Principles of Mathematics 12

August 2002 Provincial Examination

ANSWER KEY / SCORING GUIDE

CURRICULUM:

Organizers		Sub-Organizers
1. Problem Solving	A	Problem Set
2. Patterns and Relations	B	Geometric Sequences and Series
	C/D	Logarithms and Exponents
	C/D	Trigonometry
3. Shape and Space	E	Conics
	F	Transformations
4. Statistics and Probability	G	Combinatorics
	G	Probability
	G	Statistics

Part A: Multiple Choice

Q	K	C	S	CO	PLO	Q	K	C	S	CO	PLO
1.	D	K	1.5	2	D6	23.	C	K	1.5	3	E1
2.	C	U	1.5	2	C3	24.	D	U	1.5	3	E2
3.	A	K	1.5	2	D6	25.	B	U	1.5	3	E3
4.	D	U	1.5	2	C4	26.	A	H	1.5	3	E2
5.	A	U	1.5	2	C8	27.	A	K	1.5	3	F1
6.	D	U	1.5	2	C5	28.	B	U	1.5	3	F3
7.	D	U	1.5	2	C5	29.	A	U	1.5	3	F6
8.	D	U	1.5	2	D6	30.	D	H	1.5	3	F5, F6
9.	C	H	1.5	2	D7	31.	A	H	1.5	3	F6
10.	A	H	1.5	2	C5	32.	C	U	1.5	4	G5
11.	A	U	1.5	2	B1	33.	C	U	1.5	4	G6
12.	C	U	1.5	2	B1	34.	B	U	1.5	4	G8
13.	C	U	1.5	2	B1	35.	B	U	1.5	4	G4
14.	C	H	1.5	2	B1	36.	D	H	1.5	4	G7
15.	D	H	1.5	2	B3	37.	D	U	1.5	4	G11
16.	B	K	1.5	2	D2	38.	B	U	1.5	4	G11
17.	A	U	1.5	2	D3	39.	C	H	1.5	4	G12
18.	D	U	1.5	2	C1	40.	D	U	1.5	4	G2
19.	D	U	1.5	2	C2	41.	B	U	1.5	4	G1
20.	B	U	1.5	2	D4	42.	C	K	1.5	1	A1, A7
21.	B	U	1.5	2	C2	43.	B	U	1.5	1	A1, A4, A9
22.	A	H	1.5	2	D4	44.	B	H	1.5	1	A1, A4, A5

Multiple Choice = 66 marks

Part B: Written Response

Q	B	C	S	CO	PLO
1.	1	U	4	3	E2
2.	2	H	5	2	C8
3.	3	U	5	3	F4
4.	4	U	5	2	D3
5a.	5	U	2	1	A1, A2
5b.	6	U	3	1	A1, A7
6.	7	U	5	4	G11
7a.	8	U	1	4	G3
7b.	9	U	1	4	G3
7c.	10	U	3	4	G3

Written Response = 34 marks

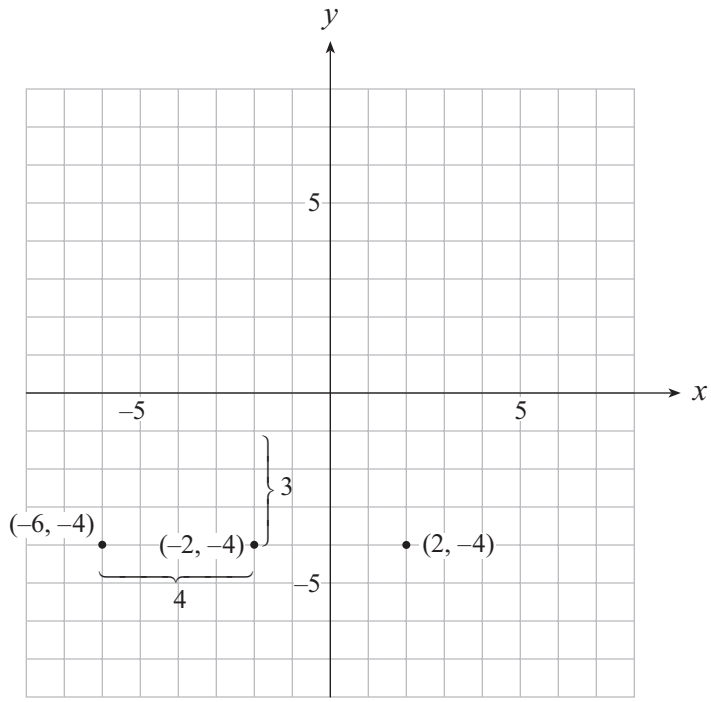
Multiple Choice = 66 (44 questions)
Written Response = 34 (7 questions)
EXAMINATION TOTAL = 100 marks

