Principles of Mathematics 12

Sample Exam 2002 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		Sub-Organizers
1. Problem Solving	A	Problem Set
2. Patterns and Relations	B	Geometric Sequences and Series
	C/D	Logarithms and Exponents
	C/D	Trigonometry
3. Shape and Space	E	Conics
	F	Transformations
4. Statistics and Probability	G	Combinatorics
	G	Probability
	G	Statistics

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	A	K	1.5	2	D5	23.	B	K	1.5	3	E1
2.	C	U	1.5	2	C3	24.	A	U	1.5	3	E2
3.	A	U	1.5	2	D6	25.	C	H	1.5	3	E2
4.	A	U	1.5	2	D6, F4	26.	B	H	1.5	3	E2
5.	B	U	1.5	2	C5	27.	D	K	1.5	3	F3
6.	A	U	1.5	2	C5	28.	A	U	1.5	3	F4
7.	A	H	1.5	2	C8	29.	C	U	1.5	3	F6
8.	B	H	1.5	2	D6	30.	C	U	1.5	3	F5
9.	A	H	1.5	2	C6	31.	B	U	1.5	3	F6
10.	B	H	1.5	2	D7	32.	C	K	1.5	4	G7
11.	A	U	1.5	2	B1	33.	B	U	1.5	4	G8
12.	C	U	1.5	2	B1	34.	C	U	1.5	4	G4
13.	C	U	1.5	2	B1	35.	C	U	1.5	4	G12
14.	D	U	1.5	2	B1	36.	D	U	1.5	4	G13
15.	D	H	1.5	2	B1, B2	37.	C	U	1.5	4	G3
16.	B	K	1.5	2	D2	38.	B	K	1.5	4	G2
17.	A	U	1.5	2	D3	39.	C	U	1.5	4	G1
18.	D	U	1.5	2	D3	40.	C	U	1.5	4	G2
19.	B	U	1.5	2	C2	41.	D	U	1.5	4	G3
20.	D	U	1.5	2	C1	42.	A	U	1.5	1	A4
21.	C	H	1.5	2	D3	43.	D	H	1.5	1	A1
22.	C	H	1.5	2	C2	44.	A	H	1.5	1	A2

Multiple Choice = 66 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1a.	1	U	1	3	F1, F2, F3
1b.	2	U	1	3	F1, F2, F3
1c.	3	U	1	3	F1, F2, F3
1d.	4	U	2	3	F1, F2, F3
2.	5	U	5	3	E3
3.	6	U	4	4	G6
4.	7	U	5	2	C8
5.	8	H	5	2	D1
6.	9	U	5	4	G12
7a.	10	U	2	1	A1
7b.	11	H	3	1	A1

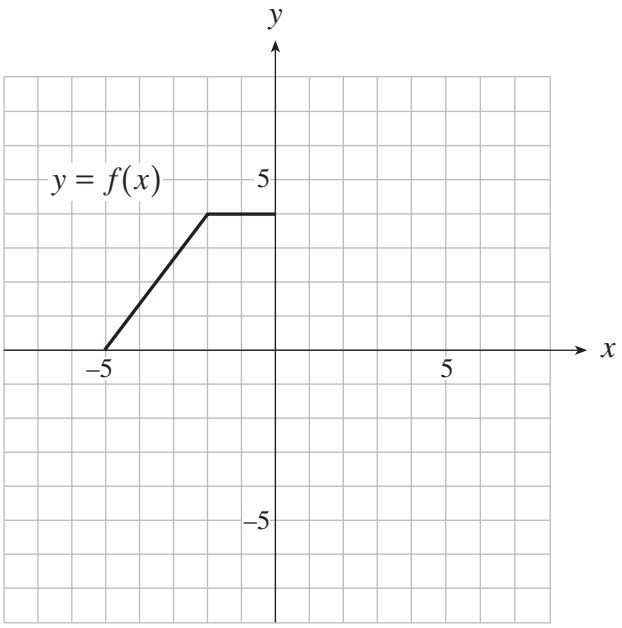
Written Response = 34 marks

Multiple Choice = 66 (44 questions)
Written Response = 34 (7 questions)
EXAMINATION TOTAL = 100 marks

LEGEND:

