000, 3\,000) \\
 = 0.3780661293 \\
 \approx 37.81\% \\
 \text{or} \\
 38\% \quad \left. \vphantom{\begin{array}{c} \approx 37.81\% \\ \text{or} \\ 38\% \end{array}} \right\} \leftarrow \frac{1}{2} \text{ mark}
 \end{array}$$

∴ 38% of the tires lasts between 49 000 km and 52 000 km.

Note: 0.38 should also be accepted for full marks.

b) If the plant makes 80 000 tires, how many tires would be expected to last more than 55 000 km? (Answer to the nearest tire.)

(2 marks)

 solution

$$\begin{array}{c}
 \frac{1}{2} \text{ mark} \quad \quad \quad \frac{1}{2} \text{ mark} \quad \quad \quad \frac{1}{2} \text{ mark} \\
 \downarrow \quad \quad \quad \downarrow \quad \quad \quad \downarrow \\
 \underbrace{\hspace{10em}} \quad \underbrace{\hspace{10em}} \quad \underbrace{\hspace{10em}} \\
 \text{normalcdf}(55\,000, 1E99, 50\,000, 3\,000) \times 80\,000 \\
 = 3823.226429 \\
 \approx 3823 \quad \leftarrow \frac{1}{2} \text{ mark}
 \end{array}$$

∴ 3 823 tires would be expected to last more than 55 000 km.

7. Solve the following equation algebraically.

(4 marks)

$$3 \cos^2 x + \cos x - 2 = 0, \quad 0 \leq x < 2\pi$$

 solution

$$\begin{array}{ccc} \frac{1}{2} \text{ mark} & \frac{1}{2} \text{ mark} & (\frac{1}{2} \text{ mark for each correct factor}) \\ \downarrow & \downarrow & \\ (3 \cos x - 2)(\cos x + 1) = 0 & & \end{array}$$

$$3 \cos x - 2 = 0 \quad \text{or} \quad \cos x + 1 = 0$$

$$\frac{1}{2} \text{ mark} \rightarrow \cos x = \frac{2}{3} \qquad \cos x = -1 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$\begin{array}{ccc} x = 0.84, 5.44 & & x = \pi \\ \uparrow \quad \uparrow & & \uparrow \\ \frac{1}{2} \text{ mark} \quad 1 \text{ mark} & & \frac{1}{2} \text{ mark} \end{array}$$

$$\begin{array}{ccc} \text{or} & \downarrow & \downarrow \\ \cos^{-1}\left(\frac{2}{3}\right) & & 2\pi - \cos^{-1}\left(\frac{2}{3}\right) \end{array}$$

$$\therefore x = 0.84, 3.14, 5.44$$

8. Prove the identity:

(5 marks)

$$(\csc \theta - \sin \theta) \tan \theta = \frac{\sin 2\theta}{2 \sin \theta}$$

 solution

LEFT SIDE	RIGHT SIDE
$(\csc \theta - \sin \theta) \tan \theta$	$\frac{\sin 2\theta}{2 \sin \theta}$
$\frac{1}{2} \text{ mark} \quad \frac{1}{2} \text{ mark}$ $\downarrow \quad \downarrow$	
$= \left(\frac{1}{\sin \theta} - \sin \theta \right) \frac{\sin \theta}{\cos \theta}$	$= \frac{2 \sin \theta \cos \theta}{2 \sin \theta} \leftarrow \frac{1}{2} \text{ mark}$
$\frac{1}{2} \text{ mark} \rightarrow = \frac{1}{\cos \theta} - \frac{\sin^2 \theta}{\cos \theta} \leftarrow \frac{1}{2} \text{ mark}$	$= \cos \theta \quad \leftarrow \frac{1}{2} \text{ mark}$
$\frac{1}{2} \text{ mark} \rightarrow = \frac{1 - \sin^2 \theta}{\cos \theta}$	
$1 \text{ mark} \rightarrow = \frac{\cos^2 \theta}{\cos \theta}$	
$\frac{1}{2} \text{ mark} \rightarrow = \cos \theta$	
LS = RS	

END OF KEY