LEGEND:

Q = Question Number
K = Keyed Response
C = Cognitive Level
B = Score Box Number
S = Score
CO = Curriculum Organizer
PLO = Prescribed Learning Outcome

1. An ellipse has vertices at $(-6, -4)$ and $(2, -4)$. If the length of the minor axis is 6, determine the equation of the ellipse in standard form. **(4 marks)**

solution



$\frac{1}{2}$ mark **$\frac{1}{2}$ mark**

$$\frac{(x+2)^2}{16} + \frac{(y+4)^2}{9} = 1$$ ← **1 mark** for ellipse equation

1 mark **1 mark**

- **if no equation, then sketch is used to determine 0, $\frac{1}{2}$ or 1 mark**

2. Prove: **(5 marks)**

$$\frac{2\cos x + 2\cos^2 x}{\sin 2x} = \frac{\sin x}{1-\cos x}$$

solution

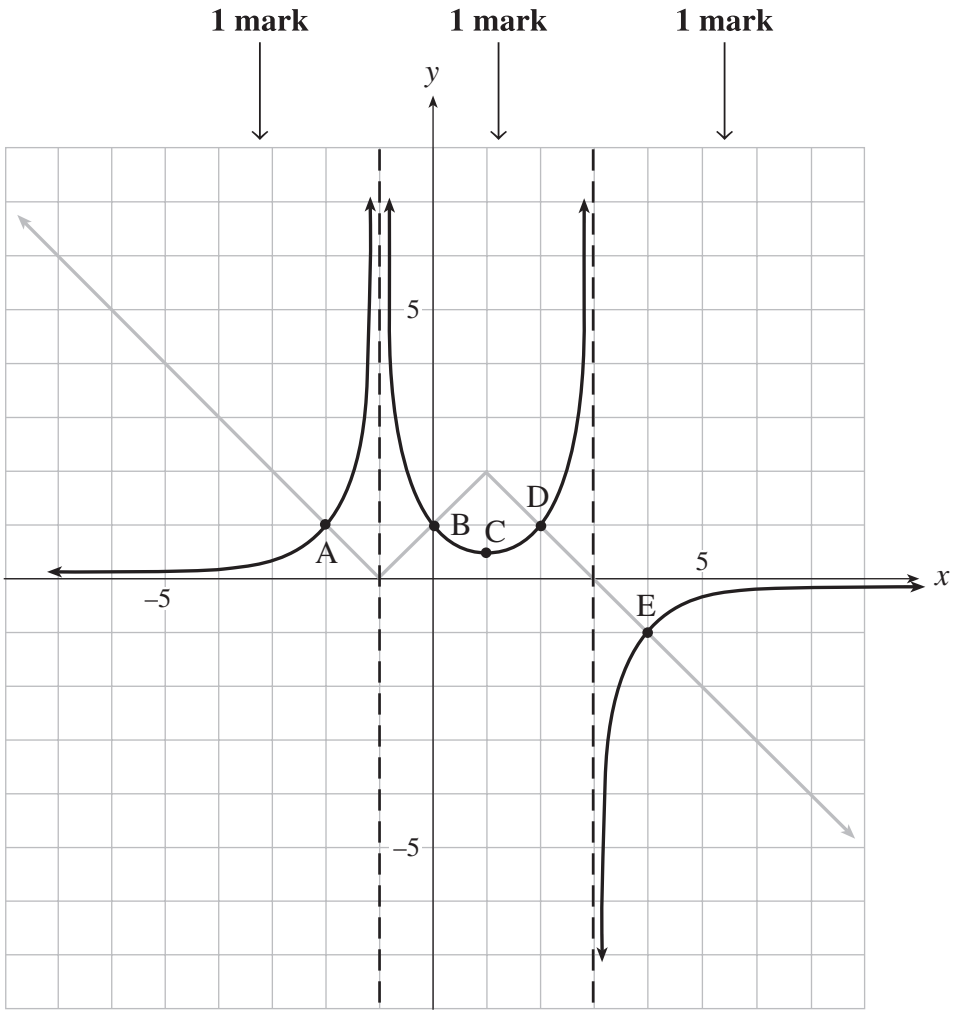
LEFT SIDE		RIGHT SIDE
	$\frac{2\cos x + 2\cos^2 x}{\sin 2x}$	$\frac{\sin x}{1-\cos x}$
1 mark → **½ mark** →	$\frac{2\cos x(1+\cos x)}{2\sin x\cos x}$	
½ mark →	$\frac{1+\cos x}{\sin x}$	
1 mark →	$\frac{(1+\cos x)(1-\cos x)}{\sin x(1-\cos x)}$	
½ mark →	$\frac{1-\cos^2 x}{\sin x(1-\cos x)}$	
1 mark →	$\frac{\sin^2 x}{\sin x(1-\cos x)}$	
½ mark →	$\frac{\sin x}{1-\cos x}$	