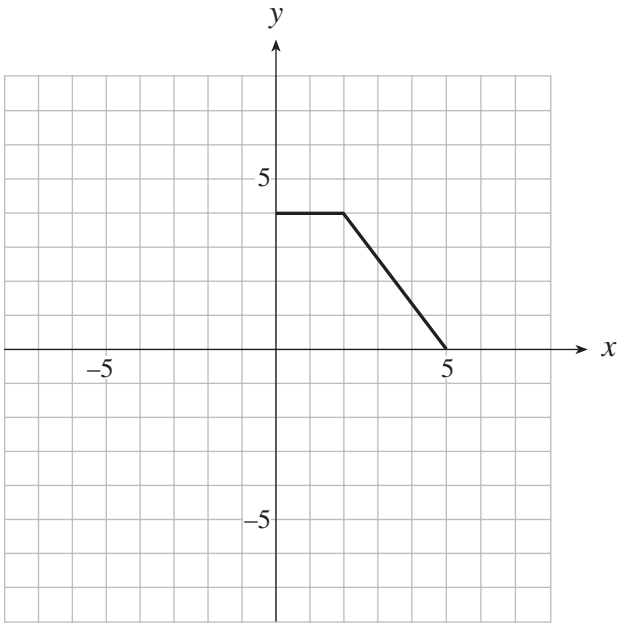
Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
CO = Curriculum Organizer
PLO = Prescribed Learning Outcome

1. Given the graph of the function $y = f(x)$ below, sketch the graph of each relation on the grids provided.



a) $y = f(-x)$ **(1 mark)**

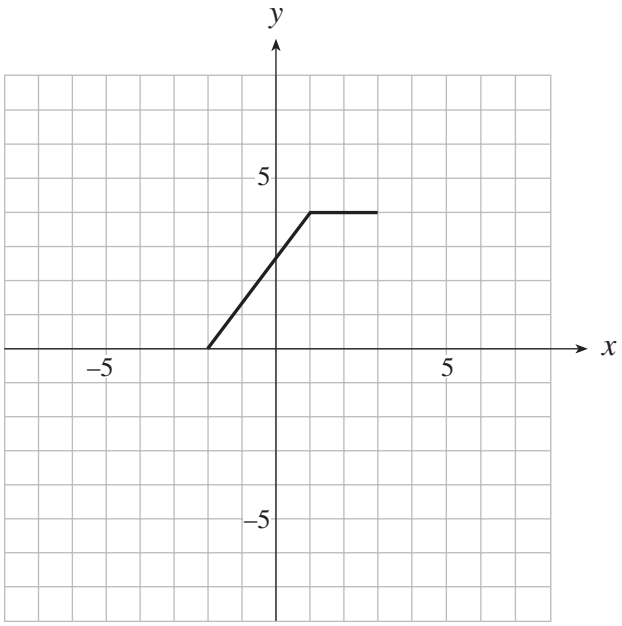
solution



← 1 mark

b) $y = f(x-3)$ **(1 mark)**

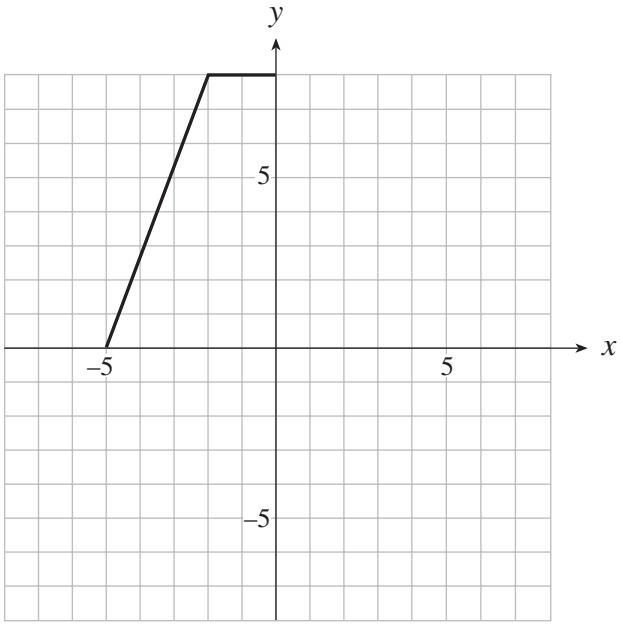
solution



← **1 mark**

c) $y = 2f(x)$ **(1 mark)**

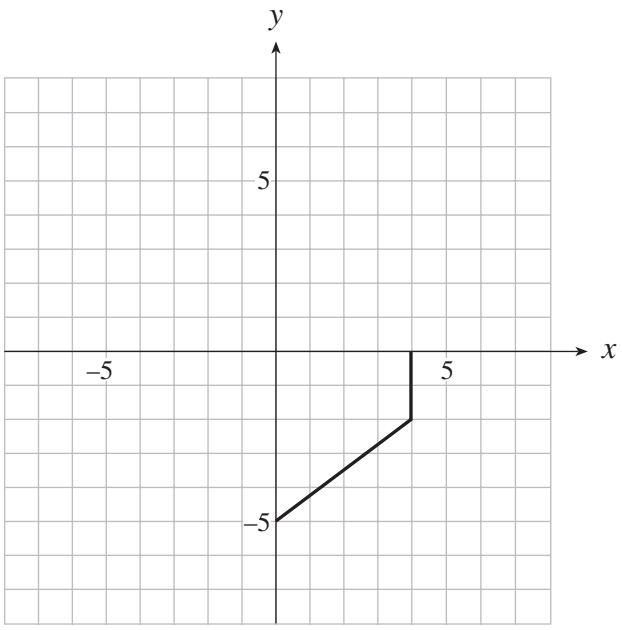
solution



← **1 mark**

d) $x = f(y)$ **(2 marks)**

solution



← **2 marks**

2. Change the following equation to standard form. **(5 marks)**

$$x + 2y^2 + 12y + 16 = 0$$

solution

$$x + 2y^2 + 12y + 16 = 0$$

$$x + 2\left(y^2 + 6y\right) = -16 \qquad \leftarrow \textbf{1 mark} \text{ for factoring 2}$$

$$x + 2\left(y^2 + 6y + 9\right) = -16 + 18 \qquad \leftarrow \textbf{1 mark} \text{ for completing square and } \textbf{1 mark} \text{ for balancing equation}$$

$$x + 2(y+3)^2 = 2 \qquad \leftarrow \textbf{1 mark} \text{ for factoring}$$

$$\left.\begin{array}{c} x = -2(y+3)^2 + 2 \\ \text{or} \\ x - 2 = -2(y+3)^2 \end{array}\right\} \leftarrow \textbf{1 mark} \text{ for standard form}$$

3. Solve algebraically: $\dfrac{n!}{(n-2)!4!} = 10$ **(4 marks)**

solution

$$\frac{n!}{(n-2)!\ 4!} = 10$$

$$\frac{n(n-1)(n-2)!}{24(n-2)!} = 10 \qquad \leftarrow \textbf{1 mark}$$

$$\frac{n(n-1)}{24} = 10 \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$n^2 - n = 240$$

$$n^2 - n - 240 = 0 \qquad \leftarrow \textbf{1 mark}$$

$$(n-16)(n+15) = 0$$

$$n = 16 \qquad n = -15 \qquad \leftarrow \textbf{1 mark}$$

$$\downarrow$$

reject

$$\therefore\ n = 16 \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

4. Prove the identity: **(5 marks)**

$$\frac{\cot\theta}{\sin\theta - \csc\theta} = -\sec\theta$$

solution

	LEFT SIDE	RIGHT SIDE
	$\dfrac{\cot\theta}{\sin\theta - \csc\theta}$	$-\sec\theta$
$\frac{1}{2}$ **mark** →	$\dfrac{\frac{\cos\theta}{\sin\theta}}{}$	
$\frac{1}{2}$ **mark** →	$\dfrac{}{\sin\theta - \frac{1}{\sin\theta}}$	
1 mark →	$\dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{\sin^2\theta - 1}{\sin\theta}}$	
1 mark →	$\dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{-\cos^2\theta}{\sin\theta}}$	
$\frac{1}{2}$ **mark** →	$\dfrac{\cos\theta}{\sin\theta} \cdot \dfrac{\sin\theta}{-\cos^2\theta}$	
1 mark →	$-\dfrac{1}{\cos\theta}$	
$\frac{1}{2}$ **mark** →	$-\sec\theta$	

LS = RS

Note: A graphical verification is not considered a proof and would receive **0 marks**.

5. If 3 150 mg of a radioactive substance decays to 450 mg in 73 weeks, determine the half-life of the substance to the nearest week. (Solve algebraically using logarithms.) **(5 marks)**

solution

$$N = C\left(\frac{1}{2}\right)^{\frac{t}{n}} \qquad \leftarrow \tfrac{1}{2}\textbf{ mark for form}$$

$\tfrac{1}{2}$ **mark** ↓

$$450 = 3\,150\left(\frac{1}{2}\right)^{\frac{73}{n}} \qquad \leftarrow \textbf{1 mark for substitution}$$

$$\frac{450}{3\,150} = \left(\frac{1}{2}\right)^{\frac{73}{n}} \qquad \leftarrow \textbf{1 mark}$$

$$\frac{1}{7} = \left(\frac{1}{2}\right)^{\frac{73}{n}}$$

$$\log\frac{1}{7} = \frac{73}{n}\log\frac{1}{2} \qquad \leftarrow \textbf{1 mark}$$