alternate solution

LEFT SIDE		RIGHT SIDE
	$\frac{2\cos x + 2\cos^2 x}{\sin 2x}$	$\frac{\sin x}{1-\cos x}$
1 mark → **½ mark** →	$\frac{2\cos x(1+\cos x)}{2\sin x\cos x}$	$\frac{\sin x}{1-\cos x}\ \frac{(1+\cos x)}{(1+\cos x)}$ ← **1 mark**
½ mark →	$\frac{1+\cos x}{\sin x}$	$\frac{\sin x(1+\cos x)}{1-\cos^2 x}$ ← **½ mark**
		$\frac{\sin x(1+\cos x)}{\sin^2 x}$ ← **1 mark**
		$\frac{1+\cos x}{\sin x}$ ← **½ mark**

3. The graph of $y = f(x)$ is shown below. Sketch the graph of $y = \dfrac{1}{f(x)}$ directly on the same grid. Accurate location of key points is necessary for full marks. **(5 marks)**

solution



- $\frac{1}{2}$ **mark** for each point A, B, C, D, E **(2.5 marks total)**
- $\frac{1}{2}$ **mark** for each asymptote **(1 mark total)**
- $\frac{1}{2}$ **mark** for shape in each region **($1\frac{1}{2}$ marks total)**

4. A radioactive substance is produced from nuclear fallout. If 250 g of this substance decays to 150 g in 30 years, what is the half-life of this substance? (Solve algebraically using logarithms.) (Answer accurate to at least 2 decimal places.) **(5 marks)**

solution

$$150 = 250\left(\frac{1}{2}\right)^{\frac{30}{n}} \quad \leftarrow \textbf{1 mark}$$

$$0.6 = \left(\frac{1}{2}\right)^{\frac{30}{n}} \quad \leftarrow \textbf{1 mark}$$

$$\log 0.6 = \log\left(\frac{1}{2}\right)^{\frac{30}{n}} \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$\log 0.6 = \frac{30}{n}\log\left(\frac{1}{2}\right) \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$n = \frac{30\log(0.5)}{\log(0.6)} \quad \leftarrow \textbf{1 mark}$$

$$n = 40.71 \quad \leftarrow \textbf{1 mark}$$

4. A radioactive substance is produced from nuclear fallout. If 250 g of this substance decays to 150 g in 30 years, what is the half-life of this substance? (Solve algebraically using logarithms.) (Answer accurate to at least 2 decimal places.) **(5 marks)**

alternate solution

$$150 = 250e^{\frac{30\ln 0.5}{n}} \quad \leftarrow \textbf{1 mark}$$

$$0.6 = e^{\frac{30\ln 0.5}{n}} \quad \leftarrow \textbf{1 mark}$$

$$\ln 0.6 = \frac{30\ln 0.5}{n} \quad \leftarrow \textbf{1 mark}$$

$$n = \frac{30\ln 0.5}{\ln 0.6} \quad \leftarrow \textbf{1 mark}$$

$$n = 40.71 \quad \leftarrow \textbf{1 mark}$$

5. A relation has the equation $\log(x-y)+\log(x+y)=\log 25$.

a) Determine an equation of the relation without logarithms. **(2 marks)**

solution

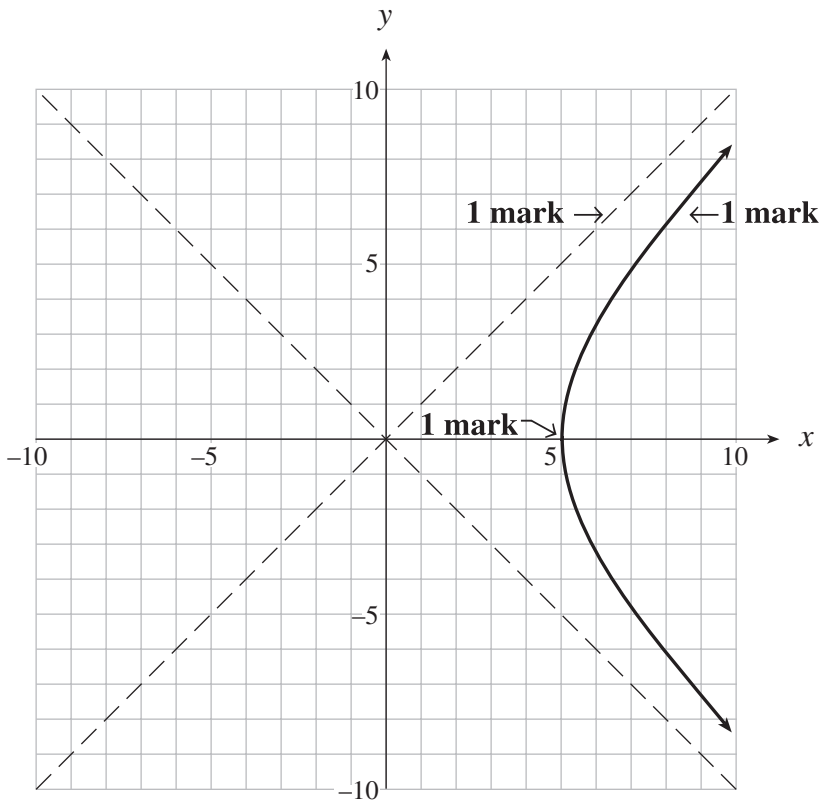
$\log(x-y)(x+y)=\log 25$ ← **1 mark**

$x^2-y^2=25$ ← **1 mark**

Note: $(x-y)(x+y)=25$ will receive full marks

b) Graph the relation $\log(x-y)+\log(x+y)=\log 25$ on the grid provided. Clearly show the asymptotes with broken lines. **(3 marks)**

solution



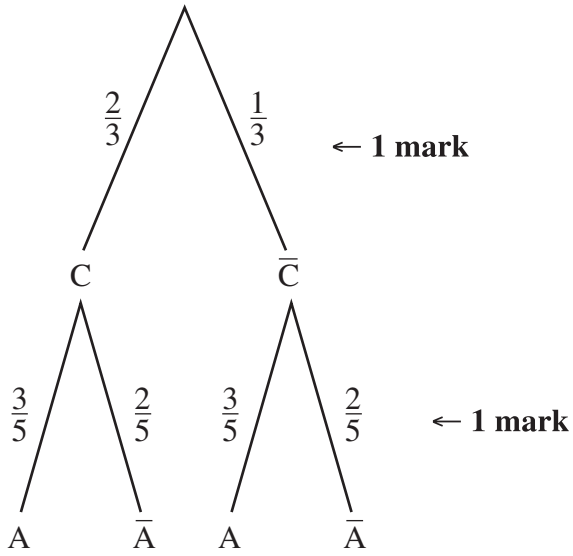
Note: Deduct **1 mark** if both halves are drawn.

6. Two basketball players, Cole and Amanda, each independently shoot a free-throw at a basket. Cole has a $\frac{2}{3}$ chance of making the free-throw and Amanda has a $\frac{3}{5}$ chance of making the free-throw. What is the probability that at least one of them will make the free-throw shot?

(5 marks)

solution

C = Cole makes the free-throw
A = Amanda makes the free-throw



← **1 mark**

← **1 mark**

$$P(\text{neither makes a free-throw}) = P(\overline{C} \text{ and } \overline{A}) = \left(\frac{1}{3}\right)\left(\frac{2}{5}\right) = \frac{2}{15} \quad \leftarrow \textbf{1 mark}$$

$$\therefore\ P(\text{at least one makes a free-throw}) = 1 - \frac{2}{15} \quad \leftarrow \textbf{1 mark}$$

$$= \frac{13}{15} \quad \leftarrow \textbf{1 mark}$$

If no diagram:

← **2 marks**

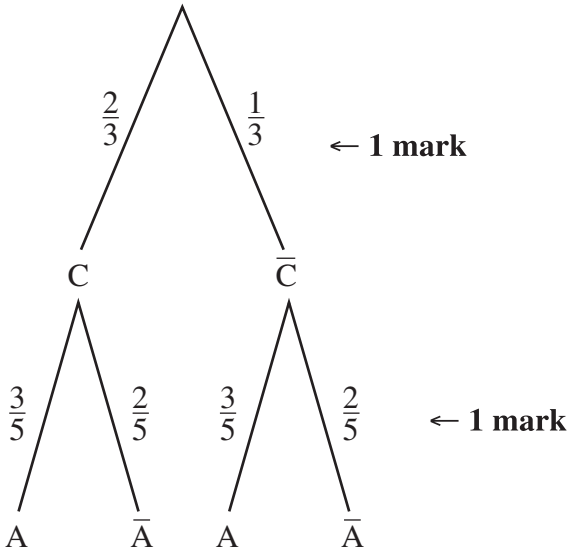
← **2 marks**

← **1 mark**

6. Two basketball players, Cole and Amanda, each independently shoot a free-throw at a basket. Cole has a $\frac{2}{3}$ chance of making the free-throw and Amanda has a $\frac{3}{5}$ chance of making the free-throw. What is the probability that at least one of them will make the free-throw shot?

(5 marks)

alternate solution 1



← **1 mark**

← **1 mark**

$\frac{1}{2}$ **mark** $\frac{1}{2}$ **mark** $\frac{1}{2}$ **mark**

$$P(\text{at least one makes a free-throw}) = \left(\frac{2}{3}\right)\left(\frac{3}{5}\right) + \left(\frac{2}{3}\right)\left(\frac{2}{5}\right) + \left(\frac{1}{3}\right)\left(\frac{3}{5}\right) \quad \leftarrow \tfrac{1}{2} \textbf{ mark for sum}$$

$$= \frac{6}{15} + \frac{4}{15} + \frac{3}{15}$$

$$= \frac{13}{15} \quad \leftarrow \textbf{1 mark}$$

Note: It is not necessary to draw the tree diagram for these solutions—no marks should be deducted.

6. Two basketball players, Cole and Amanda, each independently shoot a free-throw at a basket. Cole has a $\frac{2}{3}$ chance of making the free-throw and Amanda has a $\frac{3}{5}$ chance of making the free-throw. What is the probability that at least one of them will make the free-throw shot?

(5 marks)

alternate solution 2

C = Cole makes the free-throw
A = Amanda makes the free-throw

$P(C) = \frac{2}{3}$

$P(A) = \frac{3}{5}$

$P(C \text{ and } A) = \left(\frac{2}{3}\right)\left(\frac{3}{5}\right) = \frac{6}{15}$ ← **1 mark**

$P(C \text{ or } A) = P(C) + P(A) - P(C \text{ and } A)$ ← **1 mark**

(1) (1)
↓ ↓

$P(C \text{ or } A) = \frac{2}{3} + \frac{3}{5} - \frac{6}{15}$ ← **2 marks**

$= \frac{13}{15}$ ← **1 mark**

7. A fair die is rolled 720 times.

a) What is the mean number of  's (5's) that occur in 720 rolls of a fair die? **(1 mark)**

solution

$n = 720$ $\quad \mu = np$

$p = \frac{1}{6}$ $\quad = 720\left(\frac{1}{6}\right)$

$\uparrow$ $\quad = 120 \quad \leftarrow \frac{1}{2}$ **mark**

$\frac{1}{2}$ **mark**

b) What is the standard deviation of the number of  's that occur in 720 rolls of a fair die? **(1 mark)**

solution

$n = 720$ $\quad \sigma = \sqrt{npq}$

$p = \frac{1}{6}$ $\quad = \sqrt{(720)\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)}$

$q = \frac{5}{6} \quad \leftarrow \frac{1}{2}$ **mark** $\quad = 10 \leftarrow \frac{1}{2}$ **mark**

c) Use the normal approximation to the binomial to estimate the probability of obtaining a  between 115 and 130 times inclusive. **(3 marks)**

solution

$$P(115 \le B \le 130) \approx P(114.5 \le X \le 130.5)$$

1 mark ↓ **1 mark** ↓

$$= \text{normalcdf}\,(114.5,\ 130.5,\ 120,\ 10)$$

$$= 0.56198\ldots$$

$$\approx 0.56 \quad \leftarrow \textbf{1 mark}$$

alternate solution

Table Approach:

$$\left.\begin{aligned} Z_1 &= \frac{114.5-120}{10} = -0.55 \\ Z_2 &= \frac{130.5-120}{10} = 1.05 \end{aligned}\right\} \leftarrow \textbf{1}\tfrac{\textbf{1}}{\textbf{2}}\ \textbf{marks}$$

$$P(114.5 \le X \le 130.5) = P(-0.55 \le Z \le 1.05)$$

$$= 0.8531 - 0.2912 \quad \leftarrow \tfrac{1}{2}\ \textbf{mark}$$

$$= 0.5619$$

$$\approx 0.56 \quad \leftarrow \textbf{1 mark}$$

END OF KEY