$$n = \frac{73\log\frac{1}{2}}{\log\frac{1}{7}}$$

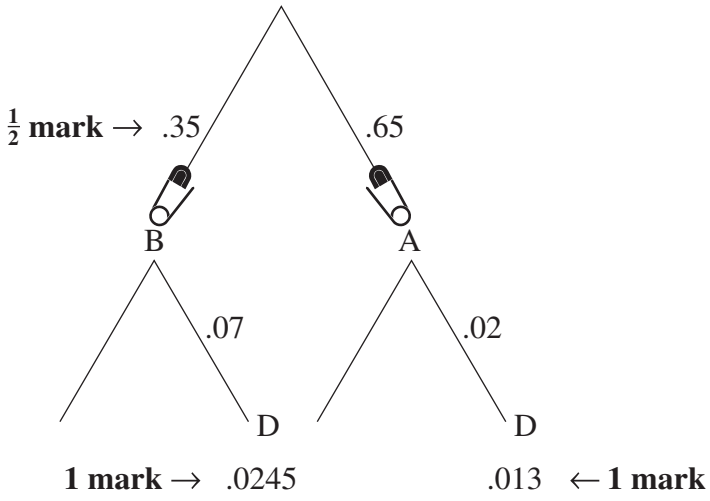
$$n = 26 \text{ weeks} \qquad \leftarrow \textbf{1 mark}$$

6. Two factories produce safety pins. 65% of the safety pins come from factory A and the rest of the safety pins come from factory B. In factory A, 2% of the pins are defective; in factory B, 7% of the pins are defective. What is the probability that a defective pin comes from factory A?

(5 marks)

solution

Let A = pins from factory A
Let B = pins from factory B
Let D = pins that are defective



$$P(\text{defective from } A) = \frac{.013}{.013 + .0245} \quad \leftarrow \tfrac{1}{2}\textbf{ mark} \quad \leftarrow \textbf{1 mark}$$

$= 0.35$
or $\frac{26}{75}$
or 35%

$\leftarrow$ **1 mark**

6. Two factories produce safety pins. 65% of the safety pins come from factory A and the rest of the safety pins come from factory B. In factory A, 2% of the pins are defective; in factory B, 7% of the pins are defective. What is the probability that a defective pin comes from factory A?

(5 marks)

alternate solution

$$P(A|D) = \frac{P(A) \times P(D|A)}{P(D)}$$

$$= \frac{P(A) \times P(D|A)}{P(D \text{ and } A) + P(D \text{ and } B)}$$

$$= \frac{P(A) \times P(D|A)}{P(A)P(D|A) + P(B)P(D|B)} \quad \leftarrow \textbf{1 mark}$$

$$= \frac{(.65)(.02)}{(.65)(.02) + (.35)(.07)} \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$\uparrow$ **1 mark** $\quad \uparrow$ $\frac{1}{2}$ **mark**

$= 0.35$
or $\frac{26}{75}$
or 35%
$\left.\right\} \leftarrow \textbf{1 mark}$

7. An organization surveyed 2 100 randomly chosen students in the province and found that 70% of them enjoyed doing mathematics.

   a) Determine the standard error for the sample proportion. **(2 marks)**

solution

$n = 2\,100$ ← $\frac{1}{2}$ **mark**

$\hat{p} = 0.70$ ← $\frac{1}{2}$ **mark**

$\hat{q} = 0.30$

$$\text{SE} = \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$\text{SE} = \sqrt{\frac{(.70)(.30)}{2\,100}} \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$= 0.01 \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

b) Use the results from this sample to find a 95% confidence interval for the actual proportion of students in the province who enjoy doing mathematics. Clearly show the substitution into the confidence interval formula. **(3 marks)**

solution

$$\hat{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

1 mark for $z_{\frac{\alpha}{2}} = 1.96$

½ mark $\rightarrow \left\{ 0.70 - 1.96\sqrt{\frac{(0.7)(0.3)}{2\,100}} < p < 0.70 + 1.96\sqrt{\frac{(0.7)(0.3)}{2\,100}} \right\} \leftarrow$ **½ mark**

½ mark $\rightarrow \left\{ \begin{array}{c} 0.68 < p < 0.72 \\ \text{or} \\ 68\% < p < 72\% \end{array} \right\} \leftarrow$ **½ mark**

alternate solution

1 mark for $z_{\frac{\alpha}{2}} = 1.96$

½ mark $\rightarrow \{ 0.70 - 1.96\,(\text{SE}) < p < 0.70 + 1.96\,(\text{SE}) \} \leftarrow$ **½ mark**

$$0.70 - 1.96\,(0.01) < p < 0.70 + 1.96(0.01)$$

½ mark $\rightarrow \left\{ \begin{array}{c} 0.68 < p < 0.72 \\ \text{or} \\ 68\% < p < 72\% \end{array} \right\} \leftarrow$ **½ mark**

Note: If students write invNorm (0.975) for $z_{\frac{\alpha}{2}}$ in the formula, this is correct.
If students use $z_{\frac{\alpha}{2}} = 1.959963986$, this is also correct.

Note: Since the accuracy requirements are that answers be correct to at least two decimal places, the answer above has been rounded to two decimal places. Greater decimal place accuracy, such as $0.6804 < p < 0.7196$, is also acceptable.

END OF KEY